Reinforcement Learning

George Konidaris
gdk@cs.duke.edu

DUKE COMPUTER SCIENCE

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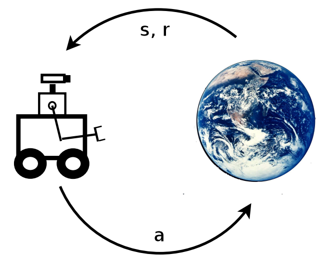
RL



Agent interacts with an environment
At each time t:

- Receives sensor signal s_t
- Executes action a_t
- *Transition:*
  - new sensor signal s_{t+1}
  - reward r_t

Goal: find policy π that maximizes expected return (sum of discounted future rewards):

$$\max_{\pi} \mathbb{E}\left[R = \sum_{t=0}^{\infty} \gamma^t r_t\right]$$

Markov Decision Processes

S : set of *states*
A : set of actions
γ : discount factor

$$< S, A, \gamma, R, T >$$

R : reward function
$R(s, a, s')$ is the reward received taking action a from state s and transitioning to state s'.

T : transition function
$T(s'|s, a)$ is the probability of transitioning to state s' after taking action a in state s.

RL: one or both of T, R unknown.

MDPs

Our target is a **policy**:

$$\pi : S \rightarrow A$$

A policy maps *states* to *actions*.

The **optimal policy** maximizes:

$$\max_{\pi} \forall s, \mathbb{E}\left[R(s) = \sum_{t=0}^{\infty} \gamma^t r_t \middle| s_0 = s\right]$$

This means that we wish to find a policy that maximizes the **return** from **every state.**

Value Functions

Value functions:

$$V_\pi(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \,\middle|\, \pi, s_0 = s\right]$$

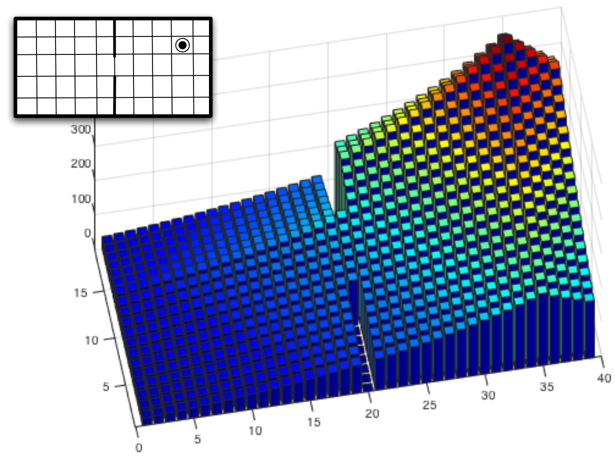
This is the value of state s **under policy** π.

State-action value functions:

$$Q_\pi(s,a) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \,\middle|\, \pi, s_0 = s, a_0 = a\right]$$

This is the value of executing a in state s, *then following* π.

Value Functions



Policy Iteration

This leads to a general policy improvement framework:

1. Start with a policy π
2. Learn Q_π
3. Improve π
   a. $\pi(s) = \max_a Q(s,a), \forall s$

Repeat

This is known as **policy iteration**.
It is guaranteed to converge to the optimal policy.

Steps 2 and 3 can be interleaved as rapidly as you like.
Usually, perform 3a *every time step*.

Value Function Learning

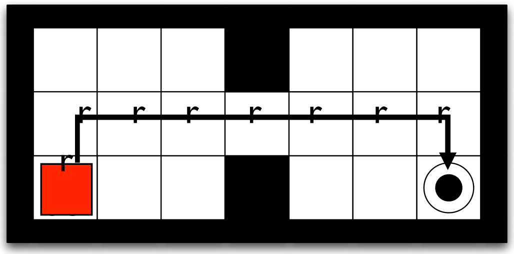
Learning proceeds by gathering samples of $Q(s,a)$.

Methods differ by:

- How you get the samples.
- How you use them to update Q.

Monte Carlo

Simplest thing you can do: sample $R(s)$.



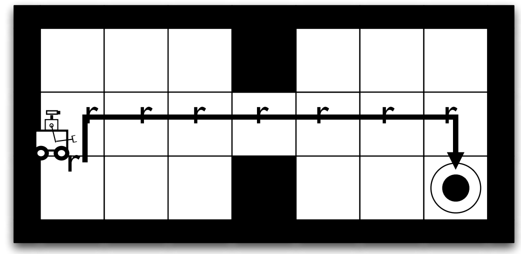
Do this repeatedly, average values:

$$Q(s,a) = \frac{R_1(s) + R_2(s) + ... + R_n(s)}{n}$$

Temporal Difference Learning

Where can we get more (immediate) samples?

Idea: *there is an important relationship between temporally successive states.*



TD Learning

Ideally and in expectation:

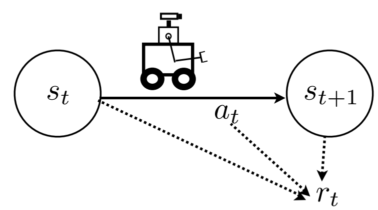
$$r_t + \gamma V(s_{t+1}) - V(s_t) = 0$$

V is correct if this holds in expectation for all states.

When it does not, it is known as a *temporal difference error.*

TD Learning

What does this look like?



$$V(s_t) \leftarrow r_t + \gamma V(s_{t+1})$$
$$Q(s_t, a_t) \leftarrow r_t + \gamma Q(s_{t+1}, a_{t+1})$$

Sarsa

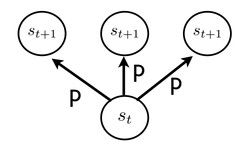
Sarsa: very simple algorithm

1. Initialize Q(s, a)
2. For *n* episodes
    - observe transition (s, a, r, s', a')
    - compute TD error $\delta = r + \gamma Q(s', a') - Q(s, a)$
    - update Q: $Q(s, a) = Q(s, a) + \alpha\delta$
    - select and execute action based on Q

TD

In Sarsa, we use a sample transition: (s, a, r, s', a')
This is a *sample backup*.

Given T, could replace with the full expectation:



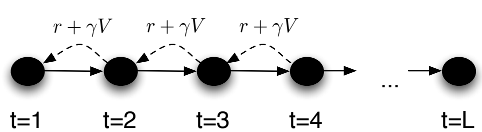
$$\delta = \mathbb{E}_{\pi,T}\left[r + \gamma Q(s', a')\right] - Q(s, a)$$

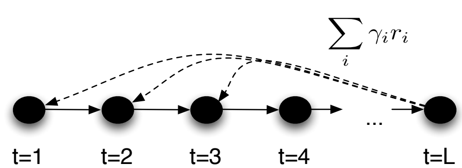
This is known as a *full backup* - **dynamic programming.**

Finds an optimal policy in time polynomial in $|S|$ and $|A|$.
(There are $|A|^{|S|}$ possible policies.)

TD vs. MC

TD and MC two extremes of obtaining samples of Q:





Generalizing TD

We can generalize this to the idea of an n-step rollout:

$$R_{s_t}^{(n)} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots + \gamma^{n-1} r_{t+n-1} + \gamma^n V(s_{t+n})$$

Each tells us something about the value function.

- We can combine *all* n-step rollouts.
- This is known as a *complex backup*.

TD(λ)

Weighted sum:

$$R^{(1)} = r_0 + \gamma V(s_1) \qquad 1$$
$$R^{(2)} = r_0 + \gamma r_1 + \gamma^2 V(s_2) \qquad \lambda$$
$$\vdots$$
$$R^{(n)} = \sum_{i=0}^{n-1} \gamma^i r_i + \gamma^n V(s_n) \qquad \lambda^n$$

weights

Estimator:

$$R_{s_t}^{\lambda} = (1-\lambda) \sum_{n=0}^{\infty} \lambda^n R_{s_t}^{(n+1)}$$

TD(λ)

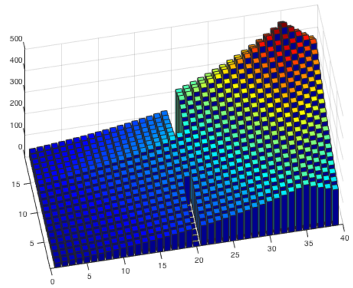
This is called the λ-return.

- At λ=0 we get TD, at λ=1 we get MC.
- Intermediate values of λ usually best.
- TD(λ) family of algorithms

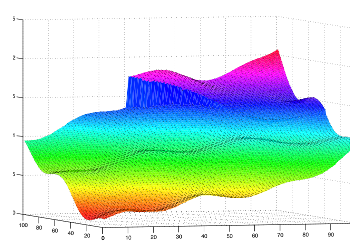
Real-Valued States

What if the states are real-valued?

- Cannot use table to represent Q.
- States may never repeat: must *generalize*.



vs



Function Approximation

How do we represent general function of state variables?

Many choices:

- Most popular is *linear value function approximation*.
- Use set of basis functions $\phi_1, ..., \phi_m$
- Define linear function of them:

$$\bar{V}(\mathbf{x}) = \sum_{i=1}^{m} w_i \phi_i(\mathbf{x})$$

Learning task is to find vector of weights **w** to best approximate V.

Function Approximation

One choice of basis functions:

- Just use state variables directly: $[1, x, y]$

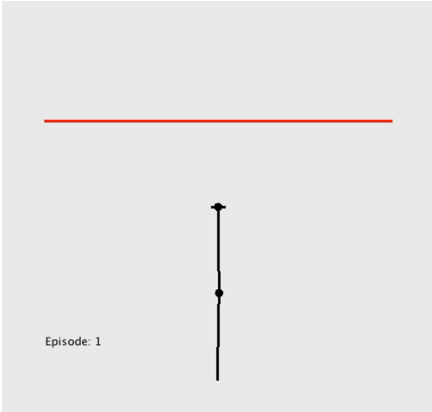
Another:

- Polynomials in state variables.
- E.g., $[1, x, y, xy, x^2, y^2, xy^2, x^2yx^2y^2]$
- This is like a Taylor expansion.

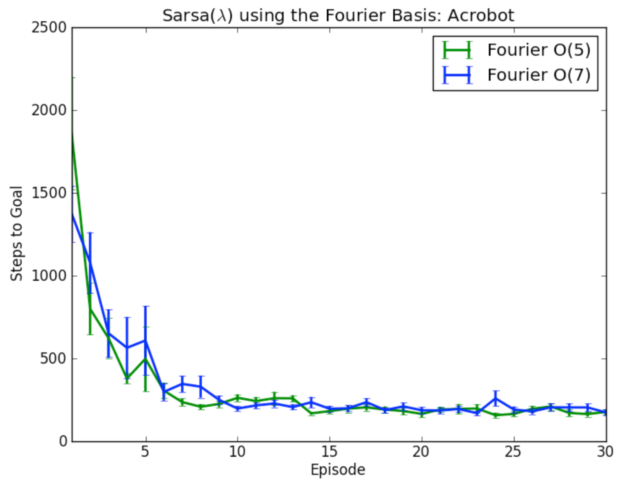
Another:

- Fourier terms on state variables.
- E.g., $[1, cos(\pi x), cos(\pi y), \cos(\pi[x + y])]$
- This is like a Fourier Series expansion.

Acrobot



Acrobot



Function Approximation

TD-Gammon: Tesauro (circa 1992-1995)

- At or near best human level
- Learn to play Backgammon through self-play
- 1.5 million games
- Neural network function approximator
- TD(λ)

Changed the way the best human players played.

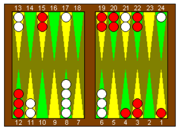


Figure 3. A complex situation where TD-Gammon's positional judgment is apparently superior to traditional expert thinking. White is to play 4-4. The obvious human play is 8-4*, 8-4, 11-7, 11-7. (The asterisk denotes that an opponent checker has been hit.) However, TD-Gammon's choice is the surprising 8-4*, 8-4, 21-17, 21-17! TD-Gammon's analysis of the two plays is given in Table 3.

Policy Search

So far: improve policy via value function.

Sometimes policies are simpler than value functions:
- Parametrized program $\pi(s, a|\theta)$

Sometimes we wish to search in space of restricted policies.

In such cases it makes sense to search directly in *policy-space* rather than trying to learn a value function.

Policy Search

Can apply *any* generic optimization method for θ.

One particular approach: policy gradient.
- Compute and ascend $\partial R / \partial \theta$
- This is the gradient of return w.r.t policy parameters

Policy gradient theorem:

$$\frac{\partial R}{\partial \theta} = \sum_s d^\pi(s) \sum_a \frac{\partial \pi(s,a)}{\partial \theta} (Q^\pi(s,a) - b(s))$$

Therefore, one way is to learn Q and then ascend gradient.
Q need only be defined using basis functions computed from θ.

Aibo Gait Optimization

from Kohl and Stone, ICRA 2004.

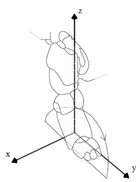
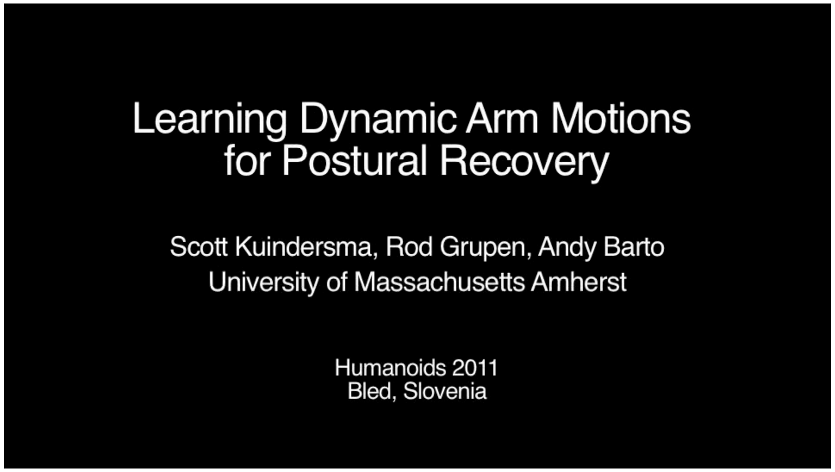


Fig. 2. The elliptical locus of the Aibo's foot. The half-ellipse is defined by length, height, and position in the x-y plane.

All told, the following set of 12 parameters define the Aibo's gait [10]:

- The front locus (3 parameters: height, x-pos., y-pos.)
- The rear locus (3 parameters)
- Locus length
- Locus skew multiplier in the x-y plane (for turning)
- The height of the front of the body
- The height of the rear of the body
- The time each foot takes to move through its locus
- The fraction of time each foot spends on the ground

Postural Recovery



Reinforcement Learning

Machine Learning for control.

Very active area of current research, applications in:

- Robotics
- Operations Research
- Computer Games
- Theoretical Neuroscience

AI

- The primary function of the brain is control.