Due Date: April 25, 2013, 11:59 PM

Problem 1: Polynomial Multiplication of FFT:
Suppose that you want to multiply the two polynomials $1+x+2x^2$ and $2+3x$ using the FFT. Choose an appropriate power of two, find the FFT of the two sequences, multiply the results componentwise, and compute the inverse FFT to get the final result.

Problem 2: Let X and Y be two sets of natural numbers such that the largest number in $X \cup Y$ is M. Let $Z = \{x+y \mid x \in X, y \in Y\}$ be the Minkowski sum of X and Y. Describe an $O(M \log M)$ algorithm to compute the set Z. ((**Hint:** *Write each X and Y as coefficients of a polynomial.*))

Problem 3: Consider a Boolean formula $\Phi(x_1, \ldots, x_n)$ with clauses $C_1, \ldots, C_m$. Φ is called *monotone* if each clause consists of only non-negated variables, i.e., no literal is of the form $\overline{x}_i$. For example,

$$\Phi(x_1, \ldots, x_4) = (x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_4) \wedge (x_2 \vee x_4).$$

Consider the following problem: Given a monotone Boolean formula and an integer $k > 0$, is there a satisfiable assignment of Φ so that at most k variables are set to 1. (Note that Φ is obviously satisfiable if all variables are set to 1.) Show that this problem is NP-Complete.

Problem 4: In the HITTING SET problem, we are given a family of sets S_1, S_2,..., S_n and a budget b, and we wish to find a set H of size $\leq b$ which intersects every S_i, if such an H exists. In other words, we want $H \cap S_i \neq \emptyset$ for all i.

Show that HITTING SET is **NP**-complete.

Problem 5: A film producer is seeking actors and investors for his new movie. There are n available actors; actor i charges s_i dollars. For funding, there are m available investors. Investor j will provide p_j dollars, but only on the condition that certain actors $L_j \subseteq 1, 2, ..., n$ are included in the cast (all of these actors L_j must be chosen in order to receive funding from investor j).
The producer's profit is the sum of the payments from investors minus the payments to actors. The goal is to maximize this profit.
(a) Express this problem as an integer linear program in which the variables take on values $(0, 1)$.
(b) Now relax this to a linear program, and show that there must in fact be an integral optimal solution (as is the case, for example, with maximum flow and bipartite matching).

Problem 6: An integer program is a linear program with the additional constraint that the variables must take only integer values. Prove that deciding whether an integer program has a feasible solution is NP-hard. (**Hint:** *Almost any NP-hard decision problem can be formulated as an integer program. Take your pick.*)