CPS 140 - Mathematical Foundations of CS
Dr. Susan Rodger
Section: Introduction (Ch. 1) (handout)

What will we do in CPS 140?

Questions

- Can you write a program to determine if a string is an integer?

  9998.89 8abab 789342

- Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)
- Can you write a program to determine if the following are arithmetic expressions?

$$((34 + 7 * (18/6)))$$

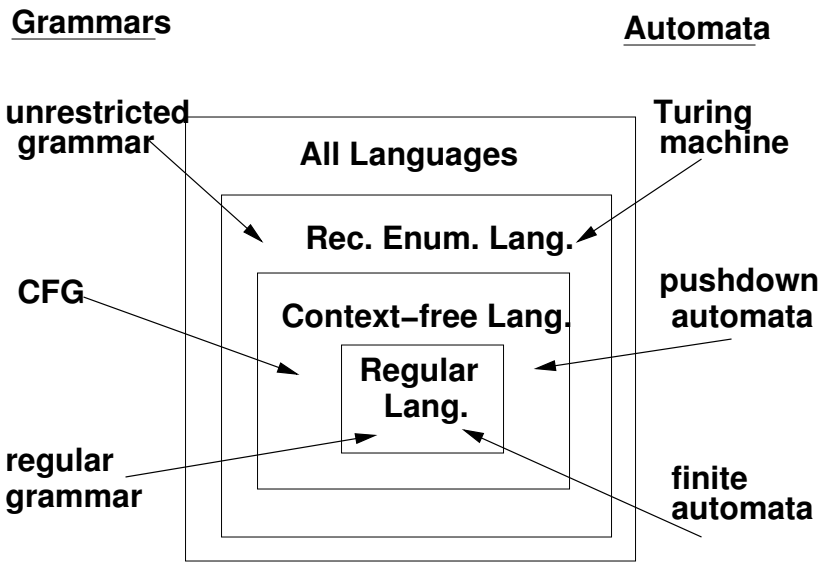
$$(((((((a + b) + c) * d(e + f)))))$$

- Can you do this if your machine had no additional memory other than the program?
- Can you write a program to determine the value of the following expression?

$$((34 + 7 * (18/6)))$$

- Can you write a program to determine if a file is a valid Java program?
- Can you write a program to determine if a Java program given as input will ever halt?

Language Hierarchy



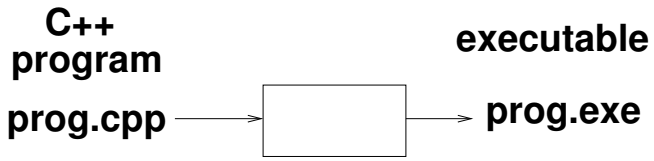
Power of Machines

automata	Can do?	Can't do?
finite automata (FA) (no memory)	integers	arith expr
pushdown automata (PDA) (only memory is stack)	arith expr	compute expr
Turing machines (TM) (infinite memory)	compute expr	decide if halts

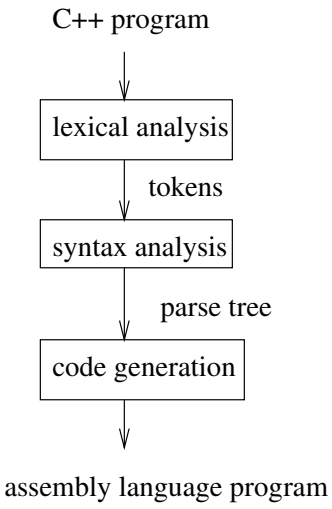
Application

Compiler

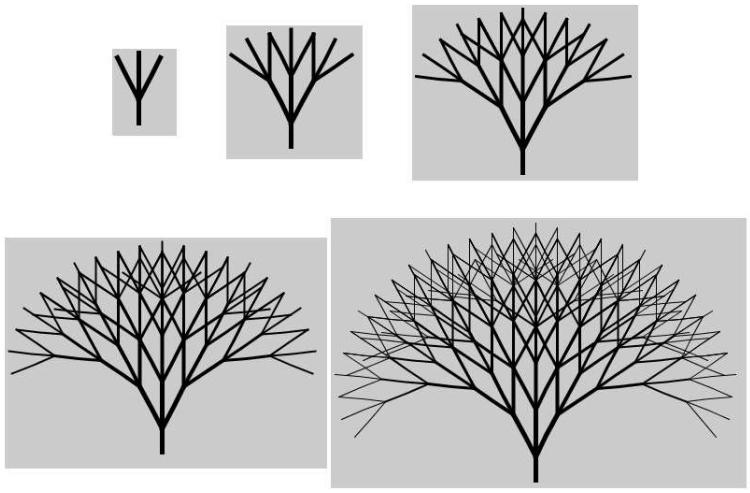
- Question: Given a Java program - is it valid?
- Question: language L, program P - is P valid?



Stages of a Compiler



L-Systems - Model the Growth of Plants



Set Theory - Read Chapter 1 Linz.

A Set is a collection of elements.

A={1,4,6,8}, B={2,4,8}, C={3,6,9,12,...}, D={4,8,12,16,...}

- (union) A∪B=
- (intersection) A∩B=
- C∩D=
- (member of) 42 ∈ C?
- (subset) B⊂C?
- B∩A ⊆D?
- (product) A×B=
- |B|=
- |A×B|=
- ∅ ∈ B∩C?
- (powerset) 2^B=

Example What are all the subsets of $\{3, 5\}$?

How many subsets does a set S have?

$\|S\|$	number of subsets
0	
1	
2	
3	
4	

How do you prove? Set S has $2^{|S|}$ subsets.

Technique: Proof by Induction

1. Basis: P(1)? Prove smallest instance is true.
2. Induction Hypothesis - I.H.
   Assume P(n) is true for 1,2,...,n
3. Induction Step - I.S.
   Show P(n+1) is true (using I.H.)

Proof of Example:

1. Basis:
2. I.H. Assume
3. I.S. Show

Ch. 1: 3 Major Concepts

- languages
- grammars
- automata

Languages

- Σ - set of symbols, alphabet
- string - finite sequence of symbols
- language - set of strings defined over Σ

Examples

- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  L=$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, ...\}$
- Σ=$\{a, b, c\}$
  L=$\{ab, ac, cabb\}$
- Σ=$\{a, b\}$
  L=$\{a^n b^n \mid n > 0\}$

Notation

- symbols in alphabet: a, b, c, d, ...
- string names: u,v,w,...

Definition of concatenation

Let w=$a_1a_2 \dots a_n$ and v=$b_1b_2 \dots b_m$

Then $w \circ v$ OR wv=

See book for formal definitions of other operations.

String Operations

strings: w=abbc, v=ab, u=c

- size of string
  $|w| + |v| =$
- concatenation
  v^3 = vvv = v∘v∘v =
- $v^0 =$
- $w^R =$
- $|vv^Rw|=$
- ab $\circ$ λ=

Definition

Σ^* = set of strings obtained by concatenating 0 or more symbols from Σ

Example

$\Sigma = \{a, b\}$

$\Sigma^* =$

$\Sigma^+ =$

Examples

$\Sigma = \{a, b, c\}$, L_1=$\{ab, bc, aba\}$, $L_2 = \{c, bc, bcc\}$

- $L_1 \cup L_2 =$
- $L_1 \cap L_2 =$
- $\overline{L_1} =$
- $\overline{L_1 \cap L_2} =$
- $L_1 \circ L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\} =$

Definition

$L^0 = \{\lambda\}$

$L^2 = L \circ L$

$L^3 = L \circ L \circ L$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots$

$L^+ = L^1 \cup L^2 \cup L^3 \ldots$

Grammars

grammar for english

<sentence> → <subject><verb><d.o.>

<subject> → <noun> | <article><noun>

<verb> → hit | ran | ate

<d.o.> → <article><noun> | <noun>

<noun> → Fritz | ball

<article> → the | an | a

Examples (derive a sentence)

Fritz hit the ball.

```
<sentence> -> <subject><verb><d.o>
           -> <noun><verb><d.o>
           -> Fritz <verb><d.o.>
           -> Fritz hit <d.o.>
           -> Fritz hit <article><noun>
           -> Fritz hit the <noun>
           -> Fritz hit the ball
```

Can we also derive the sentences?

```
The ball hit Fritz.
```

```
The ball ate the ball
```

Syntactically correct?

Semantically correct?

Grammar

G=(V,T,S,P) where

- V - variables (or nonterminals)
- T - terminals
- S - start variable (S∈V)
- P - productions (rules)
  x→y "means" replace x by y
  x∈(V∪ T)$^+$, y∈(V∪T)*
  where V, T, and P are finite sets.

Definition

w $\Rightarrow$ z	w derives z
w $\overset{*}{\Rightarrow}$ z	derives in 0 or more steps
w $\overset{+}{\Rightarrow}$ z	derives in 1 or more steps

Definition

G=(V,T,S,P)

L(G)={w∈T* | S $\overset{*}{\Rightarrow}$ w}

Example

G=({S}, {a,b}, S, P)

P={S→ aaS, S→ b}

L(G)=

Example

L(G) = $\{a^n ccb^n \mid n > 0\}$

G =

Example

G=({S}, {a,b}, S, P)

P={S→ aSb, S→ SS, S→ ab}

Which of these strings $aabb, abab, abba, babab$ can be generated by this grammar? Show the derivations.

L(G) =

Automata Abstract model of a digital computer