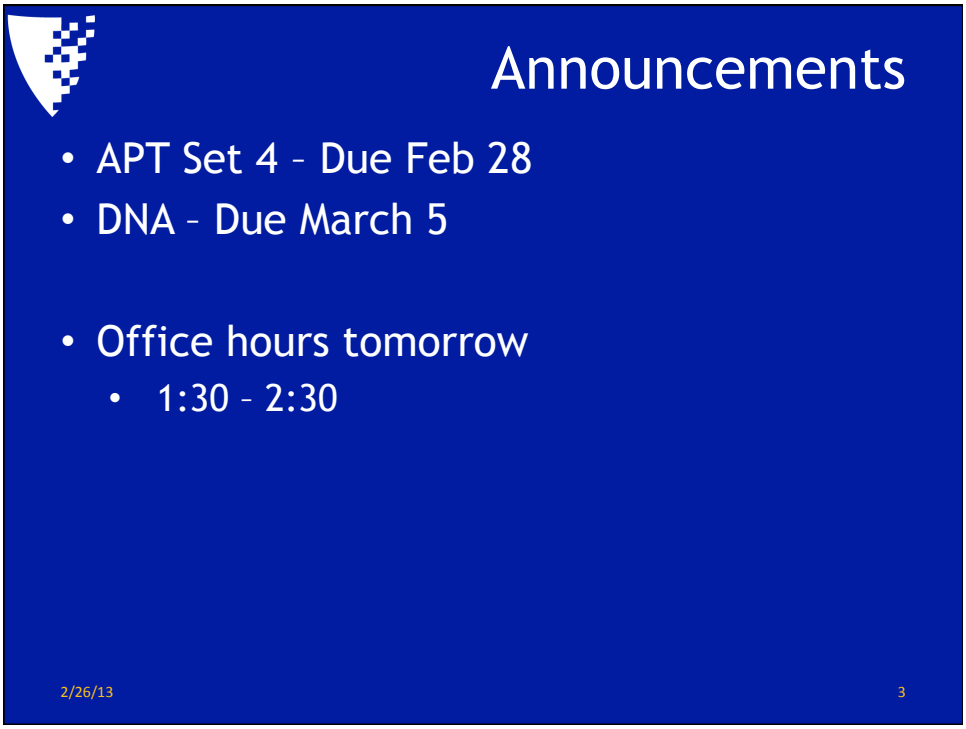
Before class

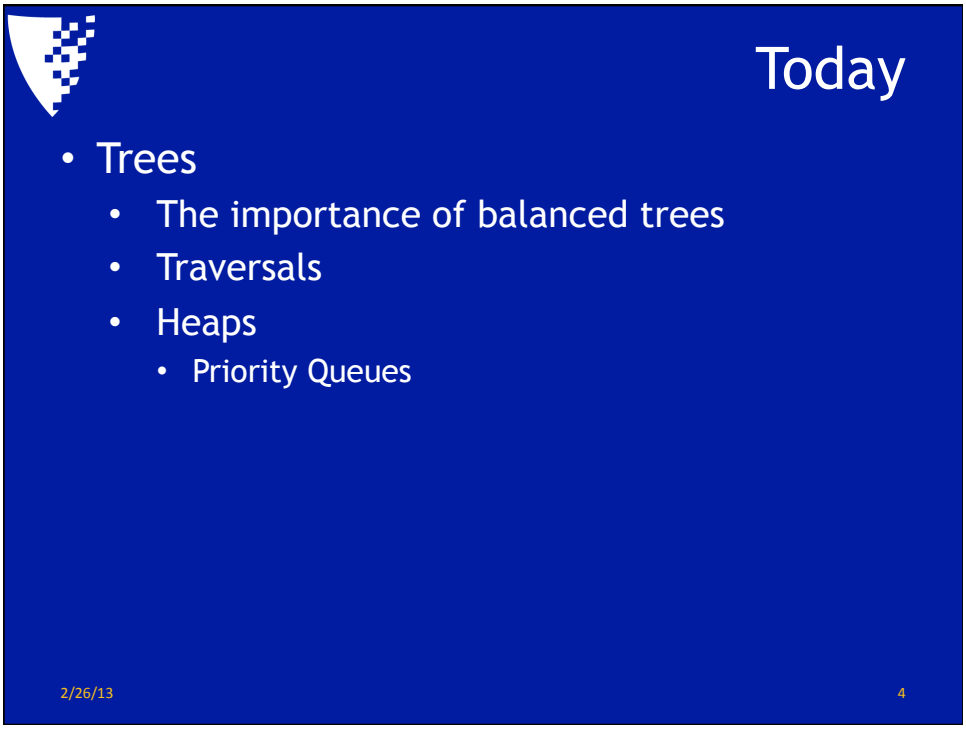
Open TreeNodeExample.java
Change main to:

```
1       public static void main(String[] args) {
2           for(int j = 4; j < 15; j++){
3               TreeNodeExample tree = new TreeNodeExample();
4               double start = System.currentTimeMillis();
5               int nodes = (int)Math.pow(2,j);
6               for(int i = 0; i < nodes; i++)
7                   tree.add(i);
8
9               double end = System.currentTimeMillis();
10              double time = (end-start)/1000.0;
11              System.out.printf("Time: %f Height: %d Nodes: %d (2^%d)
    \n", time, tree.computeHeight(), nodes, j);
12          }
13      }
```


Trees II

BSTs, Heaps, and PQs


Announcements
• APT Set 4 – Due Feb 28
• DNA – Due March 5
• Office hours tomorrow
• 1:30 – 2:30
2/26/13
3


Today
• Trees
• The importance of balanced trees
• Traversals
• Heaps
• Priority Queues
2/26/13
4

BST

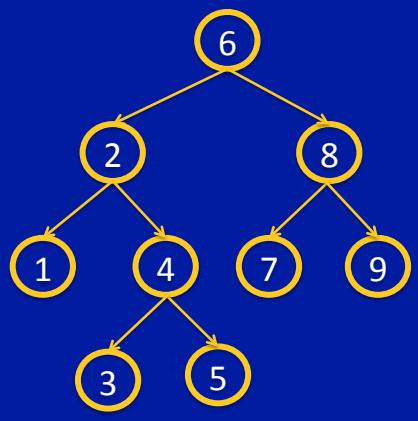
The following nodes are added to a binary search tree (BST) in order. Draw the resulting BST.

6,8,2,4,1,7,5,3,9

```
public void add(int newValue){
    if(root == null)
        root = new TreeNode(newValue);
    else
        add(newValue, root);
}
public void add(int newValue, TreeNode current) {
    if (newValue < current.myValue) {
        if (current.myLeft == null)
            current.myLeft = new TreeNode(newValue);
        else
            add(newValue, current.myLeft);
    } else
        if (current.myRight == null)
            current.myRight = new TreeNode(newValue);
        else
            add(newValue, current.myRight);
    }
}
```


BST

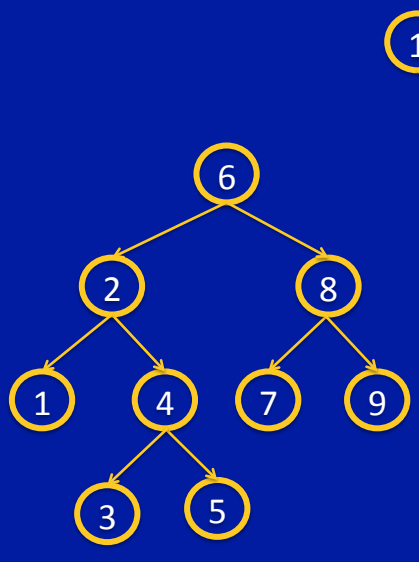
- My answer
  - 6,8,2,4,1,7,5,3,9

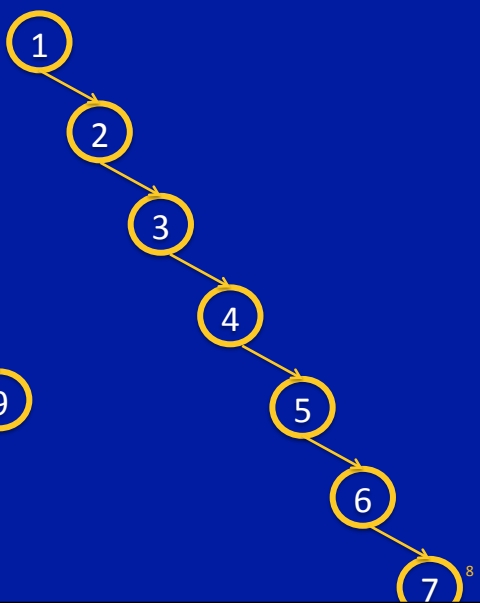


BST

- The following nodes are added to a binary search tree (BST) in order. Draw the resulting BST.
  - 1,2,3,4,5,6,7,8,9

BST





Code

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8
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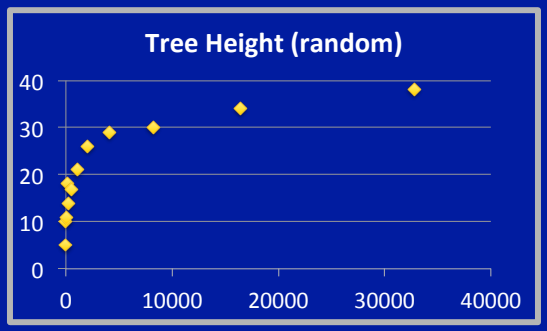
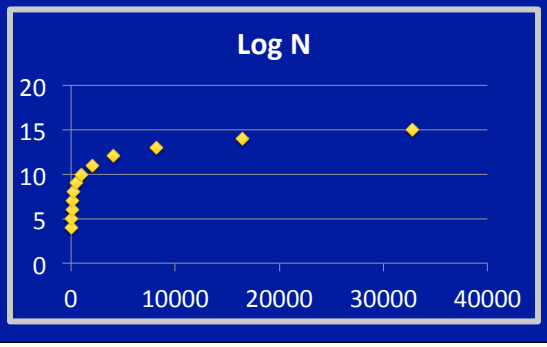
10              double time = (end-start)/1000.0;
11              System.out.printf("Time: %f Height: %d Nodes: %d (2^%d)
    \n", time, tree.computeHeight(), nodes, j);
12          }
13      }
```

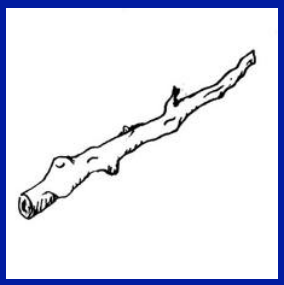

BST





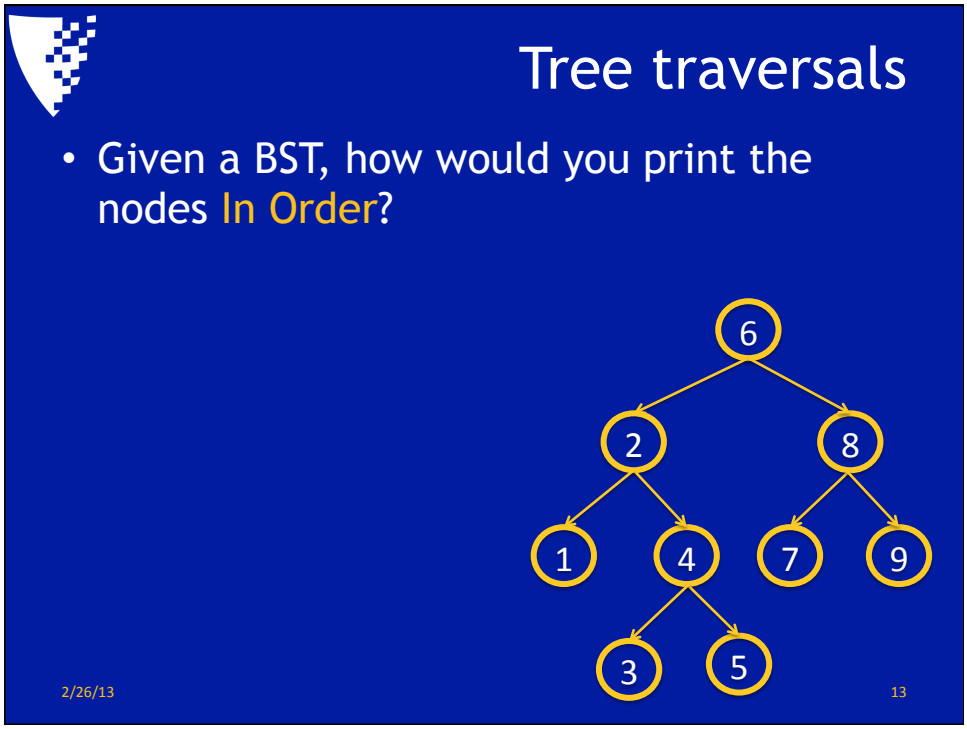
BST

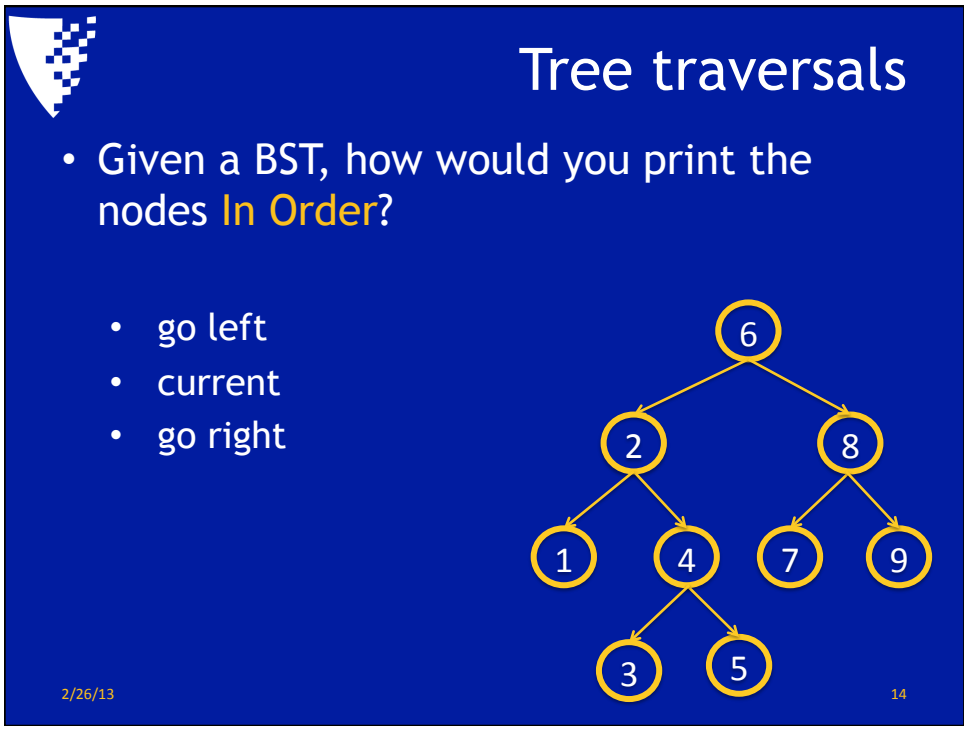
- Order matters!
  - random input ➔ height O(logN)
  - ordered input ➔ height O(N)

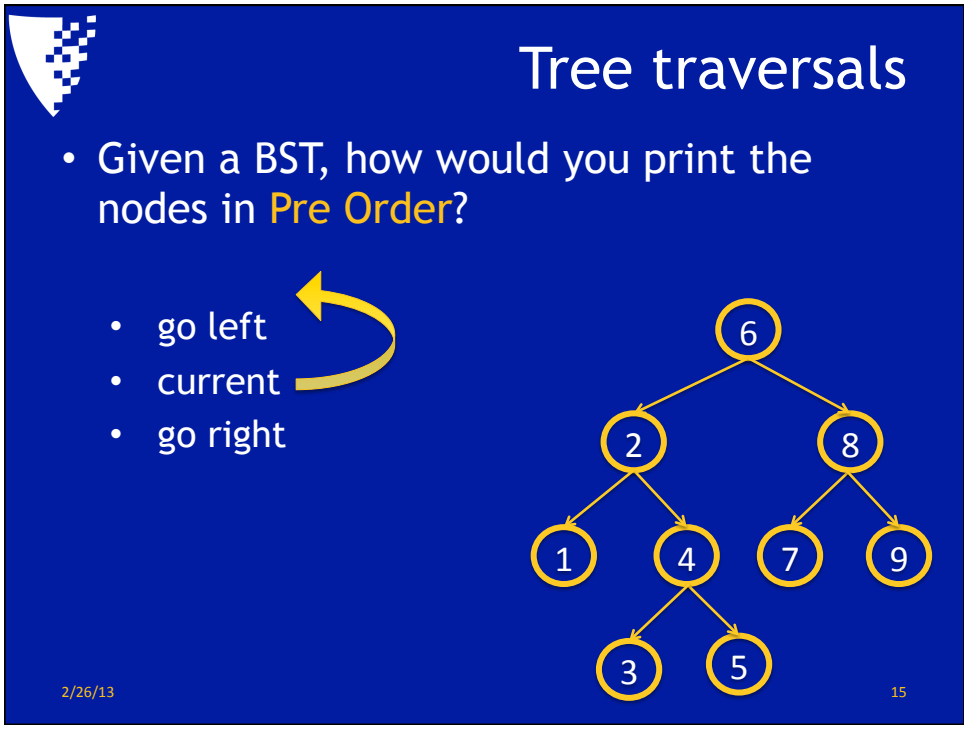


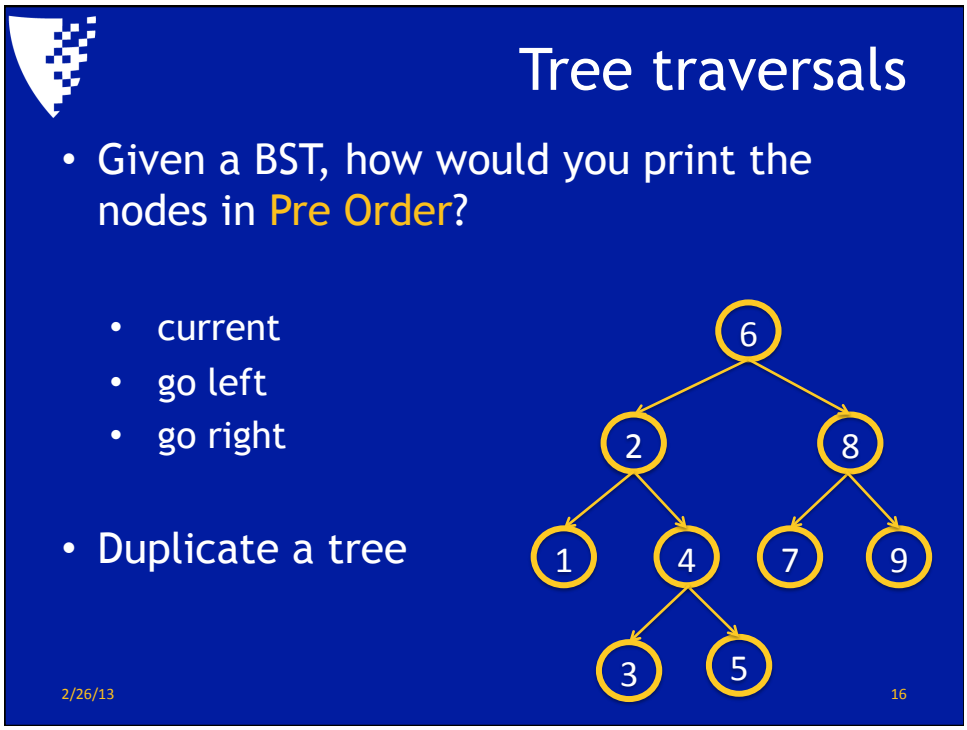
Today

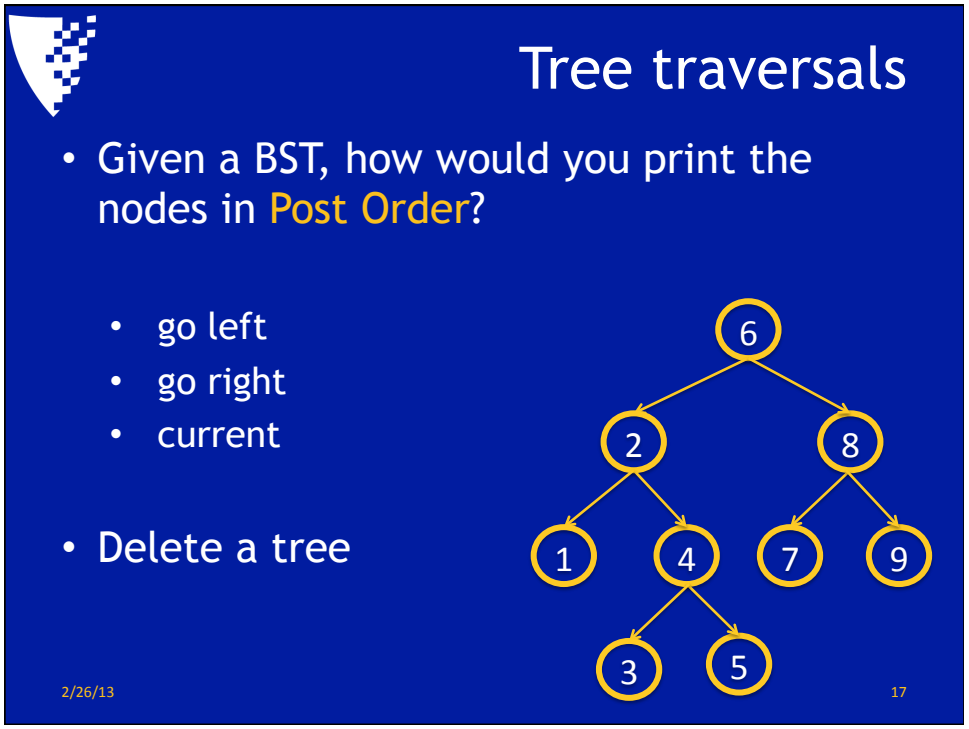
- Trees
  - The importance of balanced trees
  - Traversals
  - Heaps
    - Priority Queues


Tree traversals
• Given a BST, how would you print the nodes In Order?
6
2
8
1
4
7
9
3
5
2/26/13
13


Tree traversals
• Given a BST, how would you print the nodes In Order?
• go left
• current
• go right
6
2
8
1
4
7
9
3
5
2/26/13
14

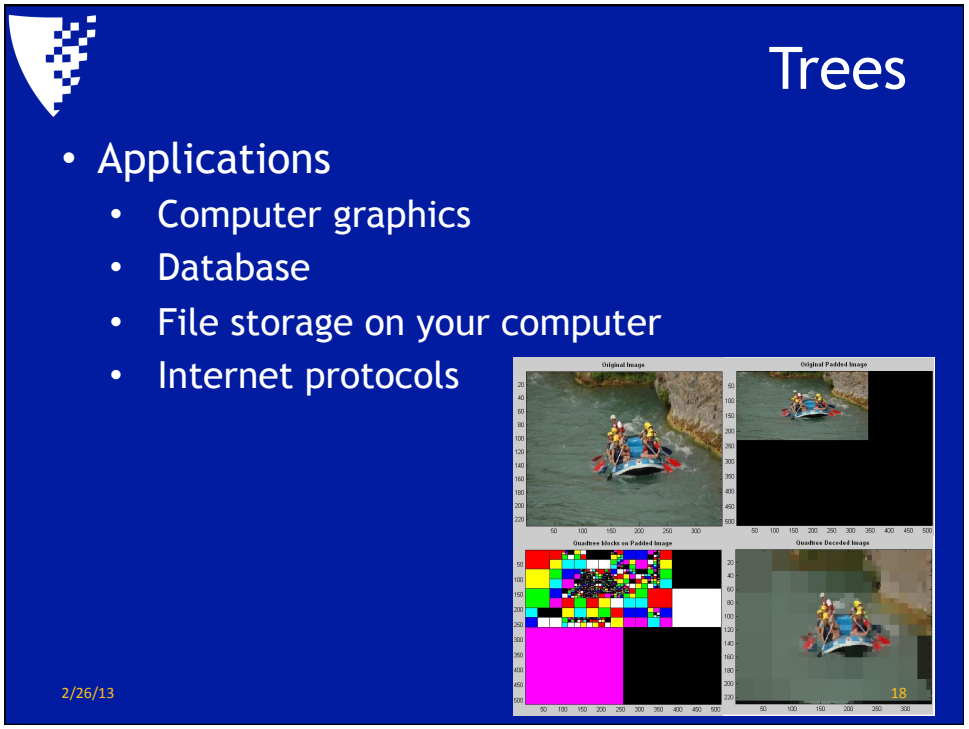

Tree traversals
• Given a BST, how would you print the nodes in Pre Order?
• go left
• current
• go right
6
2
8
1
4
7
9
3
5
2/26/13
15


Tree traversals
• Given a BST, how would you print the nodes in Pre Order?
• current
• go left
• go right
• Duplicate a tree
6
2
8
1
4
7
9
3
5
2/26/13
16



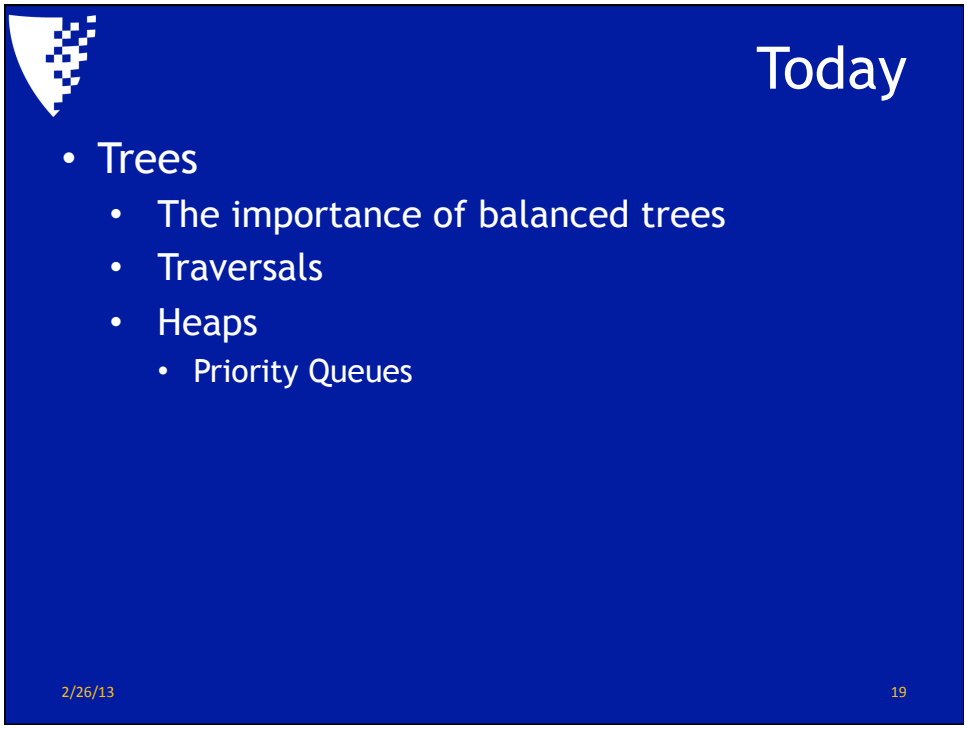
Tree traversals

- Given a BST, how would you print the nodes in Post Order?
  - go left
  - go right
  - current
- Delete a tree



Trees

- Applications
  - Computer graphics
  - Database
  - File storage on your computer
  - Internet protocols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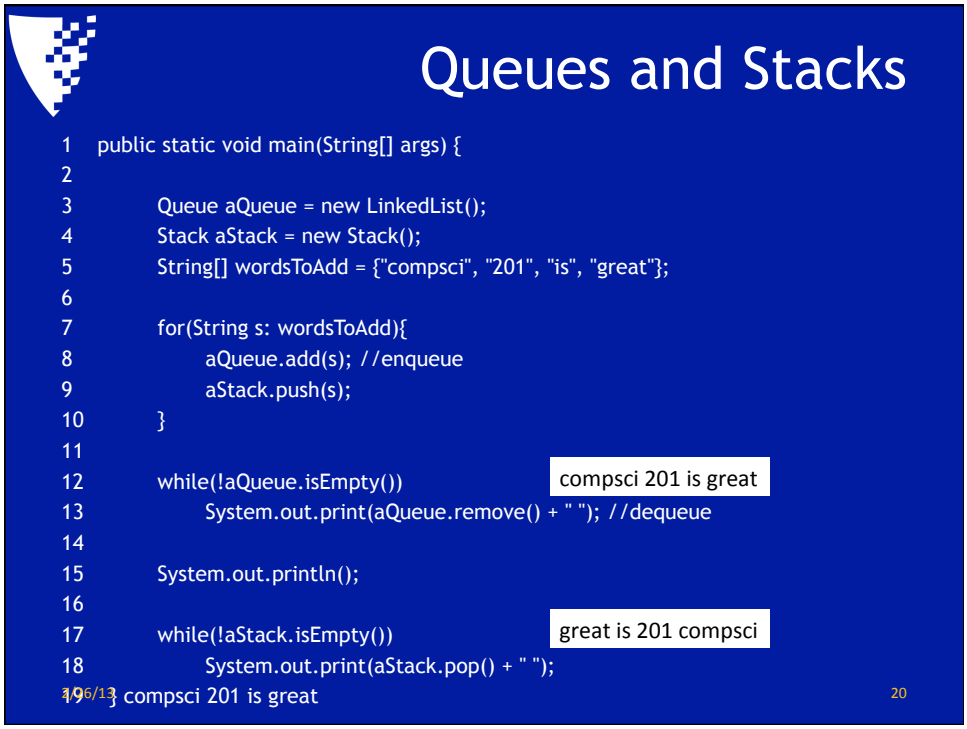

Queues and Stacks
1 public static void main(String[] args) {
2
3 Queue aQueue = new LinkedList();
4 Stack aStack = new Stack();
5 String[] wordsToAdd = {"compsci", "201", "is", "great"};
6
7 for(String s: wordsToAdd){
8 aQueue.add(s); //enqueue
9 aStack.push(s);
10 }
11
12 while(!aQueue.isEmpty())
13 System.out.print(aQueue.remove() + " "); //dequeue
14
15 System.out.println();
16
17 while(!aStack.isEmpty())
18 System.out.print(aStack.pop() + " ");
19 } compsci 201 is great
compsci 201 is great
great is 201 compsci
2/26/13
20

Priority Queue

- Airport queue
  - First class?

Priority Queue

```
public static void main(String[] args) {

    //Queue aQueue = new LinkedList();
    PriorityQueue<String> aQueue = new PriorityQueue<String>();

    String[] wordsToAdd = {"compsci", "201", "is", "great"};

    for(String s: wordsToAdd){
        aQueue.add(s);
        aStack.push(s);
    }

    while(!aQueue.isEmpty())
        System.out.print(aQueue.remove() + " ");

}
```

1. compsci 201 is great
2. great is compsci 201
3. is great compsci 201
4. 201 compsci great is

Priority Queue

- What is the output?

```
PriorityQueue<Integer> ex = new PriorityQueue<Integer>();
ex.add(2);
ex.add(13);
ex.add(9);
ex.add(75);
ex.add(4);
while(!ex.isEmpty()) {
    System.out.println(ex.remove());
}
```

- Add in any order
- Remove smallest first

Heaps

- Common implementation of priority queues
- A tree-like structure
- Almost completely filled
  - All nodes filled expect last level
- Max-Heap - Descendants have values <= to its parent
- Min-Heap - Descendants have values >= to its parent

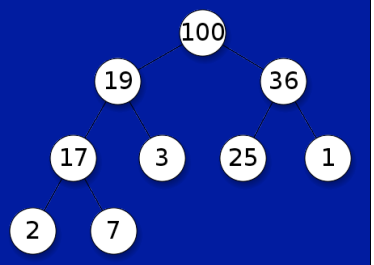
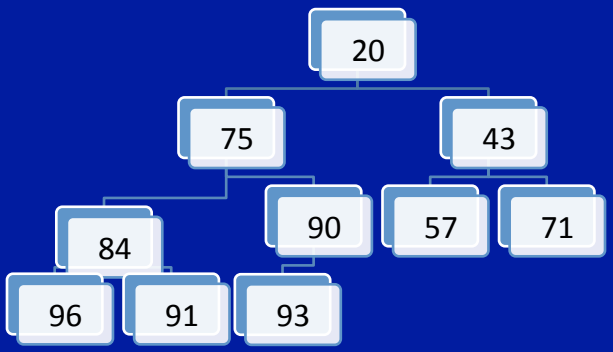


image form wikipedia

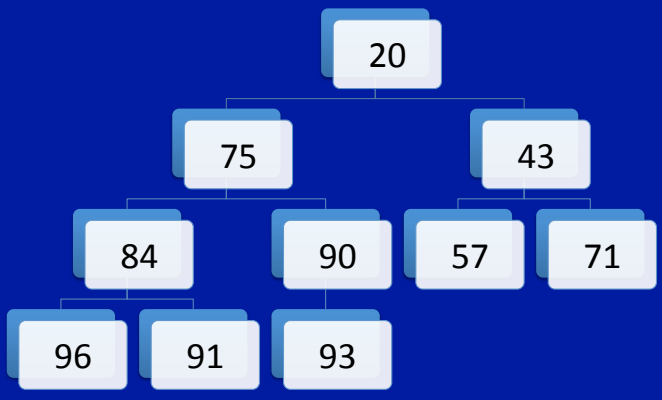
Heaps

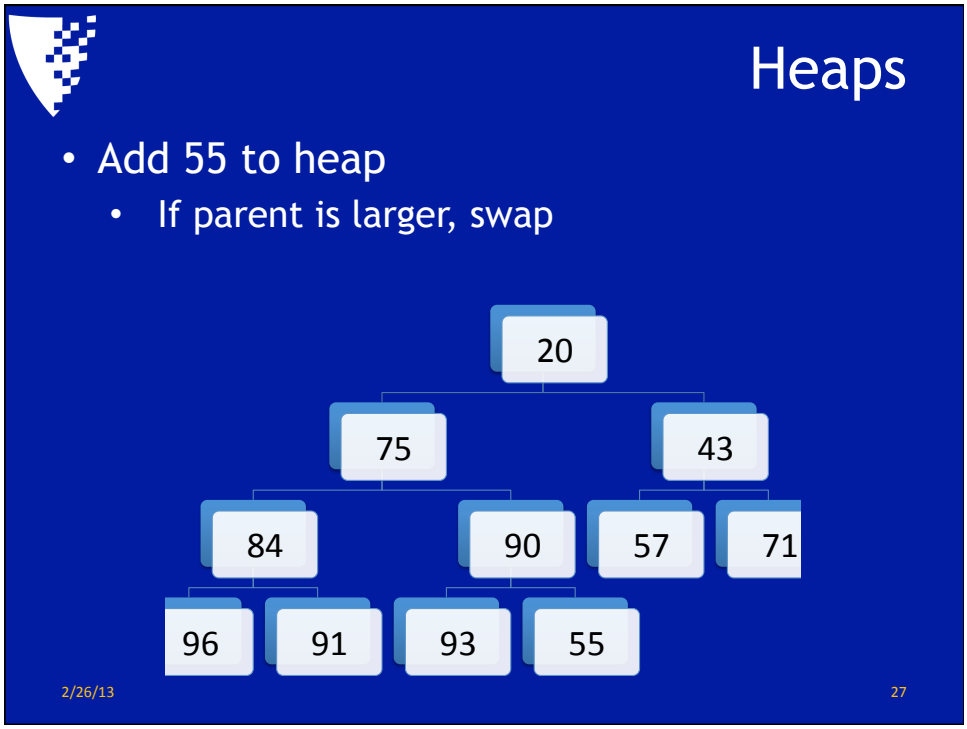
- Why is a heap implemented with a priority queue?
  - Where is the min value?

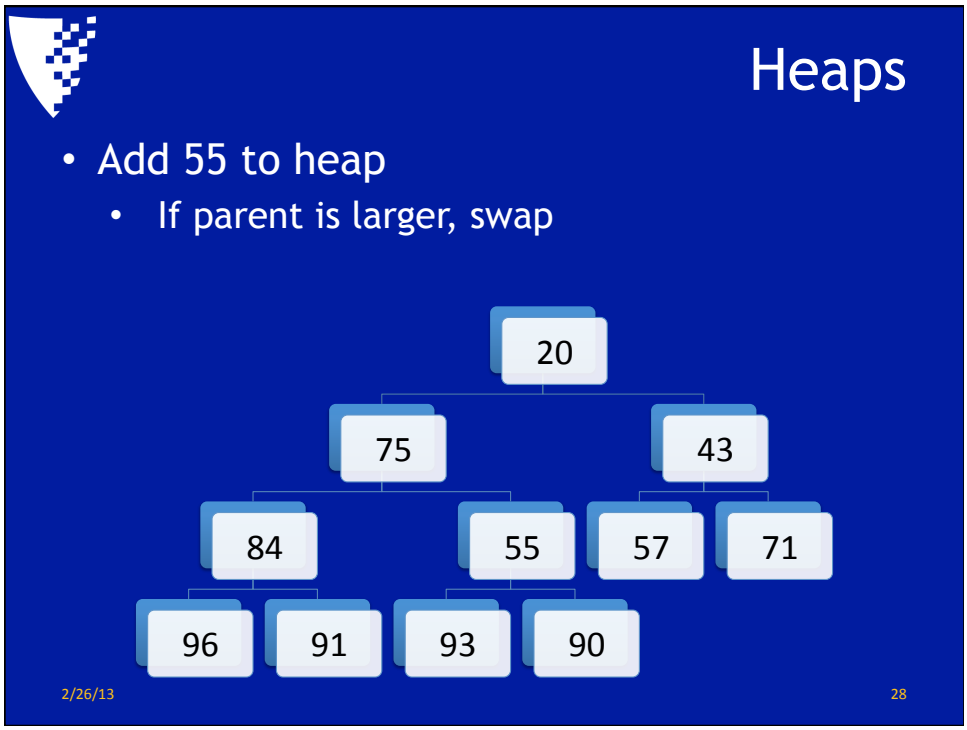


Heaps

- Add 55 to heap
  - add node to first open slot




Heaps
• Add 55 to heap
• If parent is larger, swap
20
75
43
84
90
57
71
96
91
93
55
2/26/13
27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43
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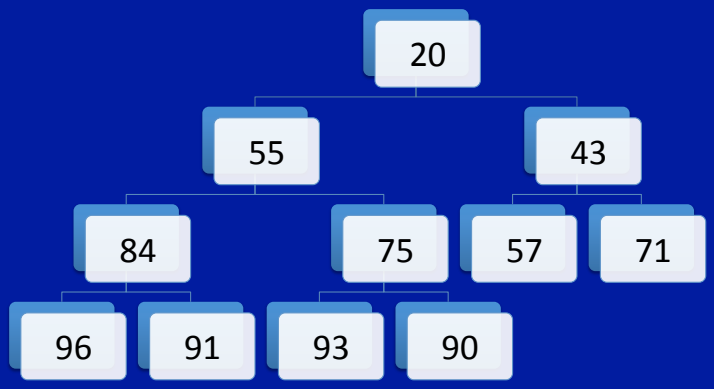
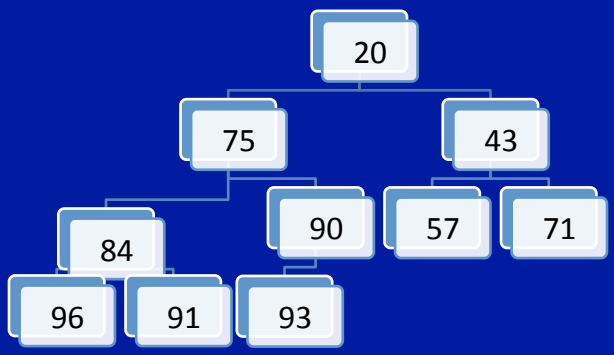
2/26/13
28

Heaps

- Add 55 to heap
  - If parent is larger, swap

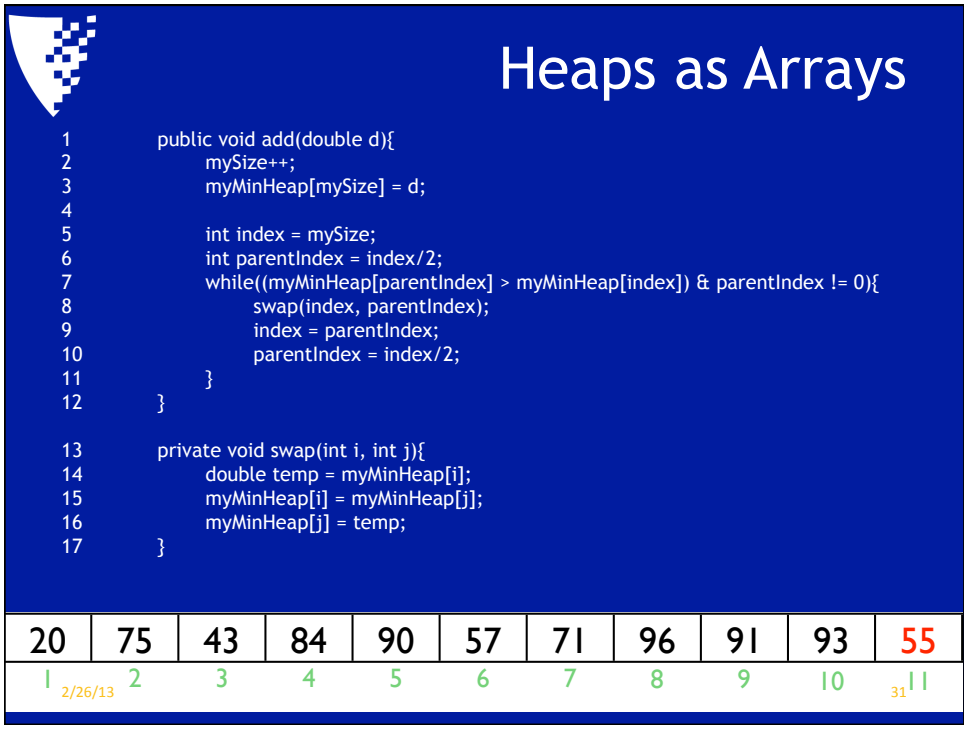


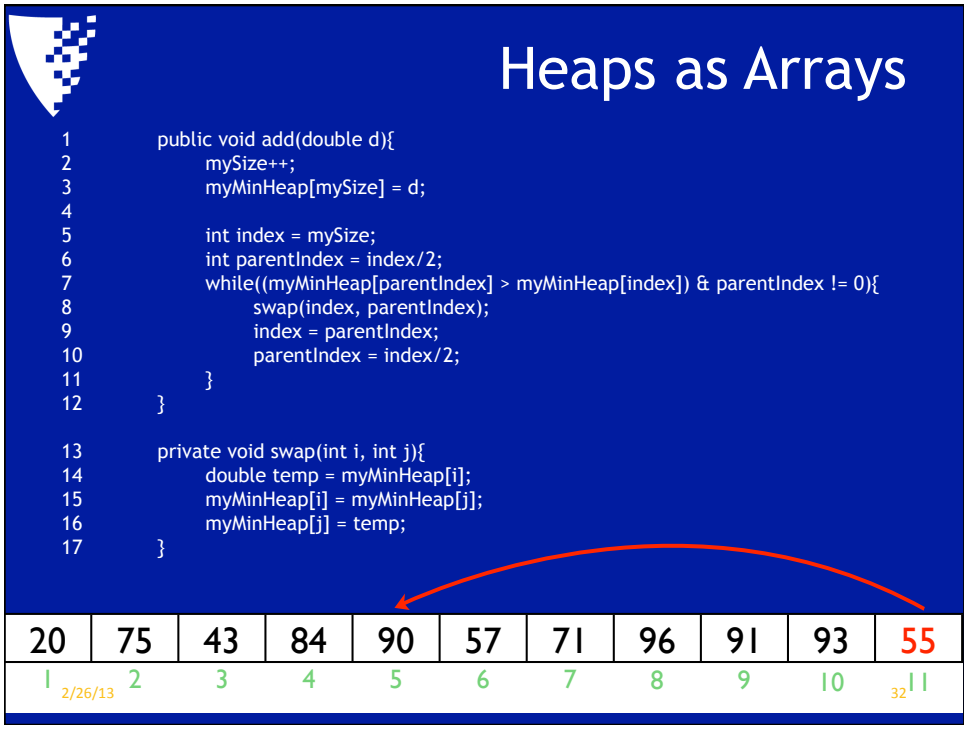
Heaps as Arrays



	20	75	43	84	90	57	71	96	91	93

Layer 1 Layer 2 Layer 3 Layer 4


Heaps as Arrays
1 public void add(double d){
2 mySize++;
3 myMinHeap[mySize] = d;
4
5 int index = mySize;
6 int parentIndex = index/2;
7 while((myMinHeap[parentIndex] > myMinHeap[index]) & parentIndex != 0){
8 swap(index, parentIndex);
9 index = parentIndex;
10 parentIndex = index/2;
11 }
12 }
13 private void swap(int i, int j){
14 double temp = myMinHeap[i];
15 myMinHeap[i] = myMinHeap[j];
16 myMinHeap[j] = temp;
17 }
20 75 43 84 90 57 71 96 91 93 55
1 2 3 4 5 6 7 8 9 10 11
2/26/13
31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8 swap(index, parentIndex);
9 index = parentIndex;
10 parentIndex = index/2;
11 }
12 }
13 private void swap(int i, int j){
14 double temp = myMinHeap[i];
15 myMinHeap[i] = myMinHeap[j];
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20 75 43 84 90 57 71 96 91 93 55
1 2 3 4 5 6 7 8 9 10 11
2/26/13
32

Heaps as Arrays

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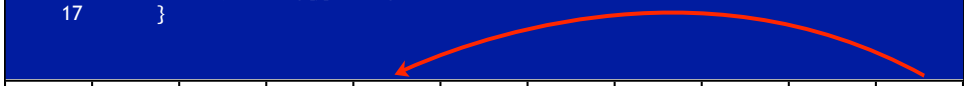
9                   index = parentIndex;
10                  parentIndex = index/2;
11            }
12      }

13      private void swap(int i, int j){
14            double temp = myMinHeap[i];
15            myMinHeap[i] = myMinHeap[j];
16            myMinHeap[j] = temp;
17      }
```



20	75	43	84	55	57	71	96	91	93	90
1	2	3	4	5	6	7	8	9	10	11

Heaps as Arrays

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Heaps as Arrays

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17      }
```

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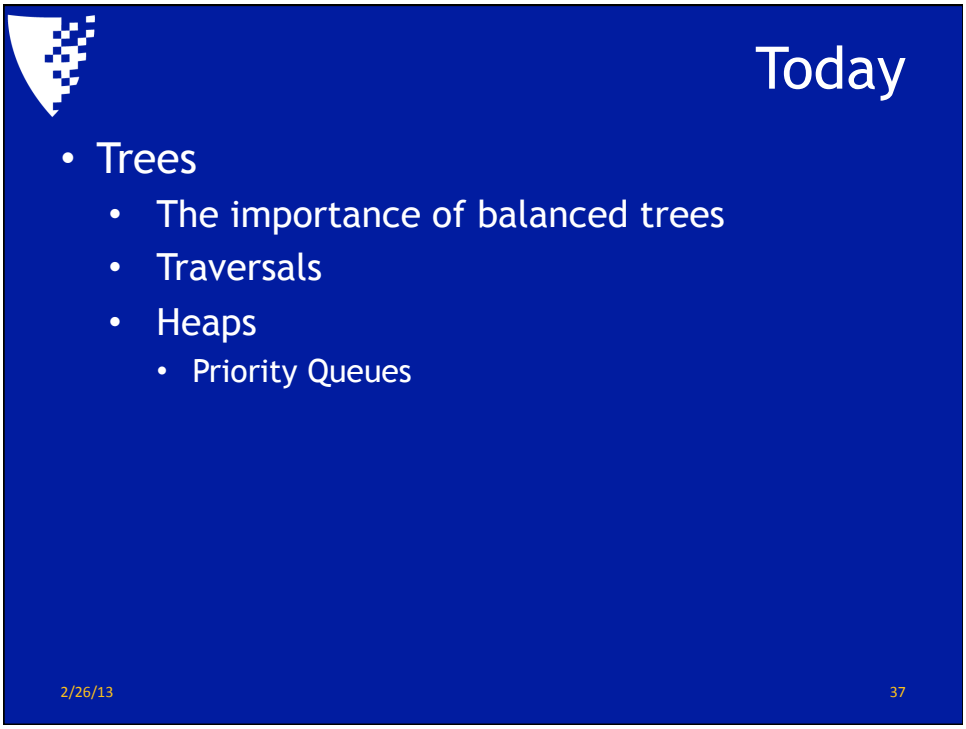
1	2	3	4	5	6	7	8	9	10	11

Remove

- Remove the root
- Move last value into root
- If a child is smaller than root
- promote the smallest child
  - What would the array look like if I called remove()?

20	55	43	84	75	57	71	96	91	93	90
1	2	3	4	5	6	7	8	9	10	11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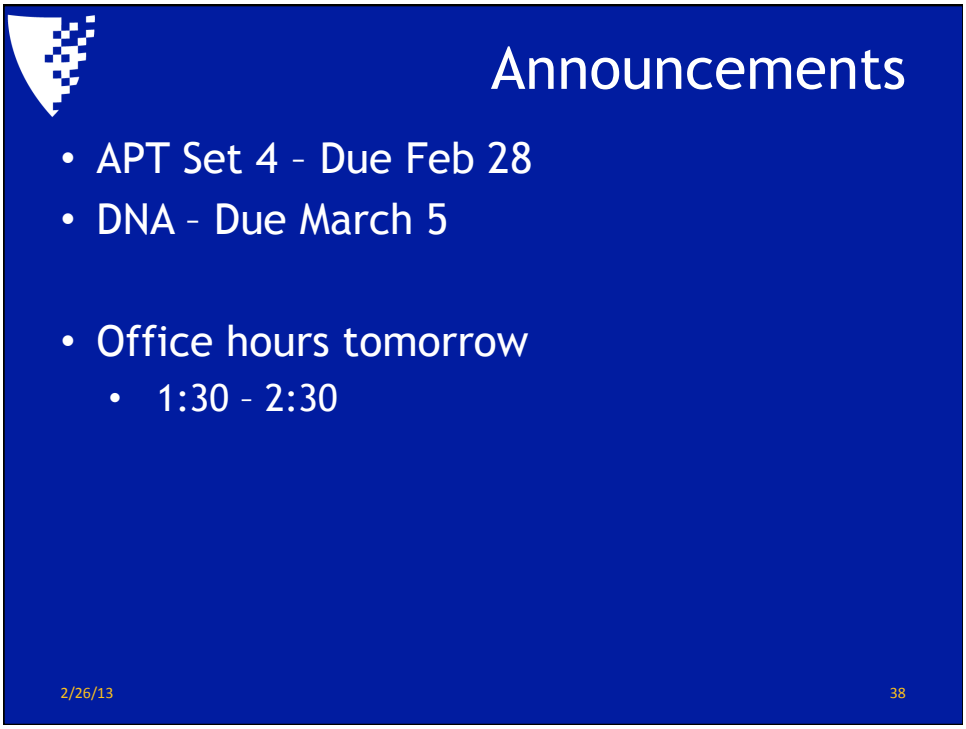
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