What will we do in CPS 140?

Questions

- **Can you write a program to determine if a string is an integer?**

  9998.89 8abab 789342

- **Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)**

- **Can you write a program to determine if the following are arithmetic expressions?**

$$((34 + 7 * (18/6)))$$

$$(((((((a + b) + c) * d(e + f)))))$$

- **Can you do this if your machine had no additional memory other than the program?**

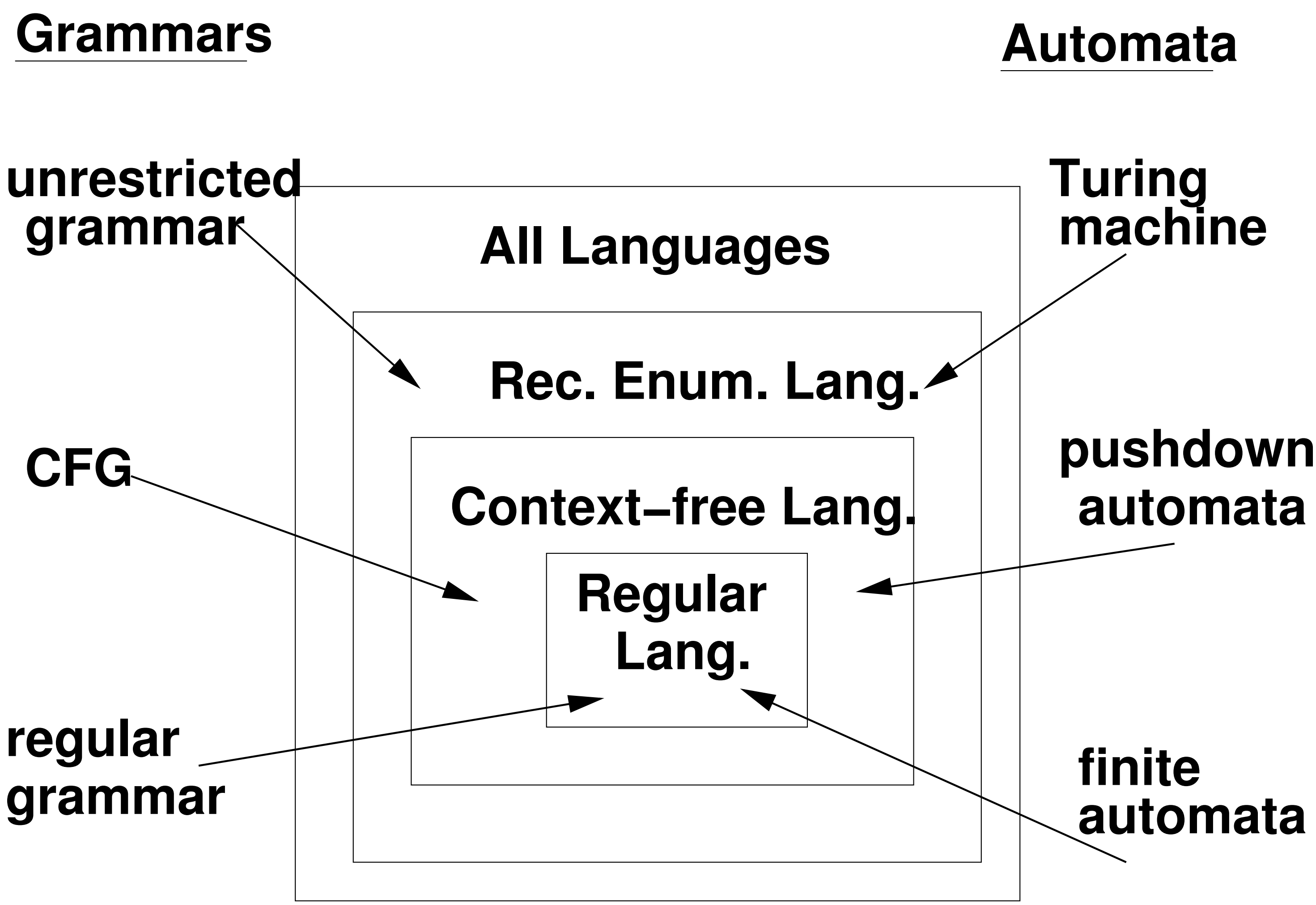
- **Can you write a program to determine the value of the following expression?**

$$((34 + 7 * (18/6)))$$

- **Can you write a program to determine if a file is a valid Java program?**

- **Can you write a program to determine if a Java program given as input will ever halt?**

Language Hierarchy



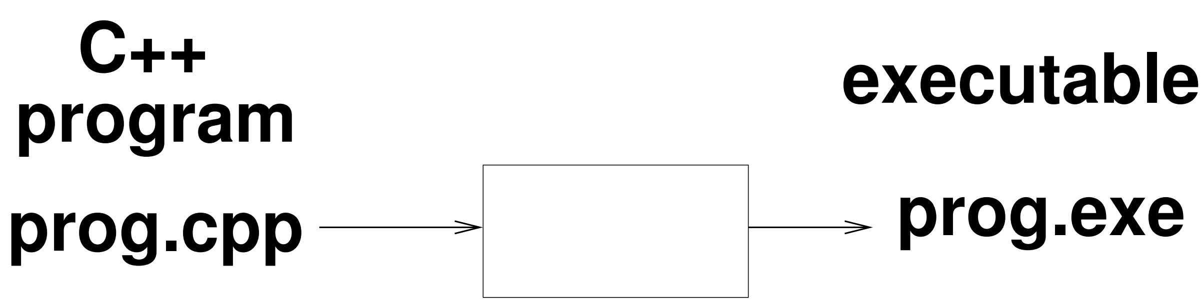
Power of Machines

automata	Can do?	Can't do?
FA (no memory)	integers	arith expr
PDA (stack)	arith expr	compute expr
TM (infinite)	compute expr	decide if halts

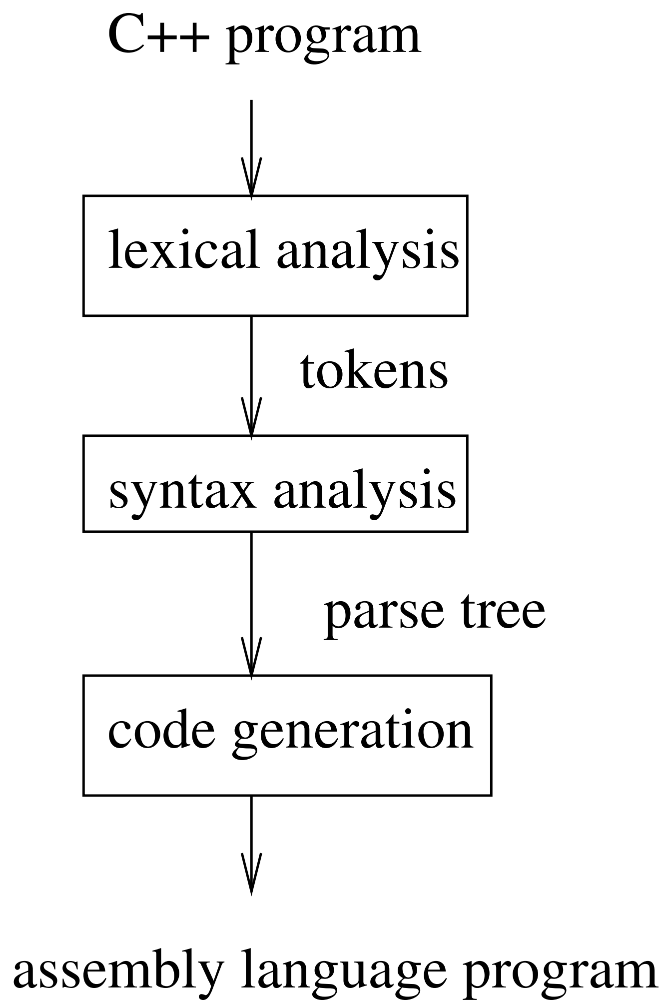
Application

Compiler

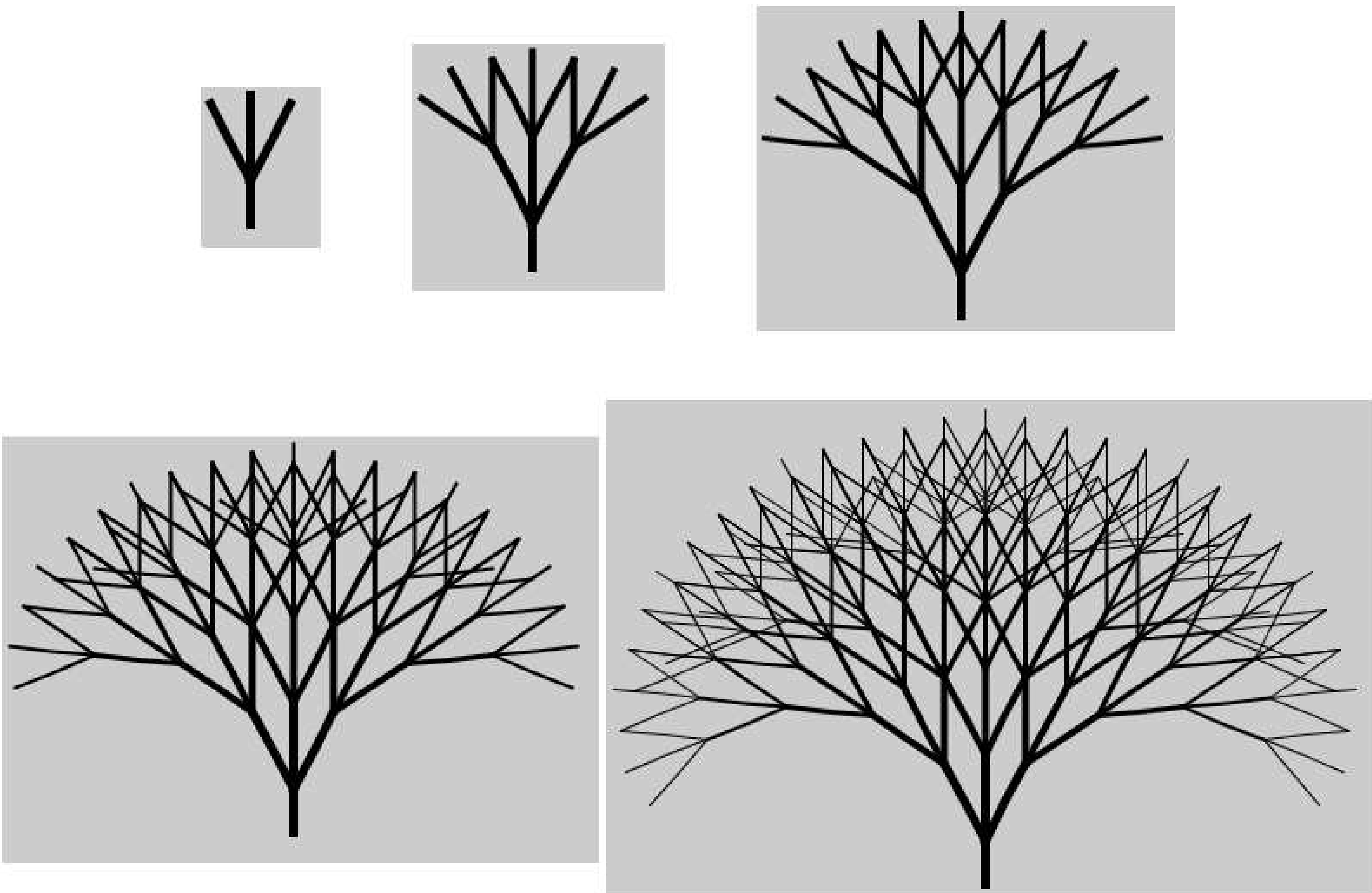
- Question: Given a Java program - is it valid?
- Question: language L, program P - is P valid?



Stages of a Compiler



L-Systems - Model the Growth of Plants



Chapter 1 - Set Theory

A Set is a collection of elements.

A={1,4,6,8}, B={2,4,8},

C={3,6,9,12,...}, D={4,8,12,16,...}

- **(union)** $A \cup B =$
- **(intersection)** $A \cap B =$
- $C \cap D =$
- **(member of)** $42 \in C$?
- **(subset)** $B \subset C$?
- $B \cap A \subseteq D$?
- **(product)** $A \times B =$
- $|B| =$
- $|A \times B| =$
- $\emptyset \in B \cap C$?
- **(powerset)** $2^B =$

Example What are all the subsets of $\{3, 5\}$?

How many subsets does a set S have?

$\|S\|$	number of subsets
0	
1	
2	
3	
4	

How do you prove? Set S has $2^{|S|}$ subsets. Technique: Proof by Induction

1. **Basis: P(1)?**
2. **I.H.**
   Assume P(n) is true for 1,2,...,n
3. **I.S.**
   Show P(n+1) is true (using I.H.)

Proof of Example:

1. **Basis:**
2. **I.H. Assume**
3. **I.S. Show**

Ch. 1: 3 Major Concepts

- **languages**
- **grammars**
- **automata**

Languages

- **Σ - set of symbols, alphabet**
- **string - finite sequence of symbols**
- **language - set of strings defined over Σ**

Examples

- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  L$=\{0, 1, 2, ..., 12, 13, 14, ...\}$
- $\Sigma=\{a, b, c\}$
  L$=\{ab, ac, cabb\}$
- $\Sigma=\{a, b\}$
  L$=\{a^n b^n \mid n > 0\}$

Notation

- **symbols in alphabet: a, b, c, d, ...**
- **string names: u,v,w,...**

Definition of concatenation

Let w=$a_1 a_2 \ldots a_n$ **and v=**$b_1 b_2 \ldots b_m$

Then $w \circ v$ **OR wv=**

String Operations

strings: w=abbc, v=ab, u=c

- **size of string**

  $|w| + |v| =$

- **concatenation**

  $v^3 = \mathbf{vvv} = \mathbf{v}\circ\mathbf{v}\circ\mathbf{v} =$

- $v^0 =$

- $w^R =$

- $|vv^Rw| =$

- $\mathbf{ab} \circ \lambda =$

Definition

Σ^* **concatenate 0 or more**

Example

$\Sigma = \{a, b\}$

$\Sigma^* =$

$\Sigma^+ =$

Examples

$\Sigma = \{a, b, c\}$, $L_1 = \{ab, bc, aba\}$,
$L_2 = \{c, bc, bcc\}$

- $L_1 \cup L_2 =$
- $L_1 \cap L_2 =$
- $\overline{L_1} =$
- $\overline{L_1 \cap L_2} =$
- $L_1 \circ L_2 = \{xy \mid x \in L_1 \textbf{ and } y \in L_2\} =$

Definition

$$L^0 = \{\lambda\}$$

$$L^2 = L \circ L$$

$$L^3 = L \circ L \circ L$$

$$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \dots$$

$$L^+ = L^1 \cup L^2 \cup L^3 \dots$$

Grammars

grammar for english

<sentence> → <subject><verb><d.o.>

<subject> → <noun> | <article><noun>

<verb> → hit | ran | ate

<d.o.> → <article><noun> | <noun>

<noun> → Fritz | ball

<article> → the | an | a

Examples (derive a sentence)

Fritz hit the ball.

<sentence> -> <subject><verb><d.o>
-> <noun><verb><d.o>
-> Fritz <verb><d.o.>
-> Fritz hit <d.o.>
-> Fritz hit <article><noun>
-> Fritz hit the <noun>
-> Fritz hit the ball

Can we also derive the sentences?

`The ball hit Fritz.`

`The ball ate the ball`

Syntactically correct?

Semantically correct?

Grammar

G=(V,T,S,P) where

- **V - variables (or nonterminals)**
- **T - terminals**
- **S - start variable (S∈V)**
- **P - productions (rules)**
  $x \rightarrow y \; x \in (V \cup T)^+, \; y \in (V \cup T)^*$

Definition

$w \Rightarrow z$ **w derives z**

$w \overset{*}{\Rightarrow} z$ **derives in 0 or more steps**

$w \overset{+}{\Rightarrow} z$ **derives in 1 or more steps**

Definition

G=(V,T,S,P)

$L(G) = \{ w \in T^* \mid S \overset{*}{\Rightarrow} w \}$

Example

G=({S}, {a,b}, S, P)

P={S→ aaS, S→ b}

L(G)=

Example

L(G) = $\{a^n ccb^n \mid n > 0\}$

G =

Example

G=({S}, {a,b}, S, P)

P={S→ aSb, S→ SS, S→ ab}

Which of these strings $aabb, abab, abba, babab$ **can be generated by this grammar? Show the derivations.**

L(G) =

Automata

Abstract model of a digital computer