Section: LR Parsing

LR PARSING

LR(k) Parser

- bottom-up parser
- shift-reduce parser
- L means: reads input left to right
- R means: produces a rightmost derivation
- k - number of lookahead symbols

LR parsing process

- convert CFG to PDA
- Use the PDA and lookahead symbols

Convert CFG to PDA

The constructed NPDA:

- **three states: s, q, f**

  start in state s, assume z on stack

- **all rewrite rules in state s, backwards**

  rules pop rhs, then push lhs

  (s,lhs) $\in \delta$(s,λ,rhs)

  This is called a reduce operation.

- **additional rules in s to recognize terminals**

  For each x$\in \Sigma$, g$\in \Gamma$, (s,xg) $\in$ δ(s,x,g)

  This is called a shift operation.

- **pop S from stack and move into state q**

- **pop z from stack, move into f, accept.**

Example: Construct a PDA.

$\mathbf{S \rightarrow aSb}$
$\mathbf{S \rightarrow b}$

LR Parsing Actions

1. shift
   transfer the lookahead to the stack
2. reduce
   For X $\rightarrow$ w, replace w by X on the stack
3. accept
   input string is in language
4. error
   input string is not in language

LR(1) Parse Table

- Columns:
  terminals, $ and variables
- Rows:
  state numbers: represent patterns in a derivation

LR(1) Parse Table Example

1) S → aSb
2) S → b

	a	b	$	S
0	s2	s3		1
1			acc	
2	s2	s3		4
3		r2	r2	
4		s5		
5		r1	r1	

Definition of entries:

- sN - shift terminal and move to state N
- N - move to state N
- rN - reduce by rule number N
- acc - accept
- blank - error

```
state = 0
push(state)
read(symbol)
entry = T[state,symbol]
while entry.action ≠ accept do
    if entry.action == shift then
        push(symbol)
        state = entry.state
        push(state)
        read(symbol)
    else if entry.action == reduce then
        do 2*size_rhs times {pop()}
        state := top-of-stack()
        push(entry.rule.lhs)
        state = T[state,entry.rule.lhs]
        push(state)
    else if entry.action == blank then
        error
    entry = T[state, symbol]
end while
if symbol ≠ $ then error
```

Example:

Trace aabbb

						5			
						b			
				3	**4**	**4**		**5**	
				b	**S**	**S**		**b**	
			2	**2**	**2**	**2**	**4**	**4**	
			a	**a**	**a**	**a**	**S**	**S**	
		2	**2**	**2**	**2**	**2**	**2**	**2**	**1**
		a	**a**	**a**	**a**	**a**	**a**	**a**	**S**
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
S:	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$	$\underline{z}$
L:	**a**	**a**	**b**	**b**	**b**	**b**	**b**	**$**	**$**
A:									

To construct the LR(1) parse table:

- **Construct a dfa to model the top of the stack**
- **Using the dfa, construct an LR(1) parse table**

To Construct the DFA

- **Add S' → S**
- **place a marker "_" on the rhs**
  S' → _S
- **Compute closure(S' → _S).**
  Def. of closure:
  1. **closure(A → v_xy) = {A → v_xy} if x is a terminal.**
  2. **closure(A → v_xy) = {A → v_xy} ∪ (closure(x → _w) for all w if x is a variable.**

- **The closure(S' → _S) is state 0 and "unprocessed".**
- **Repeat until all states have been processed**
  - **unproc = any unprocessed state**
  - **For each x that appears in A→u_xv do**
    - **Add a transition labeled "x" from state "unproc" to a new state with production A→ux_v**
    - **The set of productions for the new state are: closure(A→ux_v)**
    - **If the new state is identical to another state, combine the states Otherwise, mark the new state as "unprocessed"**
- **Identify final states.**

Example: Construct DFA

$$\begin{array}{l}\textbf{(0) S'} \rightarrow \textbf{S} \\ \textbf{(1) S} \rightarrow \textbf{aSb} \\ \textbf{(2) S} \rightarrow \textbf{b}\end{array}$$

Backtracking through the DFA

Consider aabbb

- Start in state 0.
- Shift "a" and move to state 2.
- Shift "a" and move to state 2.
- Shift "b" and move to state 3.
  Reduce by "S $\rightarrow$ b"
  Pop "b" and Backtrack to state 2.
  Shift "S" and move to state 4.
- Shift "b" and move to state 5.
  Reduce by "S $\rightarrow$ aSb"
  Pop "aSb" and Backtrack to state 2.
  Shift "S" and move to state 4.
- Shift "b" and move to state 5.
  Reduce by "S $\rightarrow$ aSb"
  Pop "aSb" and Backtrack to state 0.

Shift "S" and move to state 1.

- **Accept. aabbb is in the language.**

To construct LR(1) table from diagram:

1. **If there is an arc from state1 to state2**
   - (a) **arc labeled x is terminal or $**
     T[state1, x] = sh state2
   - (b) **arc labeled X is nonterminal**
     T[state1, X] = state2
2. **If state1 is a final state with**
   $X \rightarrow w_{\_}$
   For all a in FOLLOW(X),
   $T[state1,a] =$ reduce by $X \rightarrow w$
3. **If state1 is a final state with**
   $S' \rightarrow S_{\_}$
   T[state1,$] = accept
4. **All other entries are error**

Example: LR(1) Parse Table

$$\begin{array}{l}(0)\ S' \rightarrow S \\ (1)\ S \rightarrow aSb \\ (2)\ S \rightarrow b\end{array}$$

Here is the LR(1) Parse Table with extra information about the stack contents of each state.

Stack contents	State number	Terminals			Variables
		a	b	$	S
(empty)	0				
	1				
	2				
	3				
	4				
	5				

Actions for entries in LR(1) Parse table T[state,symbol]

Let entry = T[state,symbol].

- **If symbol is a terminal or $**
  - **If entry is "shift state$i$"**
    push lookahead and state$i$ on the stack
  - **If entry is "reduce by rule X → w"**
    pop w and k states (k is the size of w) from the stack.
  - **If entry is "accept"**
    Halt. The string is in the language.
  - **If entry is "error"**
    Halt. The string is not in the language.

- **If symbol is nonterminal**

  We have just reduced the rhs of a production X $\rightarrow$ w to a symbol. The entry is a state number, call it state$i$. Push T[state$i$, X] on the stack.

Constructing Parse Tables for CFG's with λ-rules

A $\rightarrow \lambda$ written as A $\rightarrow \lambda\_$

Example

$$\begin{aligned} S &\rightarrow ddX \\ X &\rightarrow aX \\ X &\rightarrow \lambda \end{aligned}$$

Add a new start symbol and number the rules:

$$\begin{aligned} &(0)\ S' \rightarrow S \\ &(1)\ S \rightarrow ddX \\ &(2)\ X \rightarrow aX \\ &(3)\ X \rightarrow \lambda \end{aligned}$$

Construct the DFA:

Construct the LR(1) Parse Table

	a	d	$	S	X
0					
1					
2					
3					
4					
5					
6					

Possible Conflicts:

1. **Shift/Reduce Conflict**

   Example:

$$\mathbf{A} \rightarrow \mathbf{ab}$$
$$\mathbf{A} \rightarrow \mathbf{abcd}$$

   In the DFA:

$$\mathbf{A} \rightarrow \mathbf{ab}\_$$
$$\mathbf{A} \rightarrow \mathbf{ab}\_\ \mathbf{cd}$$

2. **Reduce/Reduce Conflict**

   Example:

$$\mathbf{A} \rightarrow \mathbf{ab}$$
$$\mathbf{B} \rightarrow \mathbf{ab}$$

   In the DFA:

$$\mathbf{A} \rightarrow \mathbf{ab}\_$$
$$\mathbf{B} \rightarrow \mathbf{ab}\_$$

3. **Shift/Shift Conflict**