Due Date: February 3, 2015

Problem 1: Describe a 3-tape deterministic TM M that given two natural numbers x and y, written on tape-1 in binary, writes $x + y$ on tape-3 in the binary form. You need to describe the set of states, alphabet, initial state, and transition function of M.

You can assume that the input is of the form $\triangleright x{+}y$, where the $+$ symbol separates the two numbers. For convenience, you can assume that the least significant bit (resp. most significant bit) is the leftmost (resp. rightmost) bit. For example, if $x = 6$ and $y = 11$, then the input tape contains $\triangleright 011{+}1101\sqcup$. When the TM stops, the output tape should contain $\triangleright 10001$.

Problem 2: Suppose that we have a TM with an infinite *two-dimensional tape* (blackboard). There are now moves of the form $\uparrow$ and $\downarrow$, along with $\leftarrow$ and $\rightarrow$. The input is written initially to the right of the head position.

(a) Give a detailed definition of the transition function of such a machine. What is a configuration?

(b) Show that such a 2D TM can be simulated by a 3-tape TM with a quadratic loss of efficiency. (**Hint:** *Extend the proof of Theorem 2.1 from the Papadimitriou book.*)

Problem 3: Show that the language

$$L = \{x \in \{0,1\}^* \mid x \text{ contains at least two 0s but not two consecutive 0s}\}$$

can be accepted by a TM with just *read-only* tape.

Problem 4: Show that the following problems involving TM are not recursive:

(a) Given a TM M, is there a string on which M halts?

(b) Given a TM M, does it ever write a symbol σ?

(c) Given a TM M, is $L(M)$ empty? ($L(M)$ denotes here the language accepted by M, not decided by it.)

Which of these languages are recursively enumerable?

Problem 5: (a) Show that if L is recursively enumerable, then there is a TM M that enumerates L without repeating an element of L.

(b) Show that L is recursive if and only if there is a TM M that enumerates the strings of L in *length increasing* fashion.