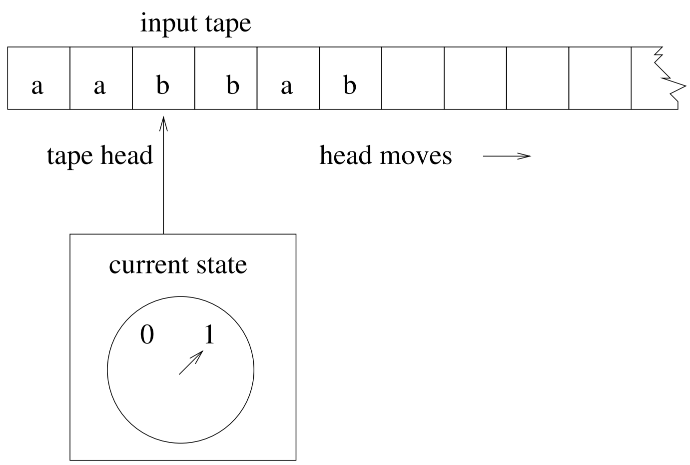
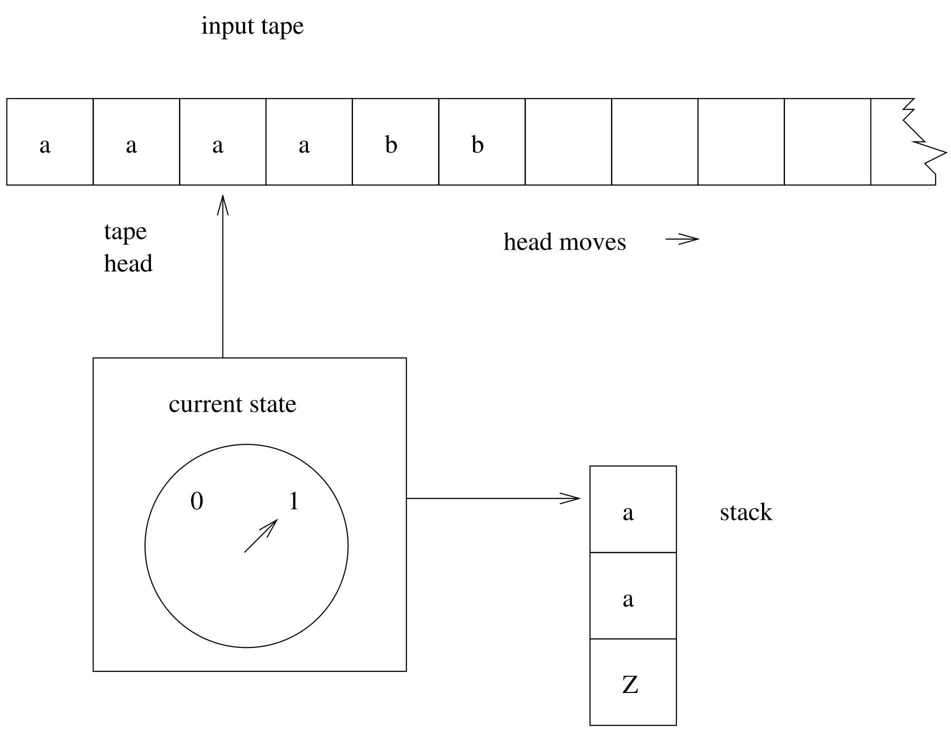
CPS 140 - Mathematical Foundations of CS
Dr. S. Rodger
Section: Pushdown Automata (Ch. 7) (handout)

**Ch. 7 - Pushdown Automata**

A DFA=(Q,$\Sigma$,$\delta$,$q_0$,F)




Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).




**Definition:** Nondeterministic PDA (NPDA) is defined by

$$M=(Q,\Sigma, \Gamma, \delta, q_0, z, F)$$

where

Q is finite set of states
$\Sigma$ is tape (input) alphabet
$\Gamma$ is stack alphabet
$q_0$ is initial state
z - start stack symbol, (bottom of stack marker), $z \in \Gamma$
$F \subseteq Q$ is set of final states.
$\delta$:Q$\times(\Sigma \cup \{\lambda\})\times\Gamma\rightarrow$ finite subsets of $Q \times \Gamma^*$

**Example of transitions**

$\delta(q_1$,a,b) = $\{(q_3$,b),$(q_4$,ab),$(q_6,\lambda)\}$

Meaning: If in state $q_1$ with "a" the current tape symbol and "b" the symbol on top of the stack, then pop "b", and either

move to $q_3$ and push "b" on stack
move to $q_4$ and push "ab" on stack ("a" on top)
move to $q_6$

Transitions can be represented using a transition diagram.

The diagram for the above transitions is:

Each arc is labeled by a triple: x,y;z where x is the current input symbol, y is the top of stack symbol which is popped from the stack, and z is a string that is pushed onto the stack.

**Instantaneous Description:**

$$(q,w,u)$$

Notation to describe the current state of the machine (q), unread portion of the input string (w), and the current contents of the stack (u).

**Description of a Move:**

$$(q_1,\text{aw},\text{bx}) \vdash (q_2,\text{w},\text{yx})$$
iff

**Definition** Let M=(Q,Σ,Γ,δ,$q_0$,z,F) be a NPDA. L(M)={w∈ $\Sigma^*$ | ($q_0$,w,z) $\overset{*}{\vdash}$ (p,λ,u), p∈F, u∈ $\Gamma^*$}. The NPDA accepts all strings that start in $q_0$ and end in a final state.

**Example:** L=$\{a^n b^n | n \geq 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$

**Another Definition for Language Acceptance**

NPDA M accepts L(M) by empty stack:

$$\text{L(M)}=\{w \in \Sigma^* | (q_0, w, z) \overset{*}{\vdash} (p, \lambda, \lambda)\}$$

**Example:** L=$\{a^n b^m c^{n+m} | n, m > 0\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$

**Example:** L=$\{ww^R | w \in \Sigma^+\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a, b\}$

**Example:** L=$\{ww | w \in \Sigma^*\}$, $\Sigma = \{a, b\}$

**Examples for you to try on your own:** (solutions are at the end of the handout).

- L=$\{a^n b^m | m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$
- L=$\{a^n b^{n+m} c^m | n, m > 0\}$, $\Sigma = \{a, b, c\}$,
- L=$\{a^n b^{2n} | n > 0\}$, $\Sigma = \{a, b\}$

**Definition:** A PDA M=(Q,$\Sigma$,$\Gamma$,$\delta$,$q_0$,z,F) is *deterministic* if for every $q \in$Q, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

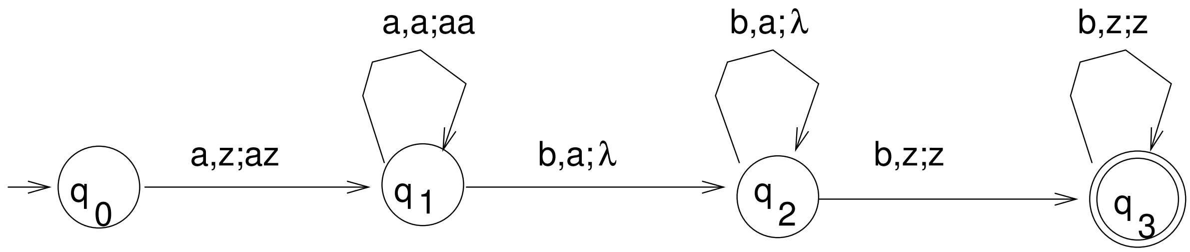
1. $\delta(q, a, b)$ contains at most 1 element
2. if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b)$=$\emptyset$ for all $c \in \Sigma$

**Definition:** L is DCFL iff $\exists$ DPDA M s.t. L=L(M).

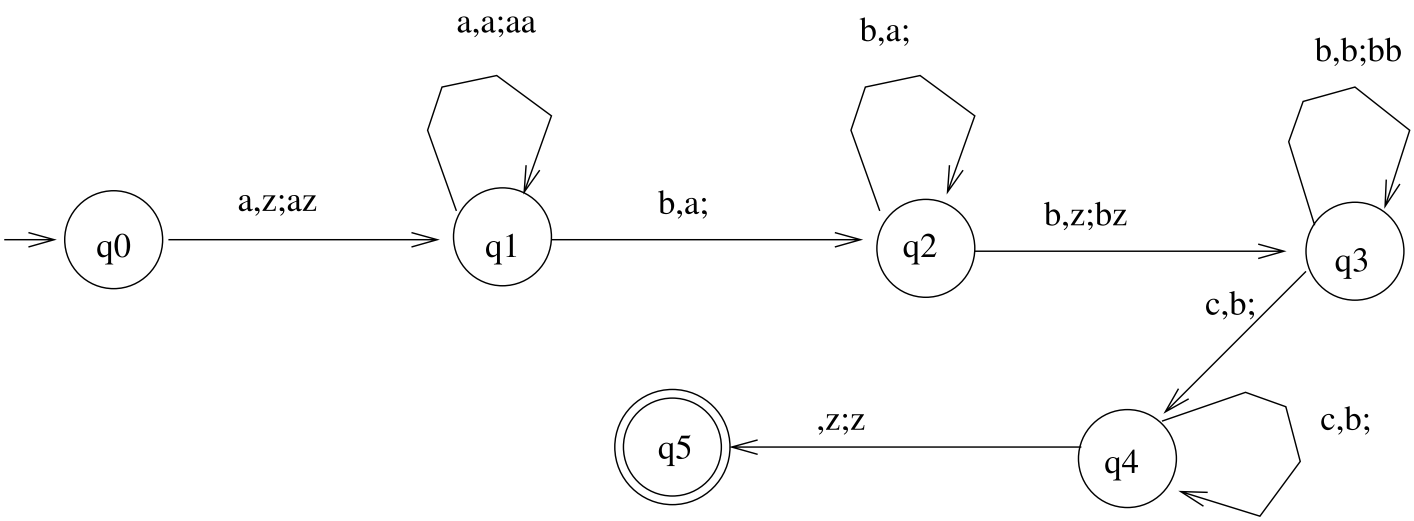
Examples:

1. Previous pda for $\{a^n b^n | n \geq 0\}$ is deterministic?
2. Previous pda for $\{a^n b^m c^{n+m} | n, m > 0\}$ is deterministic?
3. Previous pda for $\{ww^R | w \in \Sigma^+\}$,$\Sigma = \{a, b\}$ is deterministic?

**Example:** L=$\{a^nb^m|m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$




**Example:** L=$\{a^nb^{n+m}c^m|n, m > 0\}$, $\Sigma = \{a, b, c\}$,




**Example:** L=$\{a^nb^{2n}|n > 0\}$, $\Sigma = \{a, b\}$