CPS 140 - Mathematical Foundations of CS
Dr. S. Rodger
Section: Turing Machines (handout)

Review

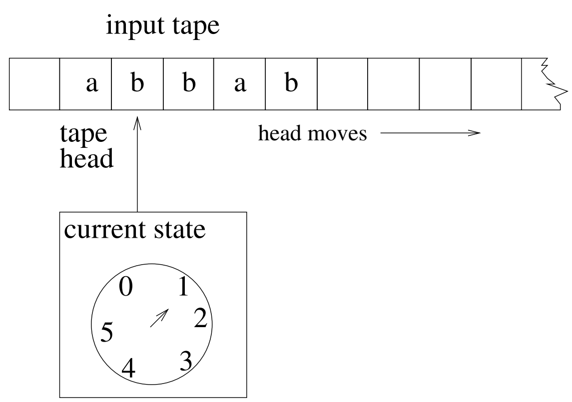
Regular Languages

- FA, RG, RE
- recognize

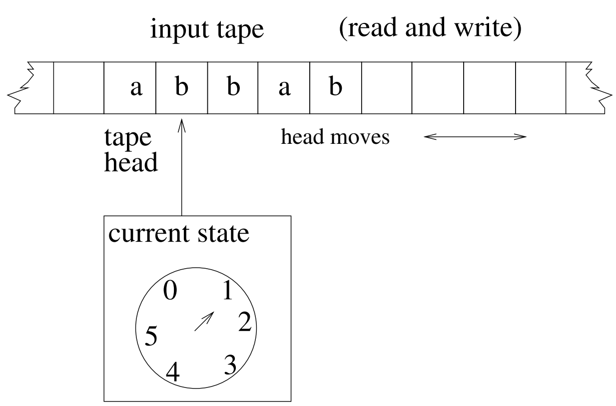
Context Free Languages

- PDA, CFG
- recognize

DFA:



Turing Machine:



Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM M is defined by M=(Q,Σ,Γ,δ,q_0,B,F) where

- Q is finite set of states
- Σ is input alphabet
- Γ is tape alphabet
- B$\in$ Γ is blank
- q_0 is start state
- F is set of final states
- δ is transition function

  δ(q,a) = (p,b,R) means "if in state q with the tape head pointing to an 'a', then move into state p, write a 'b' on the tape and move to the right".

TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.

if δ(q,a) = (p,b,R) then a move is denoted

$$\text{abaqabba} \vdash \text{ababpbba}$$

Definition: Let M be a TM, M=(Q,Σ,Γ,δ,q_0,B,F). L(M) = $\{w \in \Sigma^* | q_0 w \overset{*}{\vdash} x_1 q_f x_2$ for some $q_f \in$F, $x_1, x_2 \in \Gamma^*\}$

TM as language acceptor

M is a TM, w is in Σ^*,

- if w$\in$ L(M) then M halts in final state
- if w$\notin$ L(M) then either
  - M halts in non-final state
  - M doesn't halt

Example

$\Sigma = \{a, b\}$

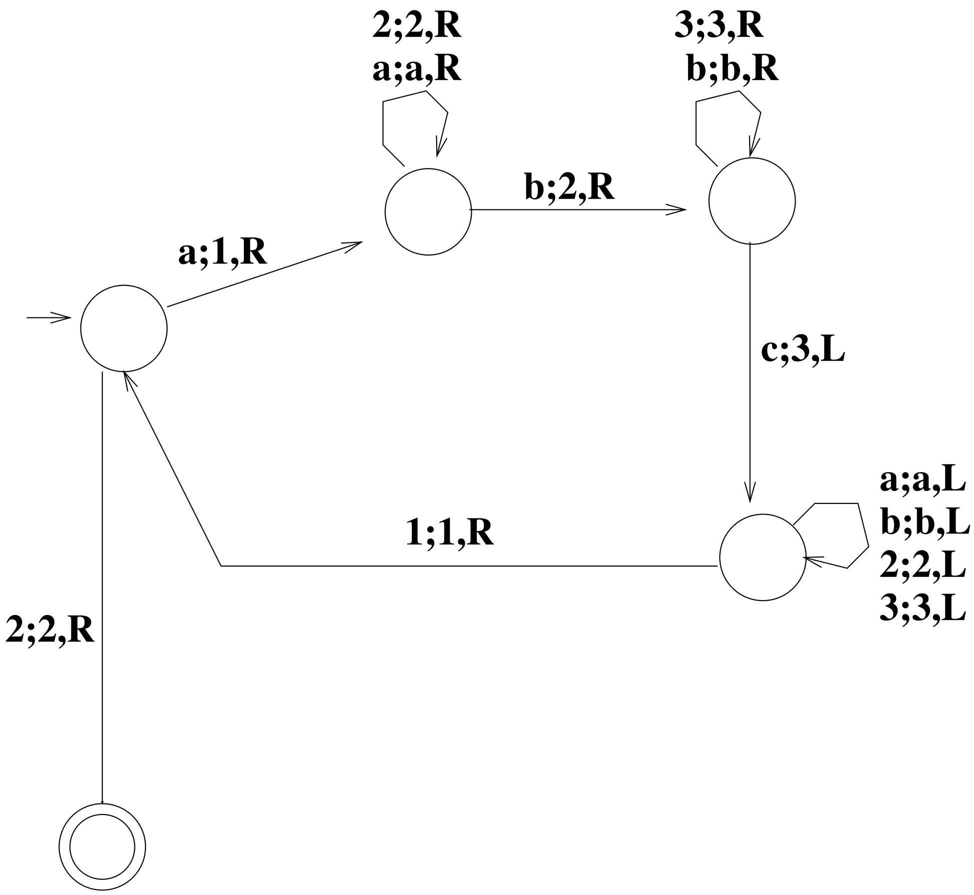
Replace every second 'a' by a 'b' if string is even length.

- Algorithm

Example:

L=$\{a^n b^n c^n | n \geq 1\}$

Is the following TM Correct?



TM as a transducer

TM can implement a function: f(w)=w'

start with:	w ↑
end with:	w' ↑

Definition: A function with domain D is *Turing-computable* or *computable* if there exists TM M=(Q,$\Sigma, \Gamma, \delta, q_0$,B,F) such that

$$q_0 w \overset{*}{\vdash} q_f f(w)$$

q_f ∈F, for all w ∈D.

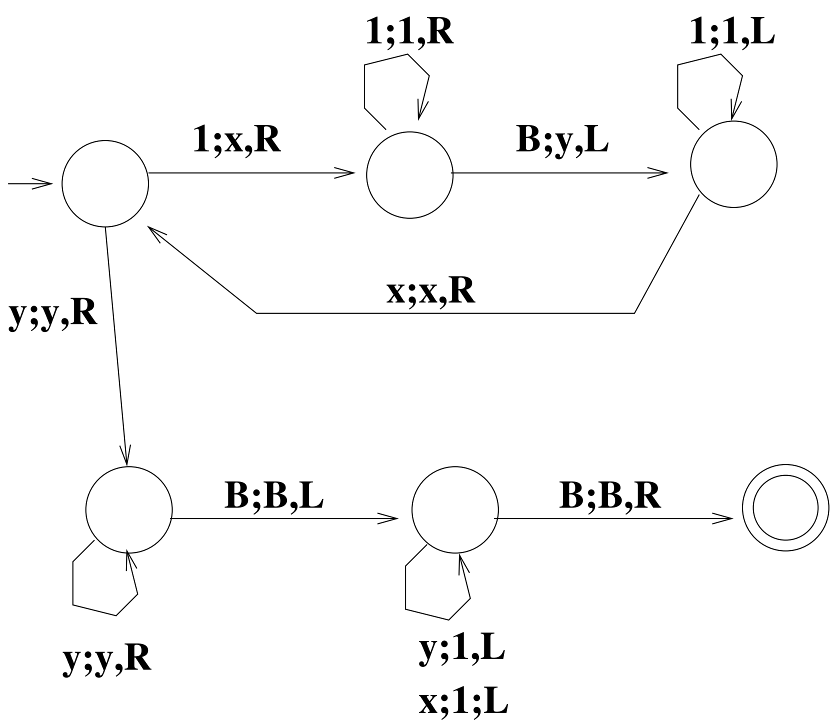
Example:

f(x) = 2x

x is a unary number

start with:	111 ↑
end with:	111111 ↑

Is the following TM correct?



Example:

L=$\{ww \mid w \in \Sigma^+\}$, Σ=$\{a, b\}$