**Due Date: January 31, 2013**

**Problem 1:** In each of the following cases, rank the functions by order of their growth. Here $\log n$ means $\log_2 n$.

- $2^{\log n}$, $(\log n)^{\log n}$, $e^n$, $4^{\sqrt{\log n}}$, $n!$, $\sqrt{\log n}$
- $(\frac{3}{2})^n$, $n^3$, $(\log n)^2$, $\log(n!)$, $2^{2^n}$, $n^{\frac{1}{\log n}}$, $n^{\frac{1}{\log \log n}}$.

**Problem 2:** Show that, if $c$ is a positive real number, then $g(n) = 1 + c + c^2 + c^3 + \cdots + c^n$ is:

- $\Theta(1)$ if $c < 1$
- $\Theta(n)$ if $c = 1$
- $\Theta(c^n)$ if $c > 1$

**Problem 3:** Solve the following recurrences by expanding the terms or using induction and give a $\Theta$ bound for each of them. If you use induction, you can use the master theorem to guess the bound. In all the cases, assume $T(k) = O(1)$ if $k$ is a constant.

- $T(n) = 5T(n/4) + n$
- $T(n) = T(\sqrt{n}) + 1$
- $T(n) = T(n-1) + n^c$

**Problem 4:** Give an efficient algorithm to compute the *least common multiple* of two $n$-bit numbers $x$ and $y$, that is, the smallest number divisible by both $x$ and $y$. What is the running time of your algorithm as a function of $n$?

**Problem 5:** The $k$th **quantiles** of an $n$-element set are the $k-1$ order statistics that divide the sorted set into $k$ equal-sized sets (to within 1). That is, compute the elements of rank $\lceil in/k \rceil$ for all $1 \leq i < k$. Give an $O(n \log k)$-time algorithm to list the $k$th quantiles of a set.