CPS 140 **Homework 1** **Spring 2012**

Due: Thursday, Jan. 19
35 points

On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You **must include the names** of any students you consulted with. Give credit where credit is due.

1. (5 pts) Given the sets of integers below, describe the new sets created using similar notation.

   $S_1 = \{0, 4, 8, 12, \ldots\}$

   $S_2 = \{n > 0 \mid \text{n is divisible by 5 }\}$

   $S_3 = \{n > 0 \mid \text{n is even }\}$

   $S_4 = \{3, 5\}$

   $S_5 = \{1, 2, 3, 4, 5, 6\}$

   (a) $S_1 \cap S_2 =$

   (b) $S_3 \cap S_4 =$

   (c) $S_1 \cup S_3 =$

   (d) $S_5 - S_4 =$

   (e) $S_3 \times S_4 =$

2. (3 pts) True or False?

   (a) $\emptyset \subseteq \{x, y, \{x, y\}\}$

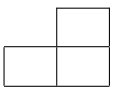
   (b) $\{x, y\} \subseteq \{x, y, \{x, y\}\}$

   (c) $\{x\} \in \{x, y, \{x, y\}\}$

3. (4 pts) Prove by induction $1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2$ for all $n > 0$. Show all steps (basis, IH, and IS).

4. (4 pts) Prove by induction $2^n < n!$, $n \geq 4$

5. (4 pts) Let $n > 0$. Prove any $2^n \times 2^n$ chessboard **with one square removed** can be completely covered by L-shaped tiles, where each tile covers 3 squares. Show all steps (basis, IH and IS). An L-shaped tile looks like:



6. (5 pts) Given the languages below, describe the new languages created using the simplest notation.

   $\Sigma = \{a, b, c\}$

   $L_1 = \{b^n \mid n \geq 1\}$

   $L_2 = \{ba^n \mid n \geq 0\}$

   $L_3 = \{b^n a^n \mid n > 0\}$

   (a) $L_1 \cap \Sigma^* =$

   (b) $L_2 \cap L_3 =$

   (c) $L_1 \circ L_1 =$

   (d) $L_2 \circ L_2 =$

   (e) $L_2 \circ L_2^R =$

7. (10 pts) Consider each of the following languages.

   (a) $L = \{b, ab, bab\}$

      i. write a grammar that generates the language.

   (b) $L = \{a^{2n}b^n \mid n > 0\}$

      i. list 3 strings in the language

      ii. write a grammar that generates the language.

   (c) $L = \{a^m b^n c^m \mid n > 0, m >= 0\}$

      i. list 3 strings in the language

      ii. write a grammar that generates the language.

   (d) $\Sigma = \{a, b\}$, $L = \{w \in \Sigma^* | n_a(w) = 2\}$, ($n_a(w)$ means number of a's in w)

      i. list 3 strings in the language

      ii. write a grammar that generates the language.