Prediction Markets

David Pennock
Microsoft Research

Thanks: Miro Dudik, Sebastien Lahaie, David Rothschild
Jake Abernethy, Rafael Frongillo, Dan Osherson, Rajiv Sethi, Rob
Schapire, Jenn Wortman Vaughan, Arvid Wang, ...

An Example
Prediction Market

- A random variable, e.g.

Will Hillary Clinton win
2016 Presidential election? (Y/N)

- Turned into a financial instrument payoff = realized value of variable

I am entitled to:

$100 if	Clinton wins	$0 if	Other wins

Trade Elections Like Stocks

betfair

2016 Presidential Election - Next President

Betting on: Hillary Clinton

Total matched on this event: **£65,227**

Betting summary - Volume: **£0**

Last price matched: **2.08**

Price/Volume over time

	Traded and Available	
Price	**To back**	**To lay**
2.02	£230	
2.04	£25	
2.06	£870	
2.1		£101
2.2		£53
2.3		£58

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2.3		£58

Between 47.6% and 48.5% chance
2015/02/23 14:44 EST

Trade Elections Like Stocks

betfair

2016 Presidential Election - Democratic Nominee

Betting on: Hillary Clinton

Total matched on this event: **£59,140**

Betting summary - Volume: **£55,182**

Last price matched: **1.43**

Price/Volume over time

1.24	£102		
1.25	£5,122		£562
1.26	£352		£1,000
1.27	£146		£52
1.28	£77		£453
1.29	£198		£317
1.30	£78		£737
1.31	£126		£64
1.32	£51		£117
1.33	£5		£458
1.34			£145
1.35			£223
1.36			£15
1.37			£32
1.38		£17	£63
1.39		£43	£159
1.40		£51	£1,138

Between 72.5% and 75.2% chance
2015/02/24 06:30 EST

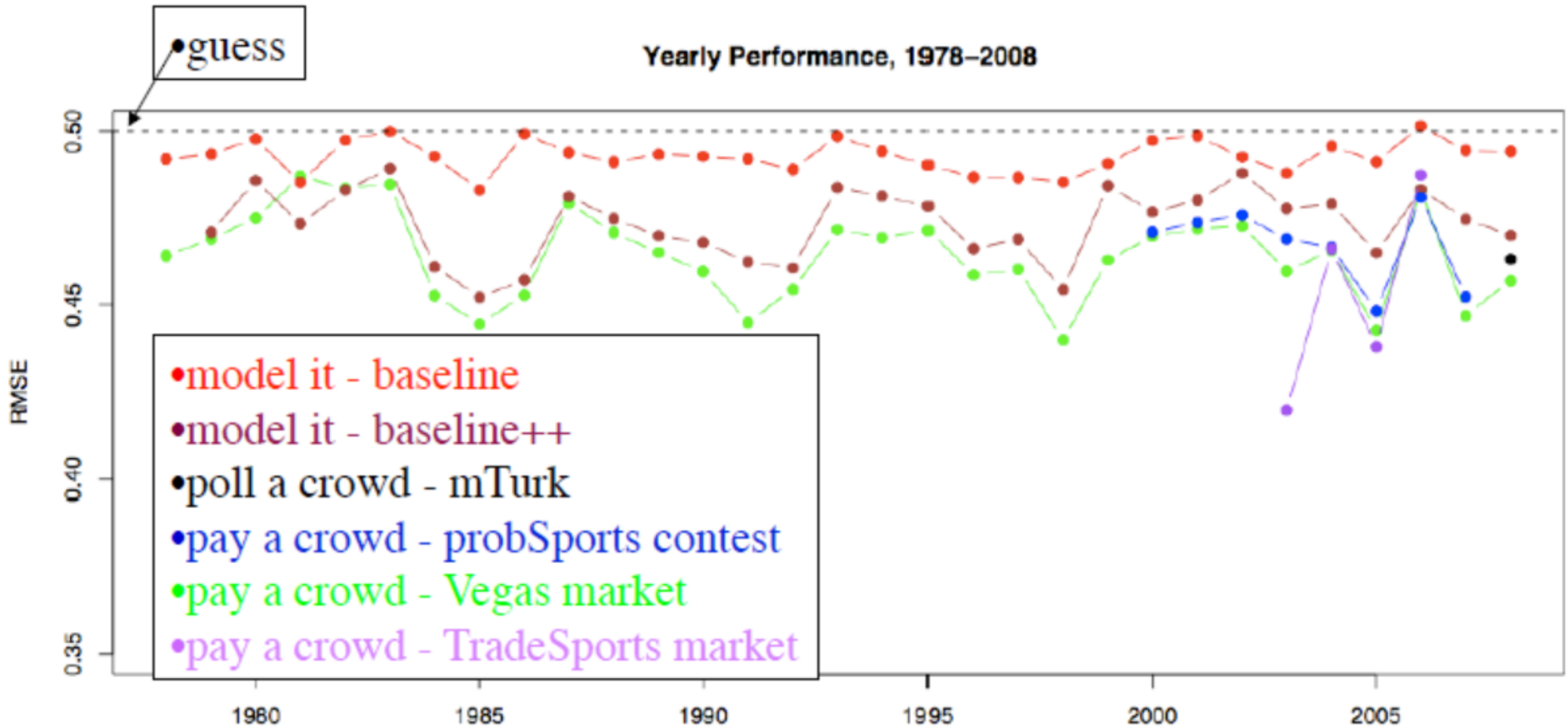
[Thanks: Yiling Chen]

Does it work?

Yes, evidence from real markets, laboratory experiments, and theory

- Racetrack odds beat track experts [Figlewski 1979]
- Orange Juice futures improve weather forecast [Roll 1984]
- I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995] [Rietz 1998][Berg 2001][Pennock 2002]
- HP market beat sales forecast 6/8 [Plott 2000]
- Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC' 03][Schmidt 2002]
- Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC' 01]
- Theory: "rational expectations" [Grossman 1981][Lucas 1972] Performance of Kelly bettors [Beygelzimer et al. 2012]
- Market games work [Servan-Schreiber 2004][Pennock 2001]

Does it work?


•guess
Yearly Performance, 1978–2008
RMSE
0.50
0.45
0.40
0.35
1980
1985
1990
1995
2000
2005
•model it - baseline
•model it - baseline++
•poll a crowd - mTurk
•pay a crowd - probSports contest
•pay a crowd - Vegas market
•pay a crowd - TradeSports market

Super Tuesday 2012

Prediction	Likelihood
VA: Romney	98.7%
MA: Romney	99.6%
VT: Romney	97.5%
ID: Romney	95.7%
GA: Gingrich	98.0%
OH: Romney	82.6%
OK: Santorum	88.6%
ND: Romney	72.1%
AK: Romney	82.6%
TN: Santorum	57.1%

- Predictions as of
  2:52 p.m. ET
  Monday, March 5, 2012

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ND: Romney	72.1%
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TN: Santorum	57.1%
total	872.5%
Expected num correct	**8.725**

- Predictions as of
  2:52 p.m. ET
  Monday, March 5, 2012

DESTINATION 2012 from YAHOO! NEWS

THE SIGNAL

YOUR FRIENDS' ACTIVITY

1-12 of 23

YOU ON YAHOO! NEWS

David Pennock

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Your Activity | Social: OFF | Options

Why we're happy we got one prediction wrong on Super Tuesday

By David Pennock | The Signal – Fri, Mar 9, 2012

Email | Recommend 3 | Tweet 3 | Share 1 | +1 0 | Print | Save this

On Super Tuesday, the Signal got 9 out of 10 election predictions correct, following up on an Oscar night where we got 3 of 4 award picks right. Not bad, but we really wish we didn't have those two annoying blemishes, right?

In fact, we're delighted! When the markets are functioning properly, we *expect* to get a certain percentage wrong every time. (The exact amount depends on how certain the probabilities are for each

ts we actutally expected to get about one wrong

have meant our confidence was too low, or what

e, but it's actually possible to get too many

Conditional probability
"logistic regression"
"combinatorial prediction markets"

MORE FROM THE SIGNAL »

Political Search Trends: The partisanship behind popular Web queries
19 hrs ago

- NCAA Predictions: Kentucky is favorite to win, Eas is wide open
- Twitter softens (slightly) on Santorum after Alabama and Mississippi wins
- Santorum's wins knock Gingrich out of the running in Louisiana
- Santorum wins Alabama and Mississippi (and you read it here first)

New Technology Lets You Learn a Language in as Little as 10 Days

Sponsored by Pimsleur_Approach

How Cruise Lines Fill All Those Unsold Cruise

Oscar Sunday 2015

Picture		Director		Best Actor		Best Actress		Supporting Actor		Supporting Actress		Adapted Screenplay		Original Screenplay	
Birdman	67%	Birdman (Alexandro G Inarritu)	57%	The Theory of Everything (Eddie Redmayne)	84%	Still Alice (Julianne Moore)	100%	Whiplash (J K Simmons)	100%	Boyhood (Patricia Arquette)	100%	The Imitation Game	77%	The Grand Budapest Hotel	74%
Boyhood	32%	Boyhood (Richard Linklater)	43%	Birdman (Michael Keaton)	15%	Gone Girl (Rosamund Pike)	0%	Birdman (Edward Norton)	0%	Birdman (Emma Stone)	0%	Whiplash	20%	Birdman	22%
American Sniper	1%	Foxcatcher (Bennett Miller)	0%	American Sniper (Bradley Cooper)	1%	Wild (Reese Witherspoon)	0%	Boyhood (Ethan Hawke)	0%	The Imitation Game (Keira	0%	The Theory of Everything	2%	Foxcatcher	2%
The Grand Budapest Hotel	0%	The Imitation Game (Morten	0%	The Imitation Game (Benedict	0%	Two Days, One Night (Marion	0%	Foxcatcher (Mark Ruffalo)	0%	Wild (Laura Dern)	0%	American Sniper	1%	Boyhood	1%
Other Movies	0	The Grand Budapest Hotel (Wes Anderson)	0%	Foxcatcher (Steve Carell)	0%	The Theory of Everything (Felicity Jones)	0%	The Judge (Robert Duvall)	0%	Into The Woods (Meryl Streep)	0%	Inherent Vice	0%	Nightcrawler	1%

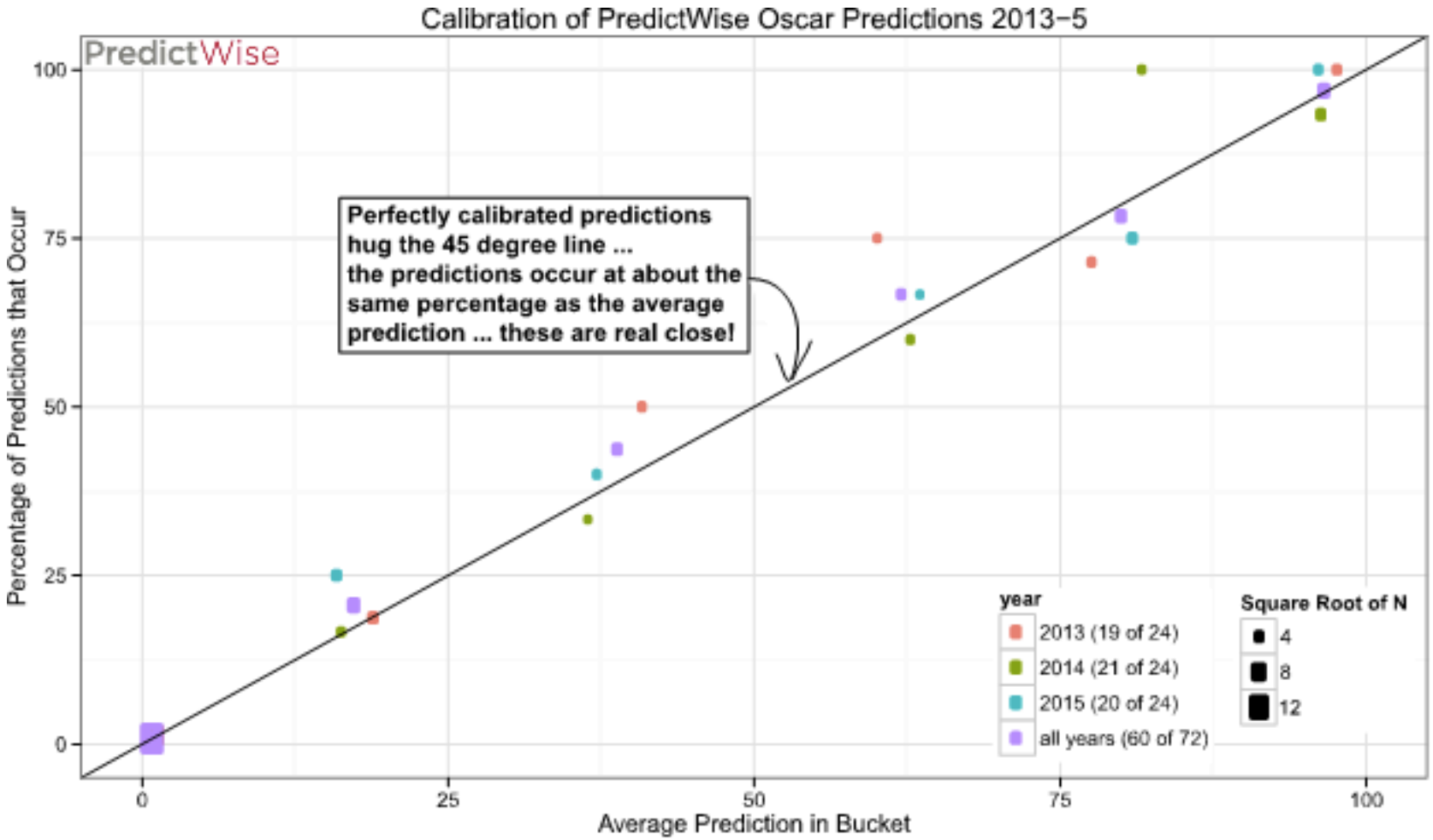
Animated Feature		Animated Short		Foreign Language		Documentary Feature		Documentary Short		Live Action Short		Original Song		Original Score	
How to Train Your Dragon 2	83%	Feast	79%	Ida	80%	Citizenfour	91%	Crisis Hotline Vets Press 1	85%	The Phone Call	78%	Selma (Glory)	90%	The Theory of Everything	67%
Big Hero 6	15%	The Dam Keeper	15%	Leviathan	14%	Virunga	8%	Joanna	13%	Boogaloo and Graham	17%	The lego Movie (Everything is Awesome)	7%	The Grand Budapest Hotel	31%
The Tale of Princess Kaguya	1%	The Bigger Picture	5%	Wild Tales	5%	Finding Vivien Maier	2%	Our Curse	1%	Parvaneh	3%	Glen Campell (Im Not Gonna Miss	2%	Interstellar	1%
Song of The Sea	1%	A Single Life	1%	Tangerines	0%	Last Days of	0%	White Earth	1%	Butter Lamp	2%	Begin Again (Lost	1%	The Imitation	1%
The Boxtrolls	0%	Me and My Moulton	0%	Timbuktu	0%	The Salt of the Earth	0%	The Reaper	1%	Aya	0%	Beyond the Lights (Grateful)	0%	Mr Turner	0%

Cinematography		Costume Design		Film Editing		Makeup and Hairstyling		Production Design		Sound Editing		Sound Mixing		Visual Effects	
Birdman	96%	The Grand Budapest Hotel	98%	Boyhood	78%	The Grand Budapest Hotel	82%	The Grand Budapest Hotel	96%	American Sniper	85%	Whiplash	43%	Interstellar	88%
The Grand Budapest Hotel	3%	Into The Woods	2%	Whiplash	21%	Guardians of the Galaxy	17%	The Imitation Game	2%	Interstellar	8%	American Sniper	37%	Dawn of the Planet of the Apes	10%
Mr Turner	0%	Maleficent	0%	American Sniper	1%	Foxcatcher	1%	Mr Turner	1%	Birdman	7%	Interstellar	12%	Guardians of the	2%
Unbroken	0%	Mr Turner	0%	Th Buda							1%	Birdman	8%	Captain America Winter Soldier	0%
Ida	0%	Inherent Vice	0%	The							0%	Unbroken	0%	X-Men Days of Future Past	0%

Feb 22, 2015: Expected **82%**
Feb 23: Got 20/24 right = **83%**

[Source: David Rothschild]

Oscar Sunday 2015

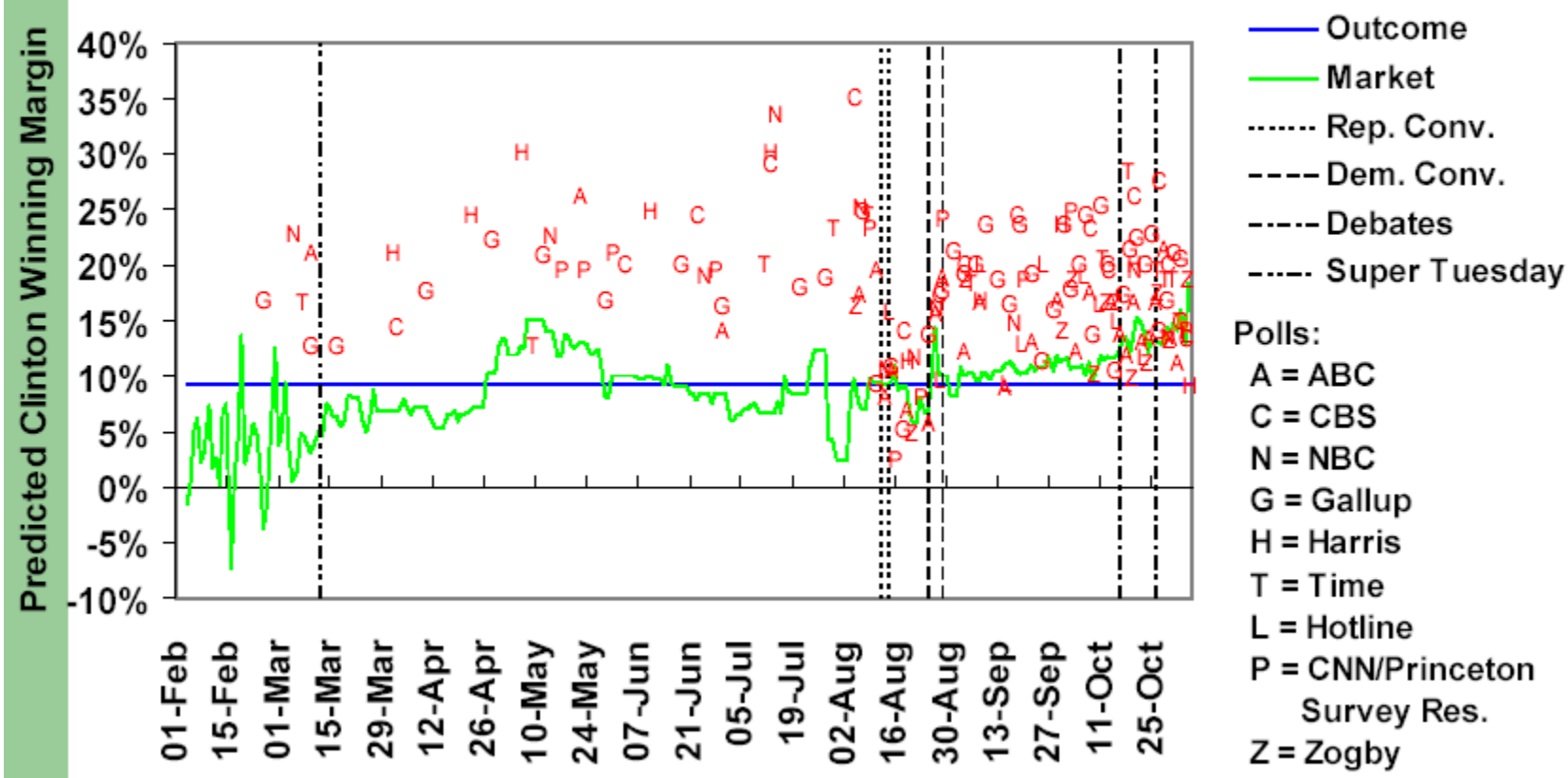


[Source: David Rothschild]

[Source: Berg, DARPA Workshop, 2002]

IEM versus Polls: 1996

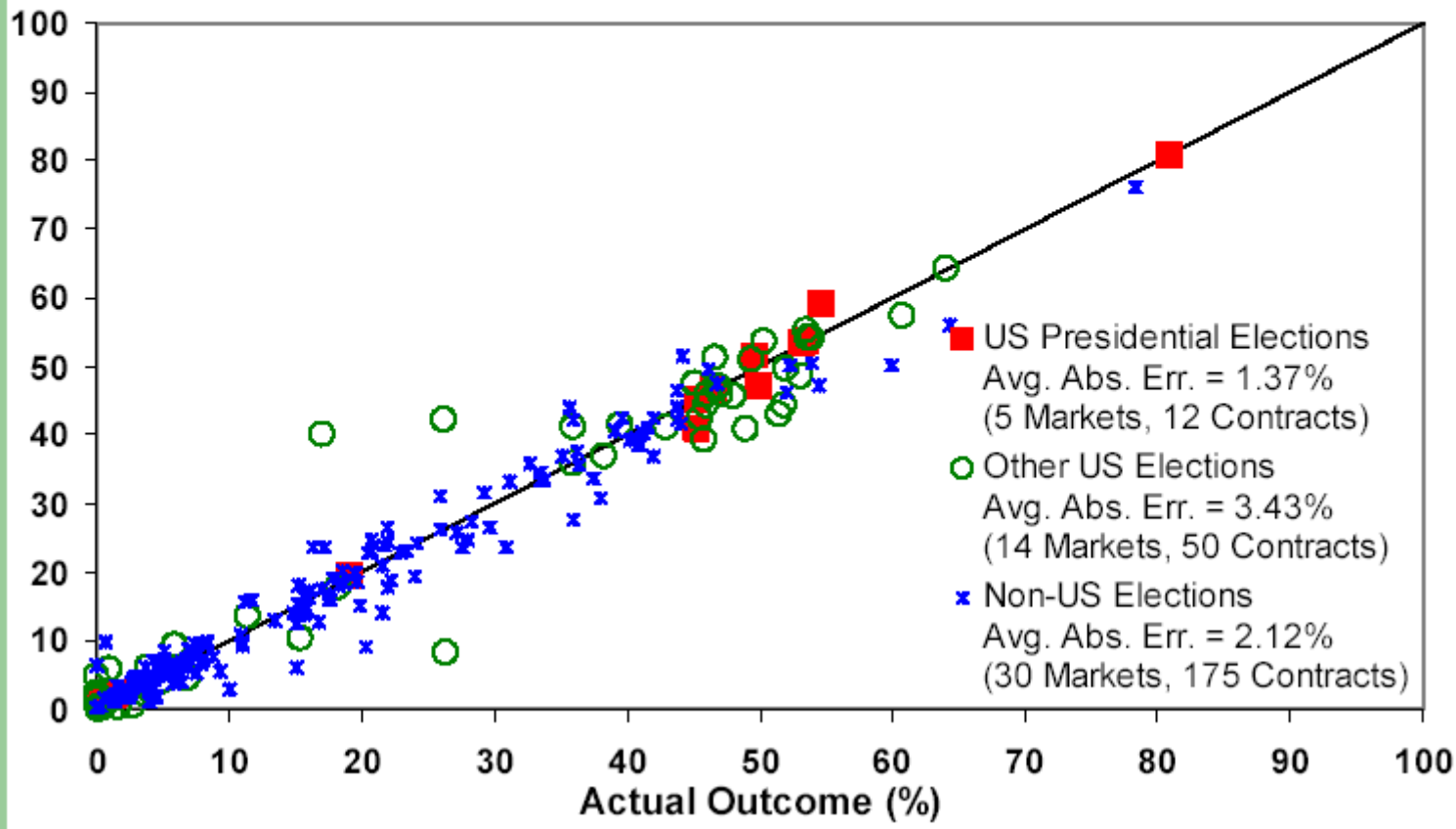
(Berg, Nelson and Rietz, 2001)



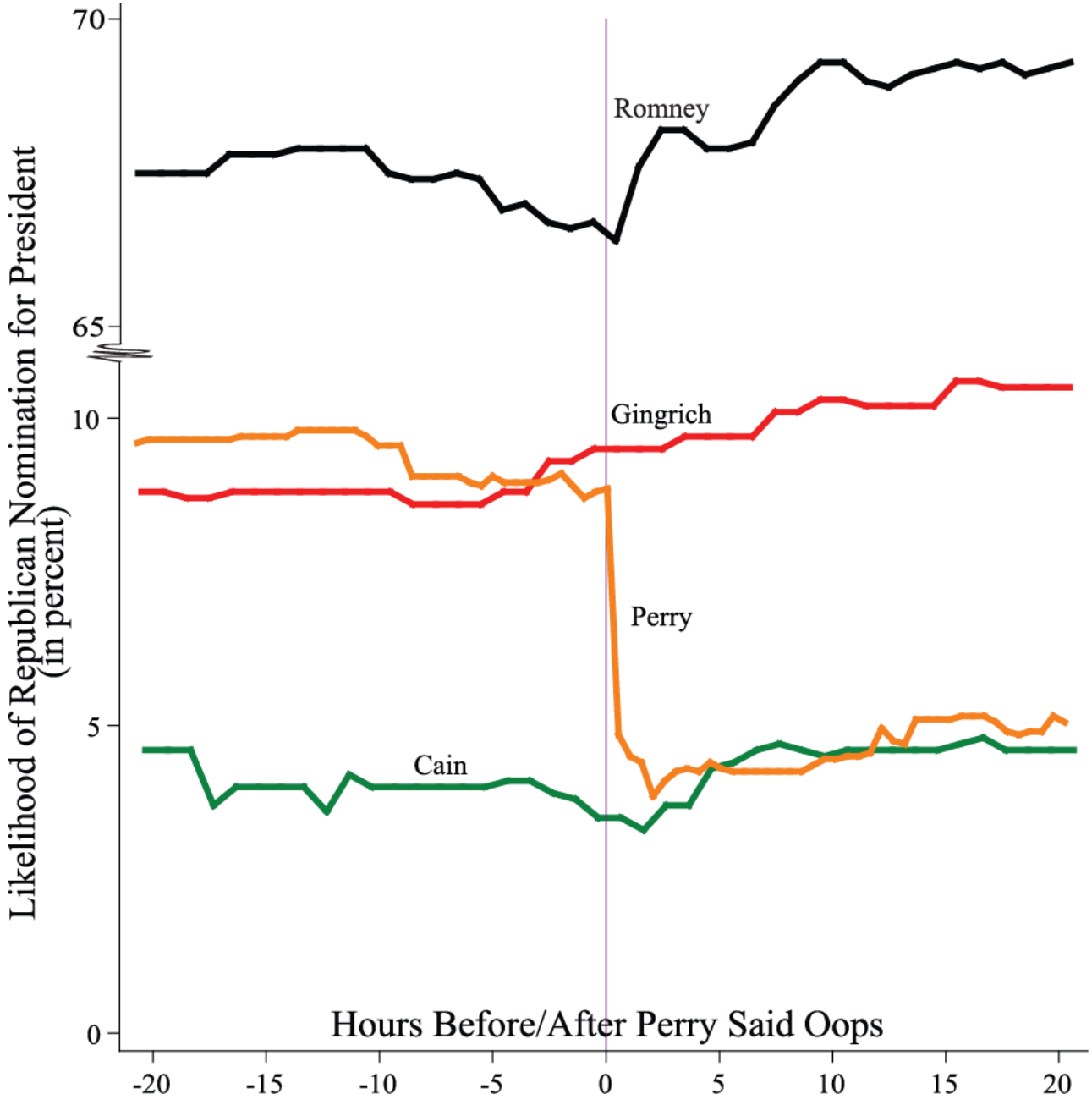
[Source: Berg, DARPA Workshop, 2002]

Predictive Accuracy

Berg, Forsythe, Nelson and Rietz (2001)



Real Time


70
65
10
5
0
Likelihood of Republican Nomination for President
(in percent)
Romney
Gingrich
Perry
Cain
Hours Before/After Perry Said Oops
-20
-15
-10
-5
0
5
10
15
20

Real Time

Prediction	Likelihood
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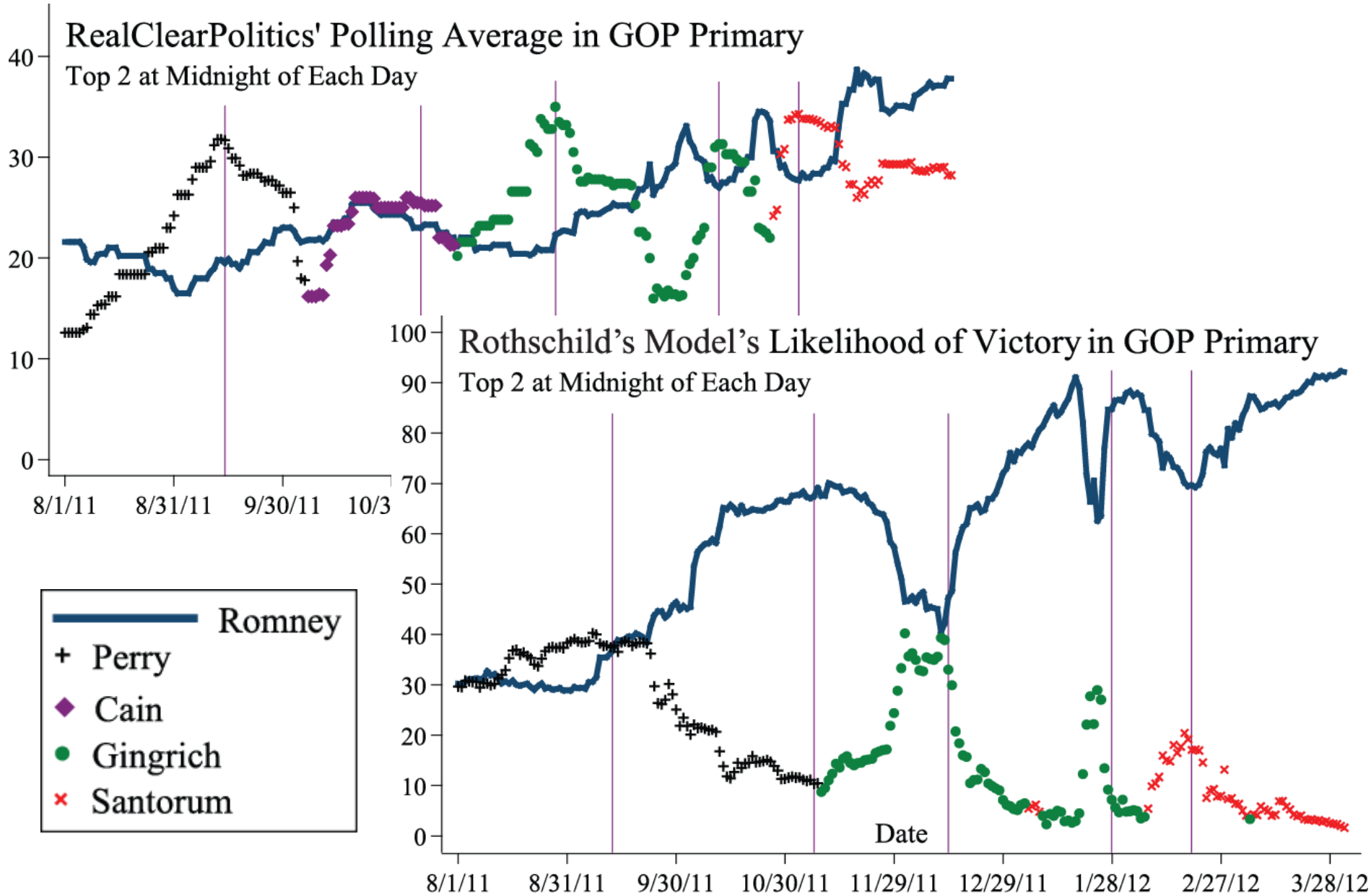
total	872.5%
Expected num correct	**8.725**

- Prior to MI primary, Santorum favored in Ohio: leading polls by wide margin
- Within hours of Romney winning MI, *OH* prediction market swung to Romney
- New York Times Nate Silver's poll-based forecast a couple days behind at least

GOP Primary: Polls vs Markets



Answering ?-mark headlines with data

After Rick Santorum's Sweep, How Worried Should Mitt Romney Be?

By George Stephanopoulos | ABC OTUS News – Wed, Feb 8, 2012

Markets still lukewarm on Santorum after candidate's three-state sweep

By David Rothschild | The Signal – Wed, Feb 8, 2012

[Santorum's] odds of winning the nomination rose slightly, to a non-negligible 11.3 percent likelihood to Romney's 79.5

- Reuters: "The dominant narrative, the day after the presidential election, is the **triumph of the quants**."
- Mashable: "here is the absolute, undoubted winner of this election: Nate Silver and his running mate, big data."
- ReadWrite: "This is about the **triumph of machines and software over gut instinct**"

Conditionals

- Pr(Clinton wins election)
  ≈ Pr(Clinton wins Dem nomination & election)
  = 47.6 - 48.5%
- Pr(Clinton wins Dem nomination)
  = 72.5 - 75.2%
- Pr(election|nomination) =elec&nom/nom
  = **Probability she wins election *if* nominated**
  = 0.48/0.74
  = **63.3 - 66.9%**

Conditionals

- Causes of win
  - Probability she wins election *if* nominated
  - Probability she wins if unemployment <7%
  - Probability she wins if ISIS controls Mosul
- *Effects* of win
  - Unemployment rate if she wins
  - War casualties if she wins
  - Tax rates, S&P, GDP, currency, global temps, consumer confidence, inflation, deficit, ...

Conditionals

- Cancer rates if landfill reduced
- Cancer rates if carbon emissions reduced
- GDP if carbon tax
- global temps if carbon tax
- ...
- “happiness” if X

Decision Markets

- Condition on a *decision* [Hanson 1999]
- Example 1: election
- Example 2: fire CEO?
  Microsoft stock price if CEO fired
  >? =? <?
  Microsoft stock price if CEO not fired
- Guidance to actually make the decision

Correlation ≠ Causation

- In a world where CEO is fired, company likely doing poorly
- In a world where Ron Paul wins Republican nomination, some new development (e.g., Snowden II) may make his general election candidacy especially strong. Different than simply anointing him the nominee.

Thought Experiment = Causation

- If CEO firing is random, conditional does reflect (beliefs about) causation
- If CEO firing is deterministic based on market, incentives get distorted
- As long as CEO firing is partially random, can recover (beliefs about) causation [Chen et al. 2011]

Prediction Markets

Work for events that are

- Objective
- Verifiable
- Completely describable ("clarity test")
  Harder than it seems
  - Reform party nom
  - Bush v. Gore
  - North Korean missile

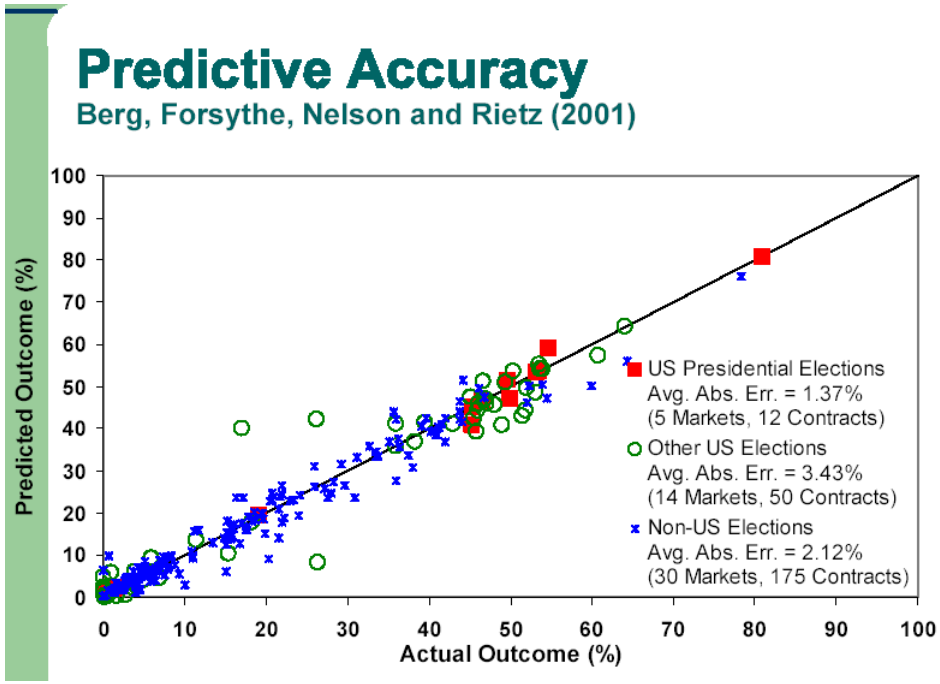
Don't Work for events that are

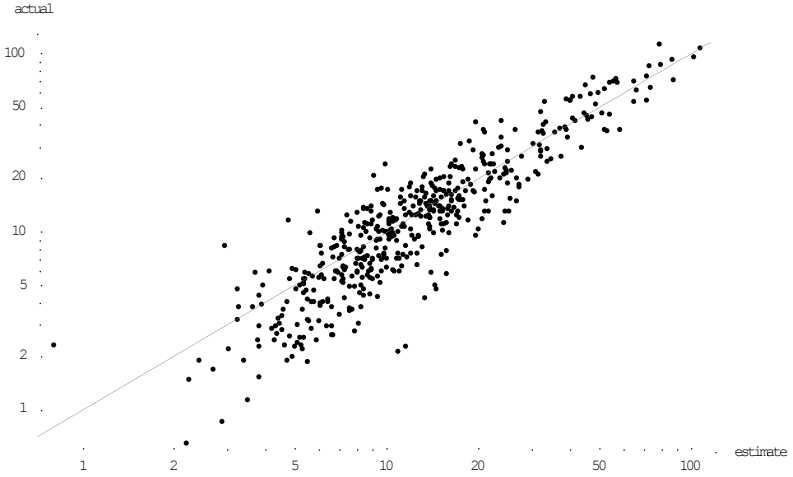
- Subjective (movie, restaurant)
- End of the world
- e.g. "First female US Pres: 2016? 2024? 3000? Never"
- Unverifiable
- Hard to describe
- See: *Peer prediction*
  "Bayesian truth serum"
  And be skeptical

Pay a crowd

With *money* With *points*

IEM: 237 Candidates HSX: 489 Movies

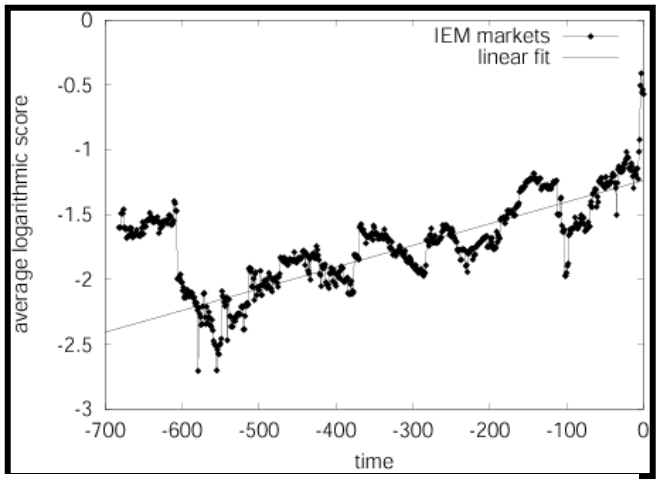


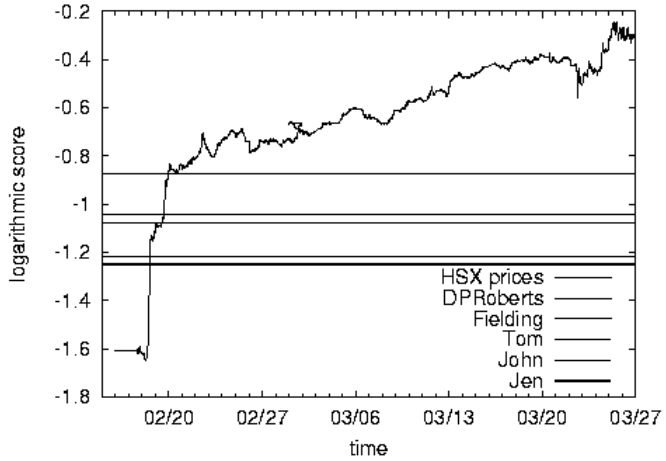


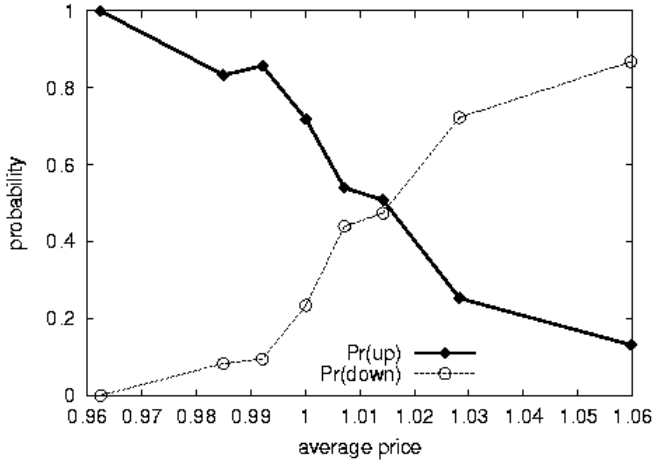
Foresight Exchange, HSX.com, InklingMarkets.com, Microsoft Prediction Lab, CrowdCast, Yahoo! Predictalot, Newsfutures.com, CasualObserver.net, FTPredict.com, ProTrade.com, StorageMarkets.com, TheSimExchange.com, TheWSX.com, Alexadex, Celebdaq, Cenimar, BetBubble, Betocracy, CrowdIQ, MediaMammon,Owise, PublicGyan, RIMDEX, Smarkets, Trendio, TwoCrowds, ...

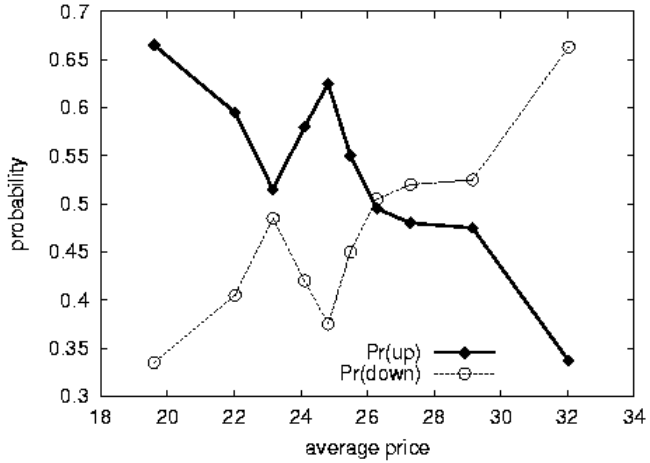
Pay a crowd

With *money* With *points*





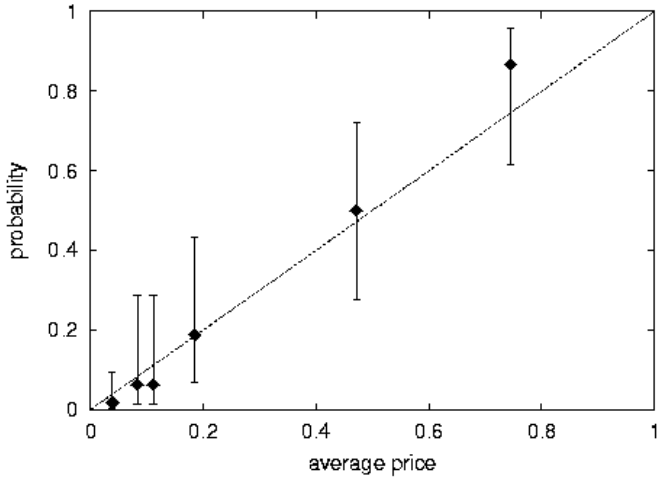




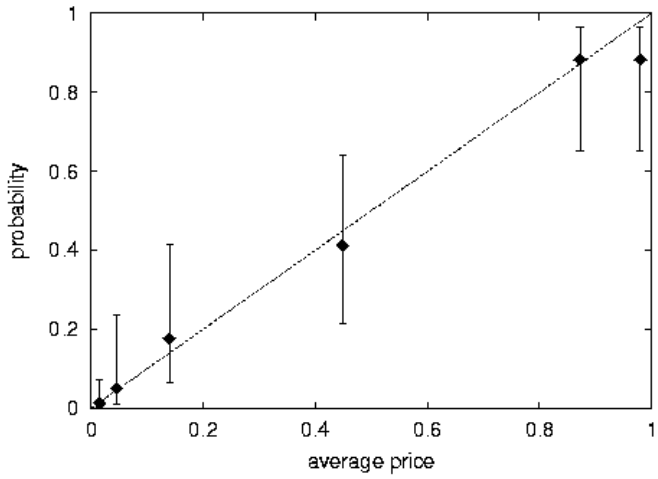
Real markets vs. market games

probabilistic forecasts

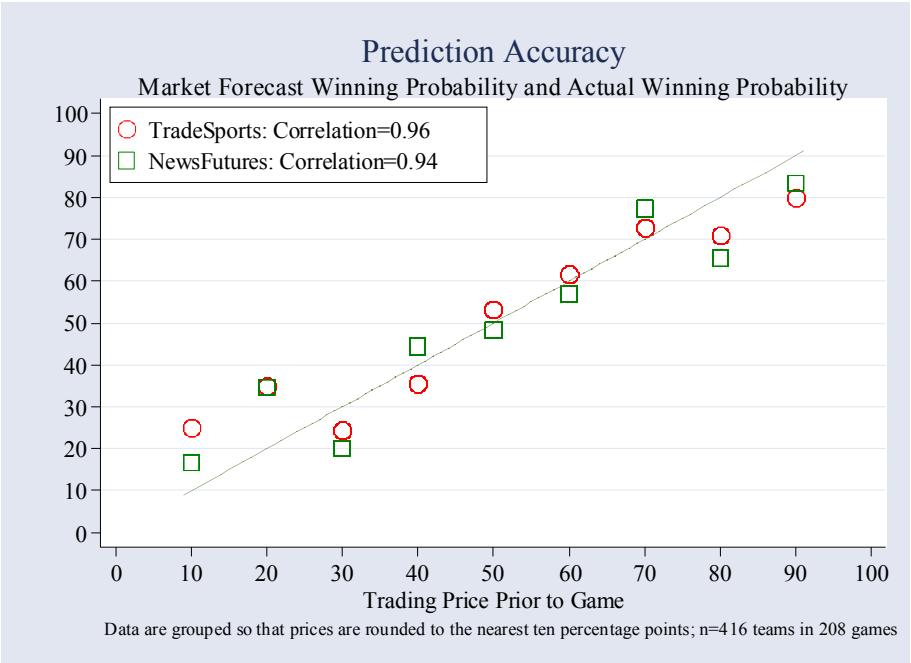
HSX

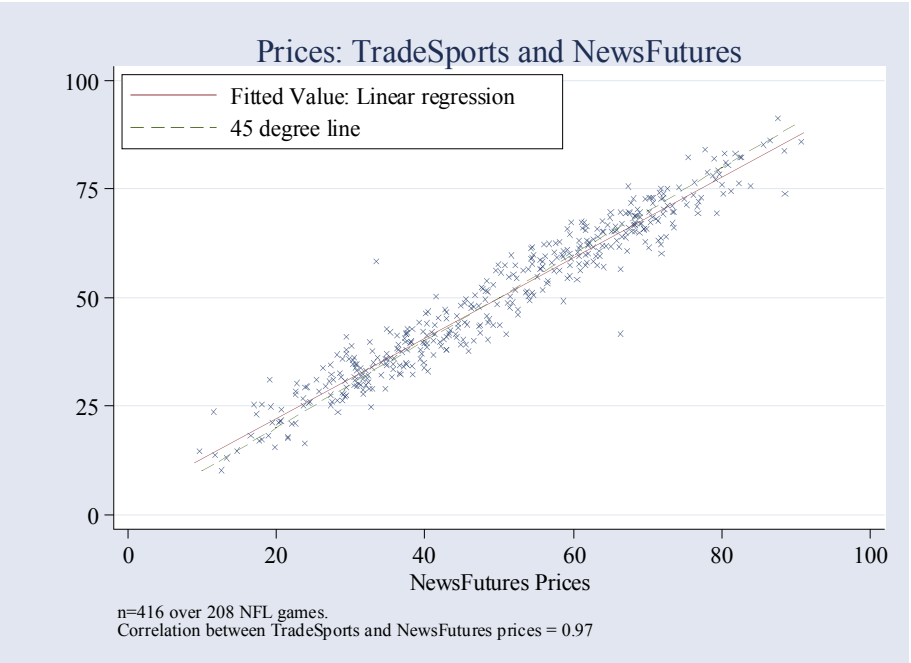


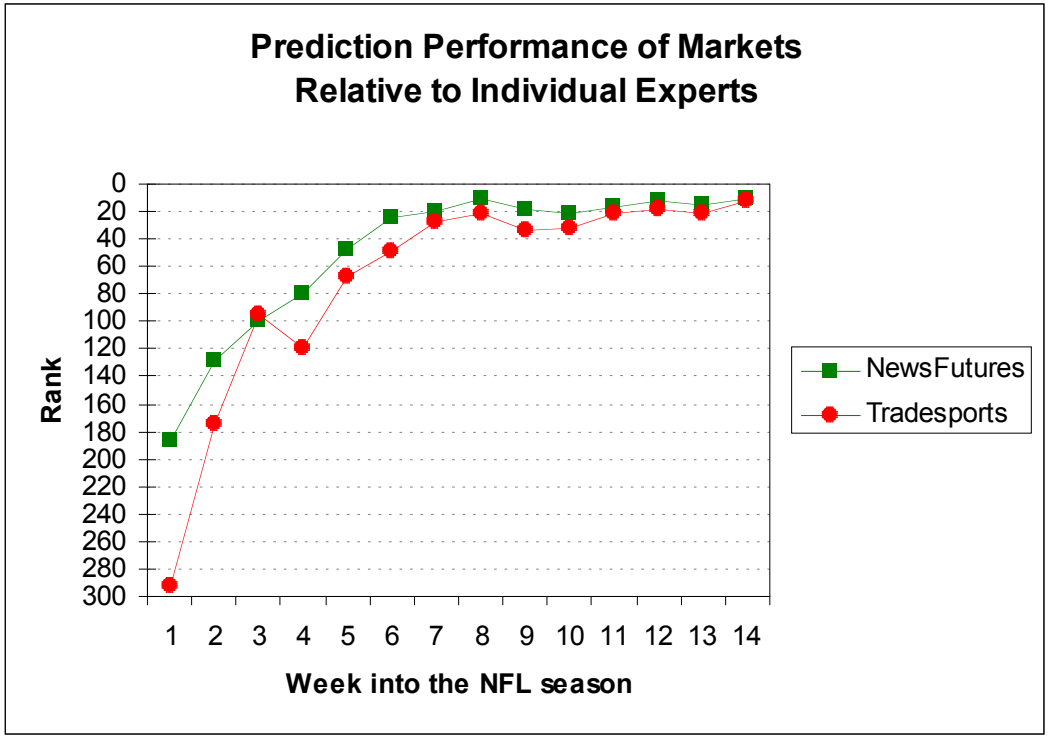
FX, F1P6



forecast source	avg log score
F1P6 linear scoring	**-1.84**
F1P6 F1-style scoring	**-1.82**
betting odds	-1.86
F1P6 flat scoring	-2.03
F1P6 winner scoring	-2.32


Prediction Accuracy
Market Forecast Winning Probability and Actual Winning Probability
TradeSports: Correlation=0.96
NewsFutures: Correlation=0.94
0
10
20
30
40
50
60
70
80
90
100
Trading Price Prior to Game
Data are grouped so that prices are rounded to the nearest ten percentage points; n=416 teams in 208 games


Prices: TradeSports and NewsFutures
Fitted Value: Linear regression
45 degree line
0
25
50
75
100
NewsFutures Prices
n=416 over 208 NFL games.
Correlation between TradeSports and NewsFutures prices = 0.97


Prediction Performance of Markets
Relative to Individual Experts
Rank
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Week into the NFL season
NewsFutures
Tradesports

Combinatorial Predictions

- 22% chance Romney will win in Iowa but Obama will win the national election
- 75.7% chance the same party will win both Michigan and Ohio
- 48.3% chance Obama gets 300 or more Electoral College votes
- 12.3% chance Obama will win between 6 and 8 states that begin with the letter M
- Path of blue from Canada to Mexico

Some Counting

- 54 “states”: 48 + DC + Maine (2), Nebraska (3)
- 2^{54} = 18 quadrillion possible outcomes
- $2^{2^{54}} \sim 10^{18008915383333485}$ distinct predictions
  More than a googol, less than a googolplex
- NOT independent

Real Conditionals: How pivotal was Ohio?

- Day after 1st debate (October 4)
- If Obama wins Ohio, 90% he'll win the election
- If Obama wins Florida, 95%; Virginia, 92%
- If Romney wins Ohio, 41% he'll win election; Florida, 32%; Virginia 31%

PredictWiseQ

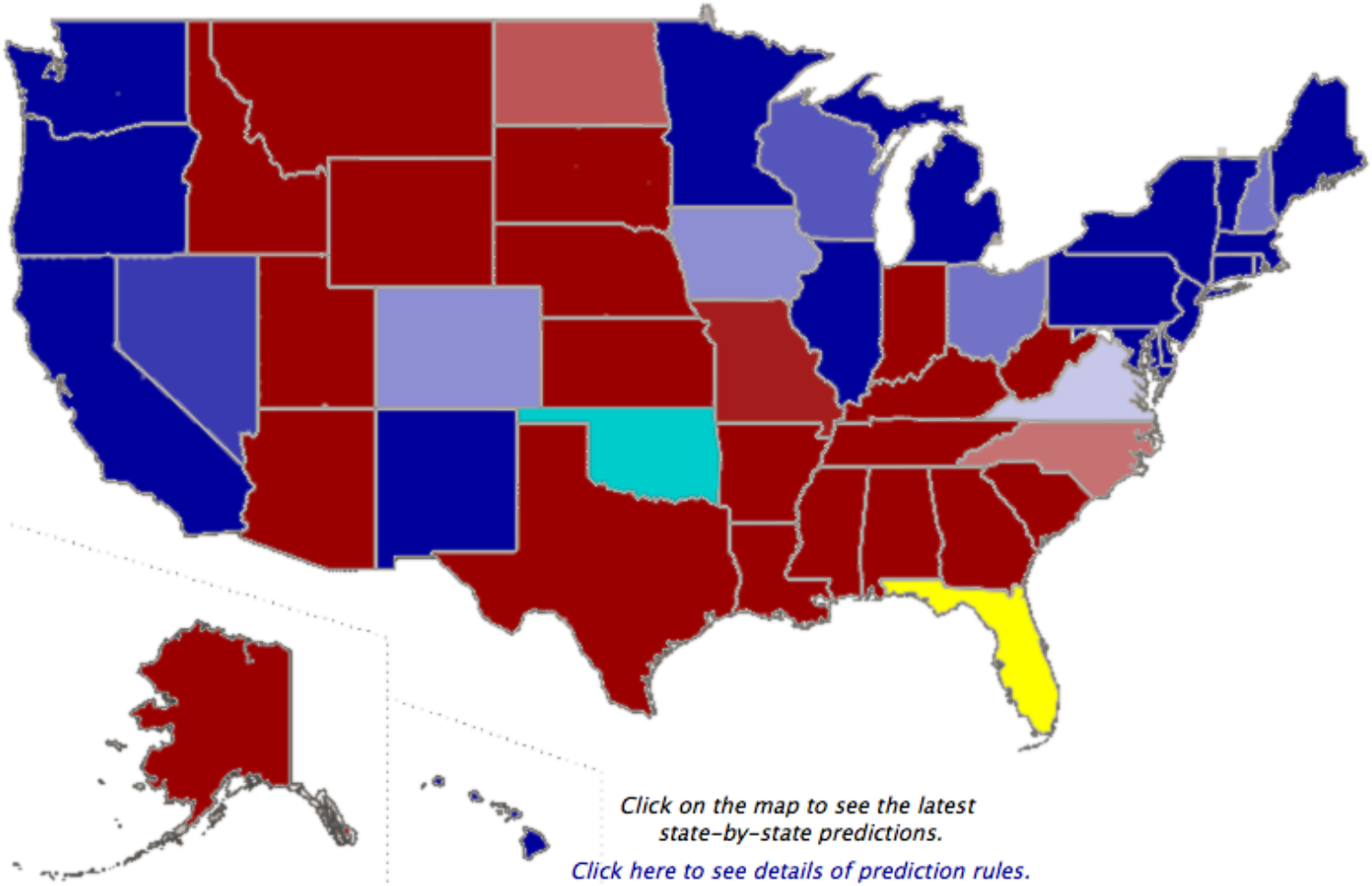
- In theory, combo PM yields billions more predictions "for free". What about in practice?
- A combinatorial prediction market for the election: **http://PredictWiseQ.com**
- Fully working beta & field test
  - Sep 16 – Nov 6, 2012
  - 680 users; 437 made at least one trade
  - 3137 trades in all 23 possible prediction categories
  - 514 distinct security types were bought
    261 of these were traded by a unique user
  - Empirical paper in ACM EC'13
  - Market maker algorithm details in ACM EC'12

WiseQ Game – Elections 2012 *(Beta)*

MAKE A PREDICTION

President | Senate | Governors | Trending | My Portfolio | My Leagues | Forum | FAQ

Welcome,
pennock!
Logout



Click on the map to see the latest state-by-state predictions.

Click here to see details of prediction rules.

CURRENT ODDS

OK President

REPUBLICAN
99.9 %

DEMOCRAT
0.1 %

YOUR POINTS

EXPECTED RETURN
1011.5

AVAILABLE
799.9

WISEQ SCORE
102.19

My Portfolio »

http://PredictWiseQ.com

WiseQ Game – Elections 2012 *(Beta)*

MAKE A PREDICTION

President | Senate | Governors | Trending | My Portfolio | My Leagues | Forum | FAQ

Welcome,
pennock!
Logout

MAKE A PREDICTION

Make a prediction on:

President – 2 States Same

State: Ohio

State: Pennsylvania

My Prediction:

the same party will win both **Ohio** and **Pennsylvania**

Current Odds	Investment	Return if Correct
74.4 %	50	66.08

Buy Prediction

CURRENT ODDS

OK President

REPUBLICAN
99.9 %

DEMOCRAT
0.1 %

YOUR POINTS

EXPECTED RETURN
1011.5

AVAILABLE
799.9

WISEQ SCORE
102.19

My Portfolio »

Click on the map to see the latest state-by-state predictions.

Click here to see details of prediction rules.

http://PredictWiseQ.com

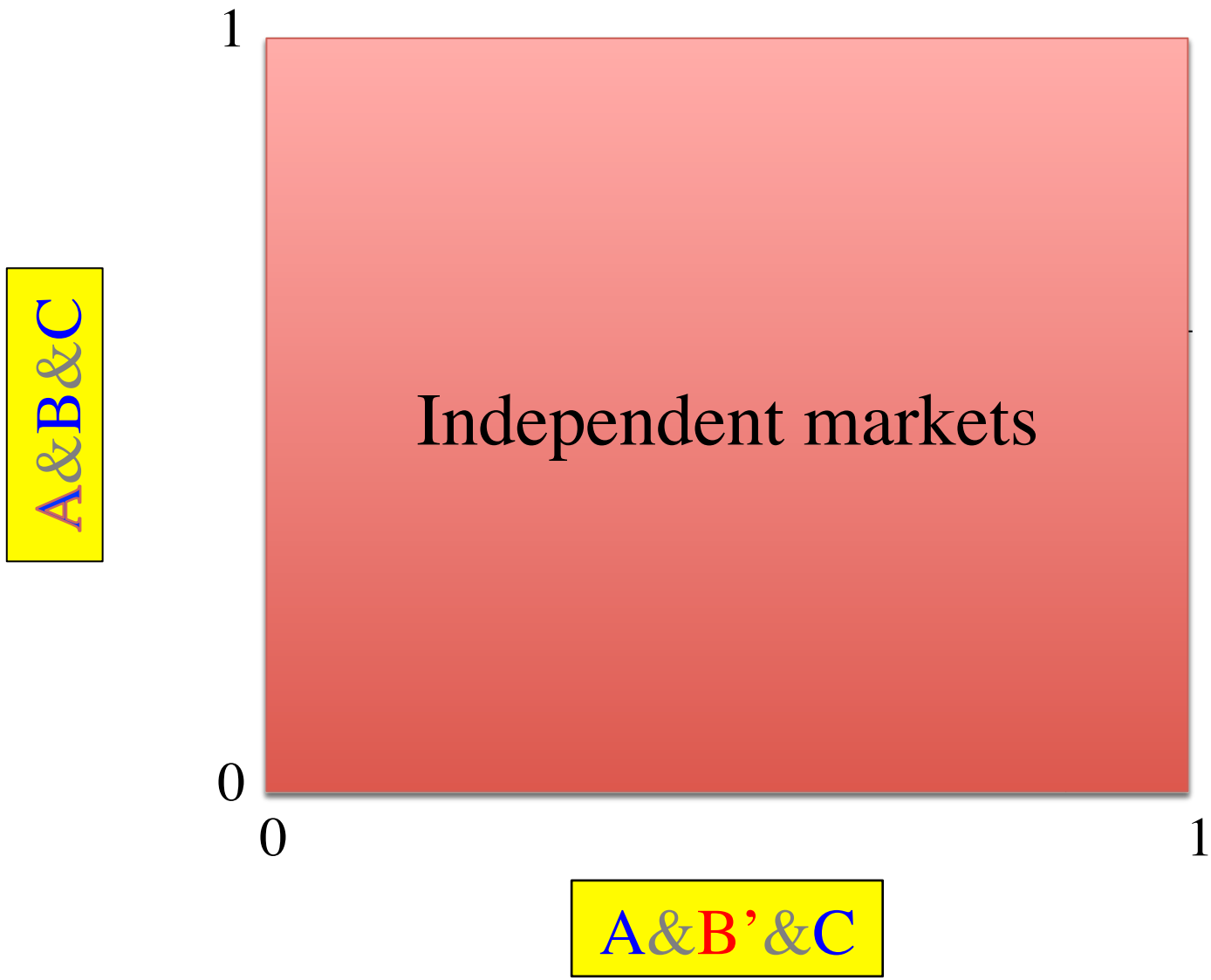
Why Expressiveness?

- Dem Pres, Dem Senate, Dem House
  Dem Pres, Dem Senate, GOP House
  Dem Pres, GOP Senate, Dem House
  Dem Pres, GOP Senate, GOP House...
- Dem Pres
  Dem House
  Dem wins >=270 electoral votes
  Dem wins >=280 electoral votes...

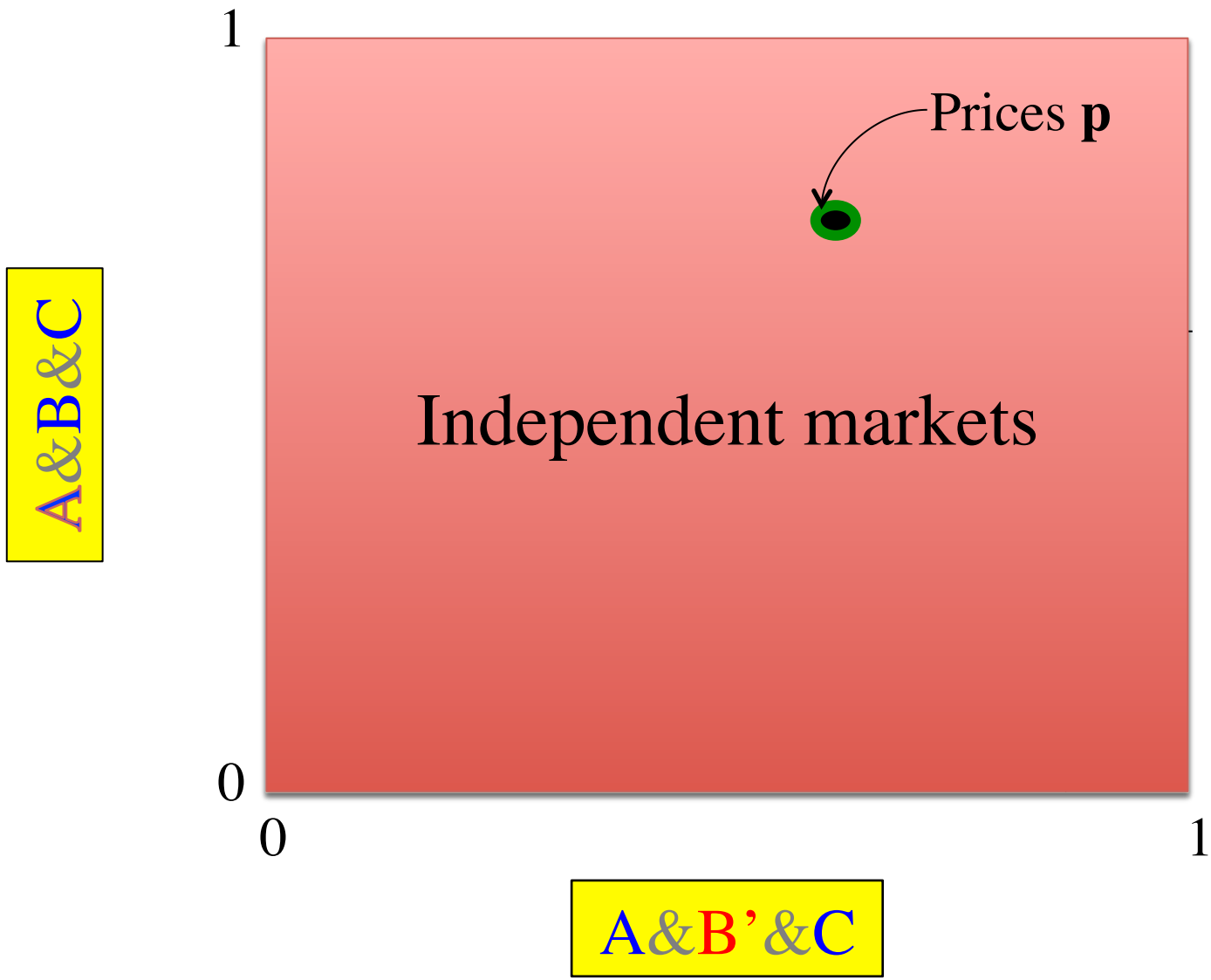
Standard vs Combinatorial

- Industry standard: Ignore relationships
  Treat them as independent markets
- Las Vegas sports betting
  Kentucky horseracing
  Wall Street stock options
  High Street spread betting
- Combinatorial
  - Automatic information propagation: reward traders for information, not computational horsepower

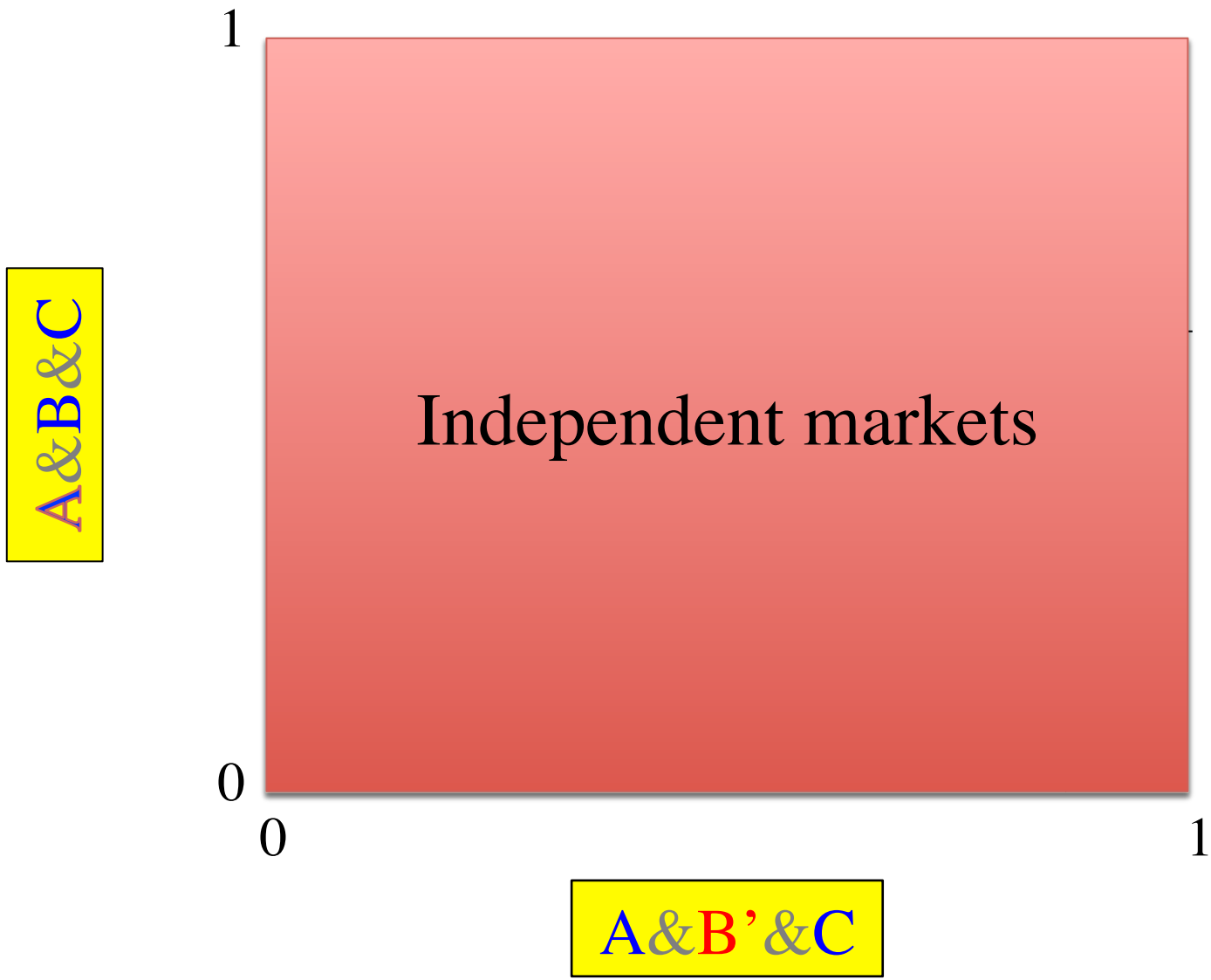
Consistent pricing



Consistent pricing



Consistent pricing



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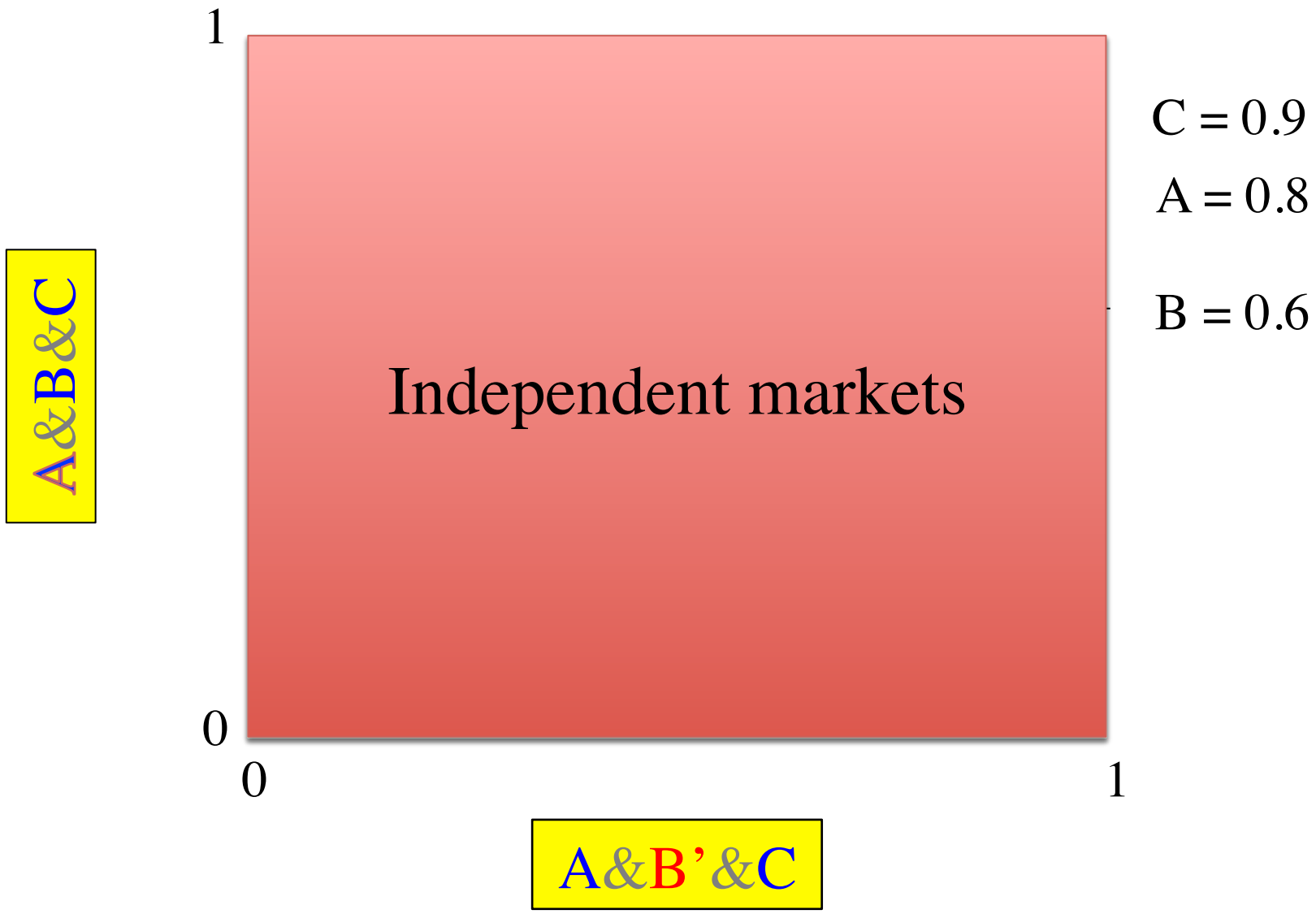
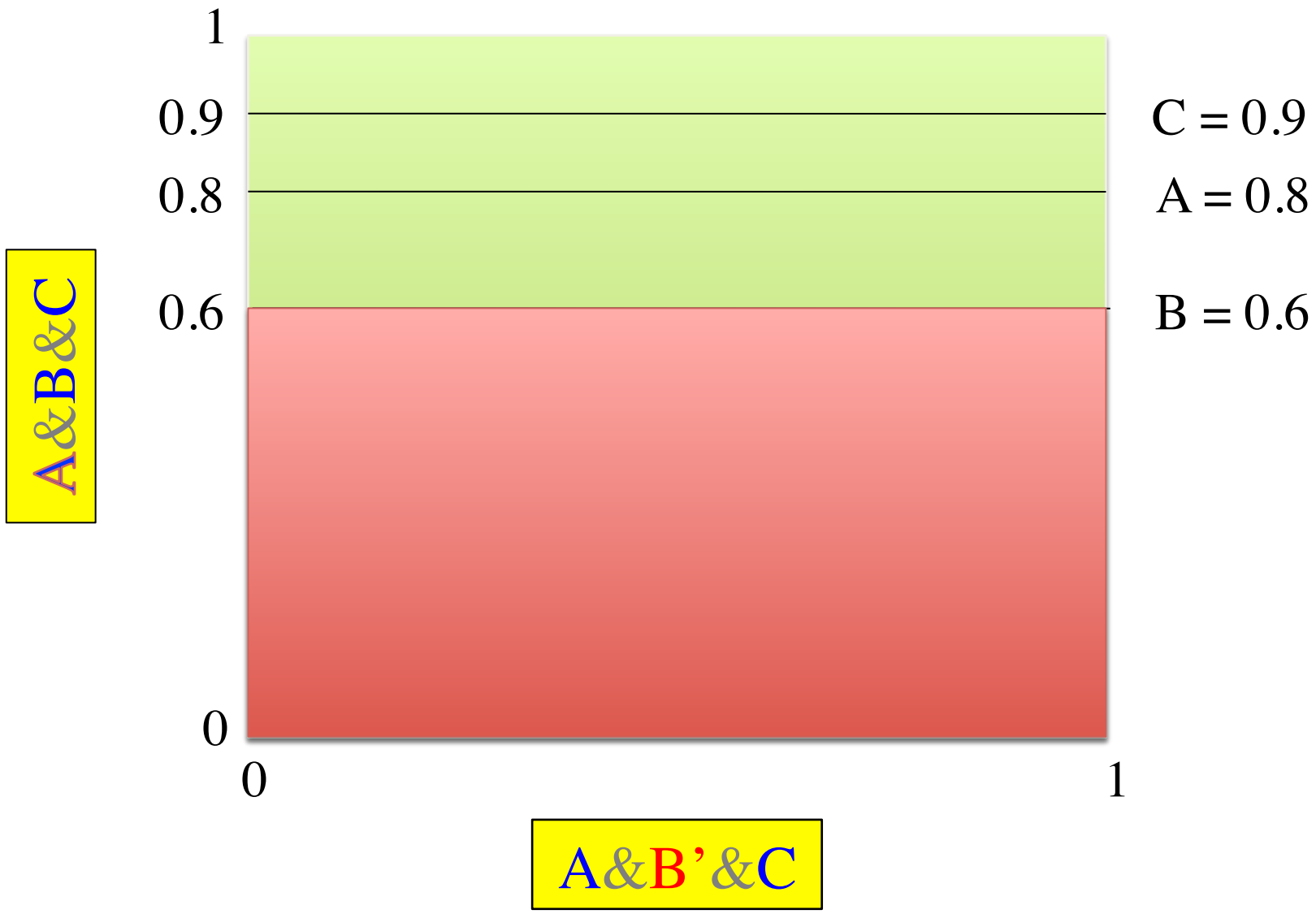
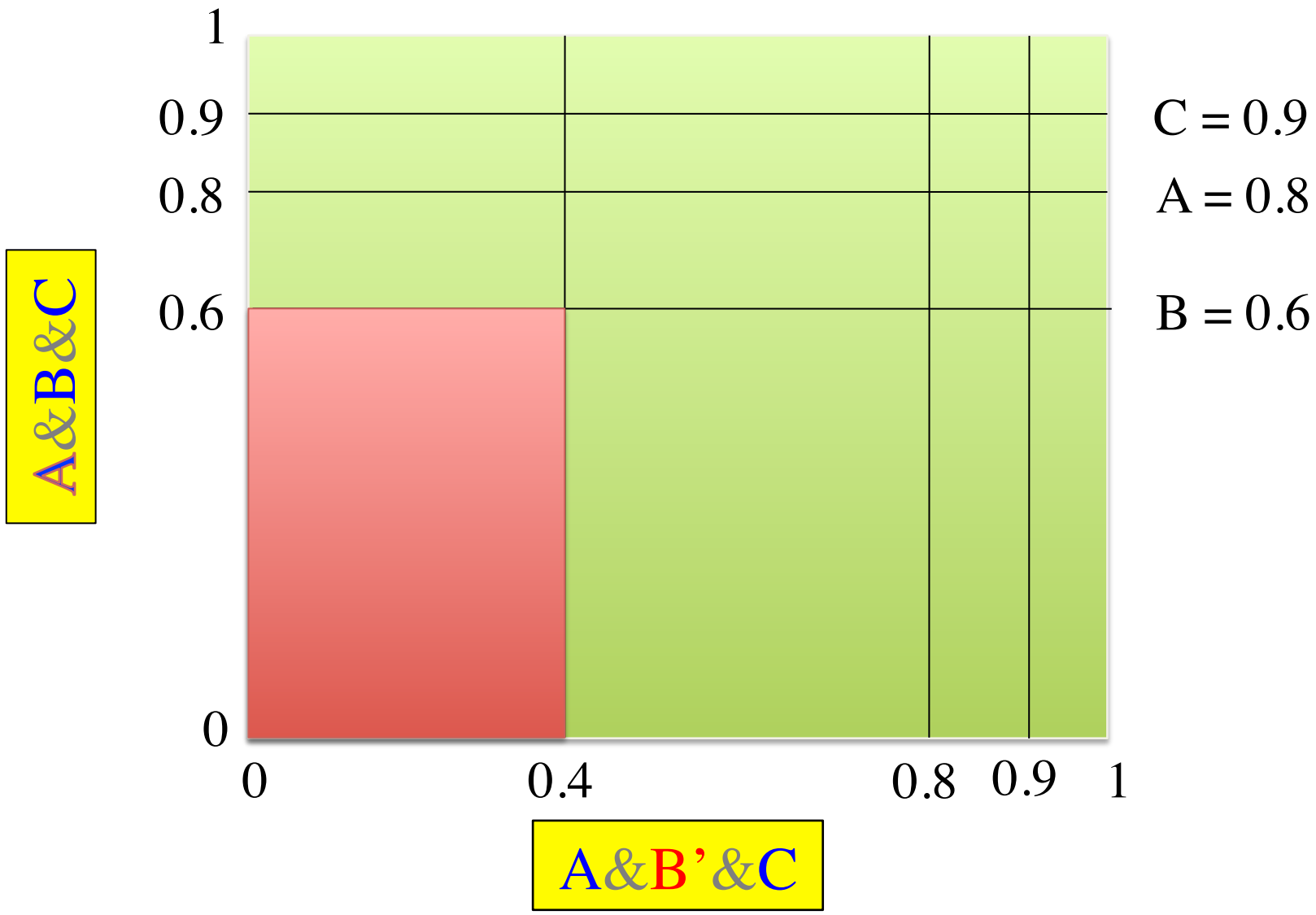


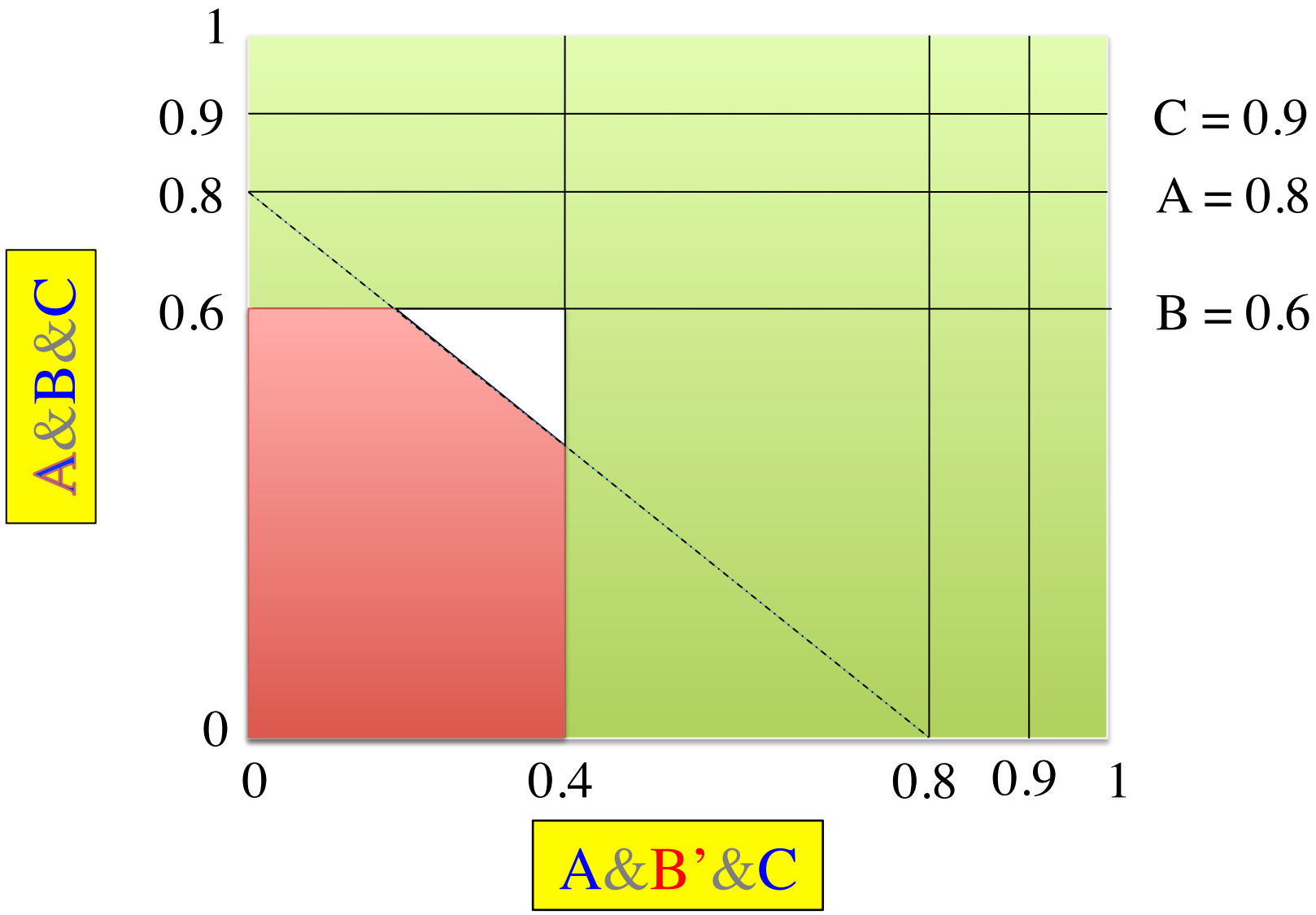
Consistent pricing



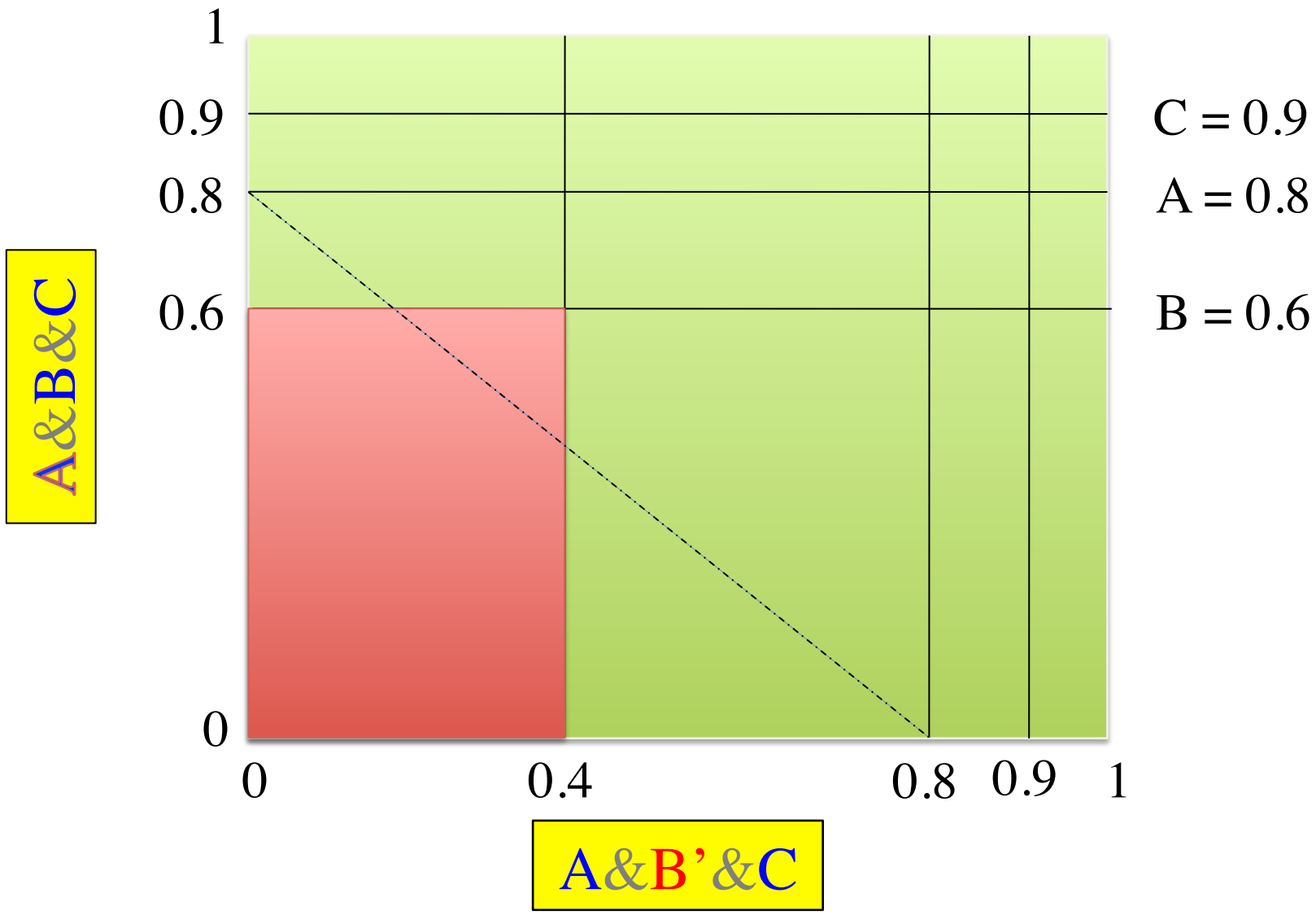
Consistent pricing



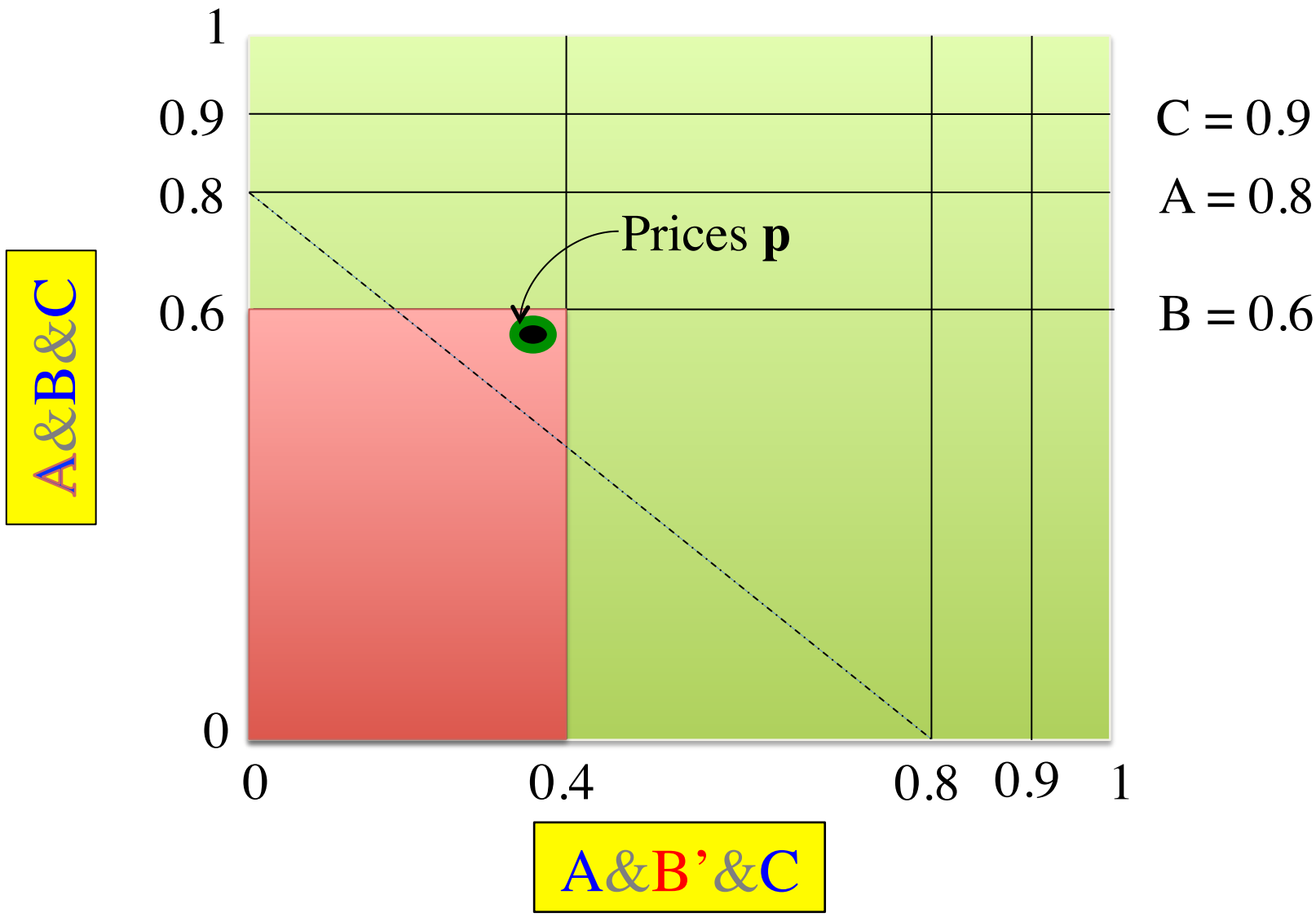
Consistent pricing



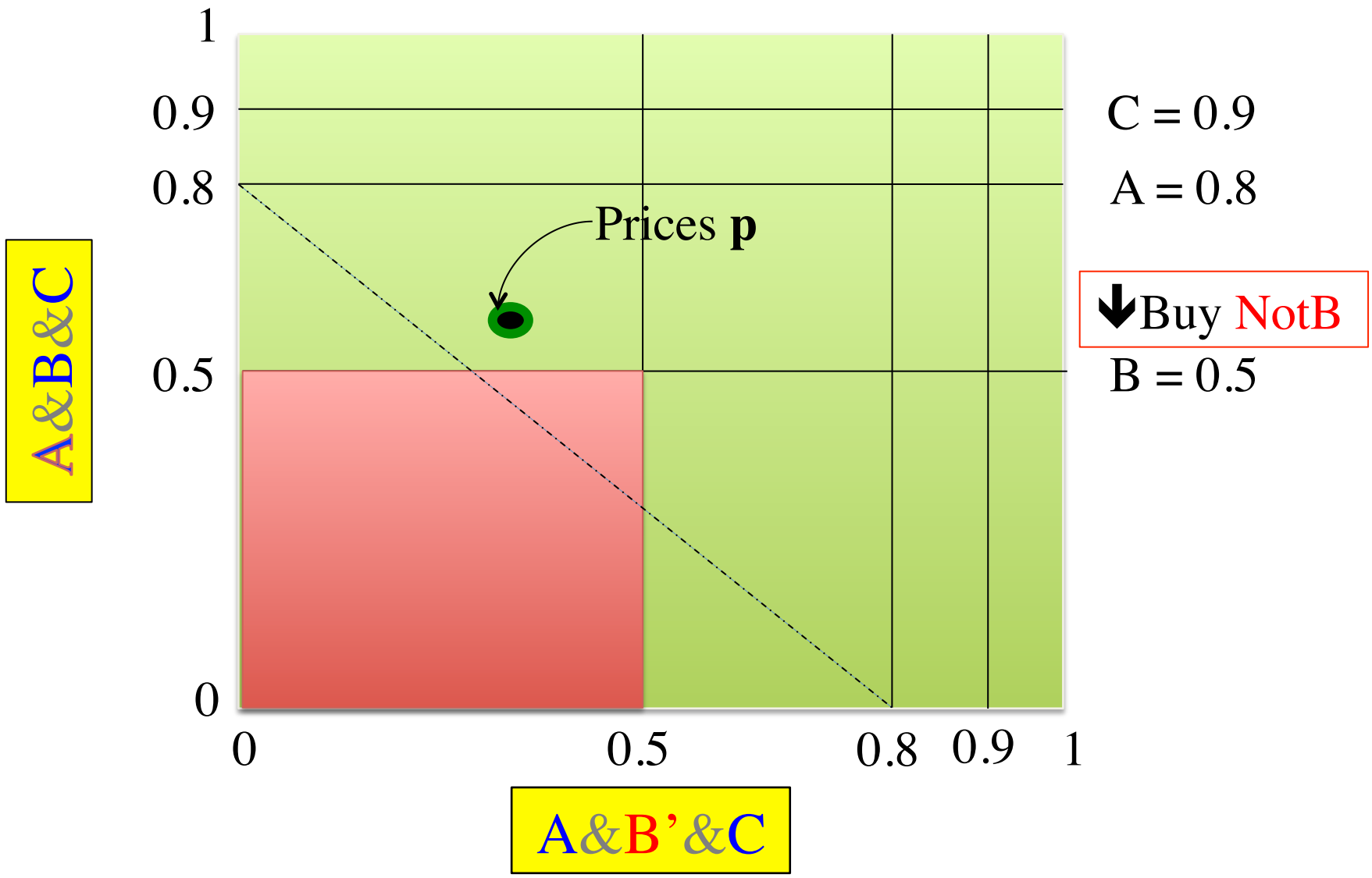
Approximate pricing



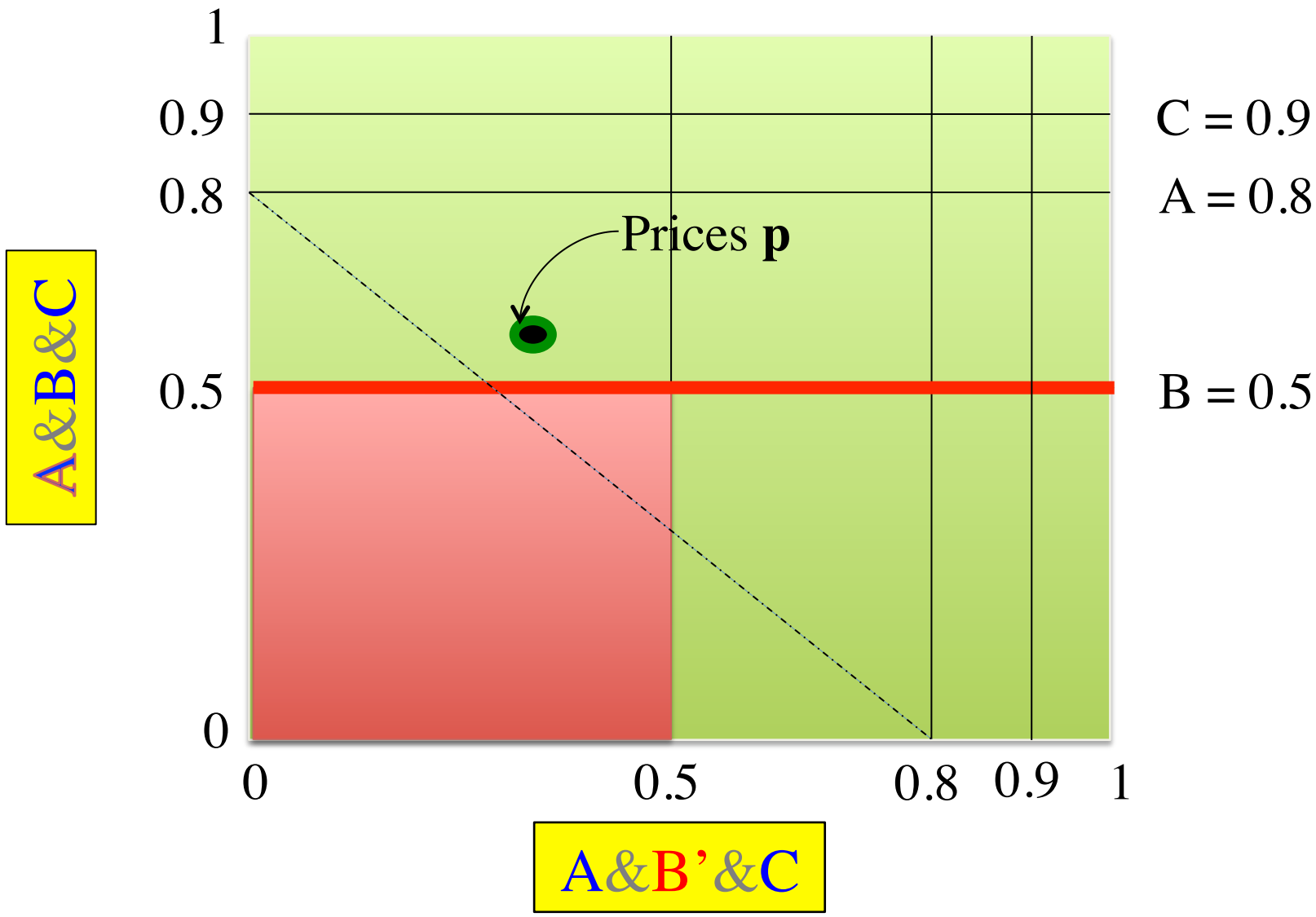
Approximate pricing



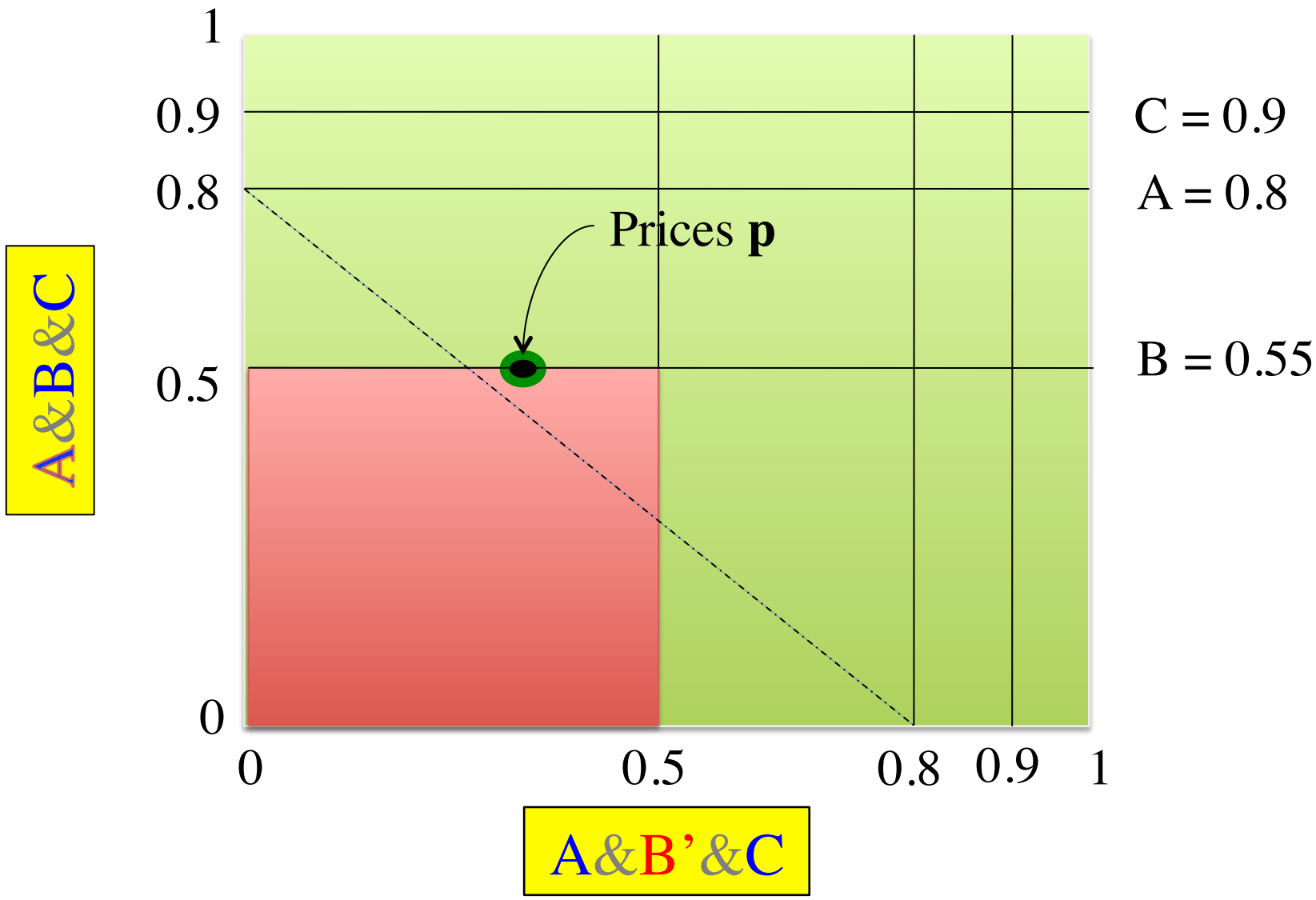
Approximate pricing



Approximate pricing

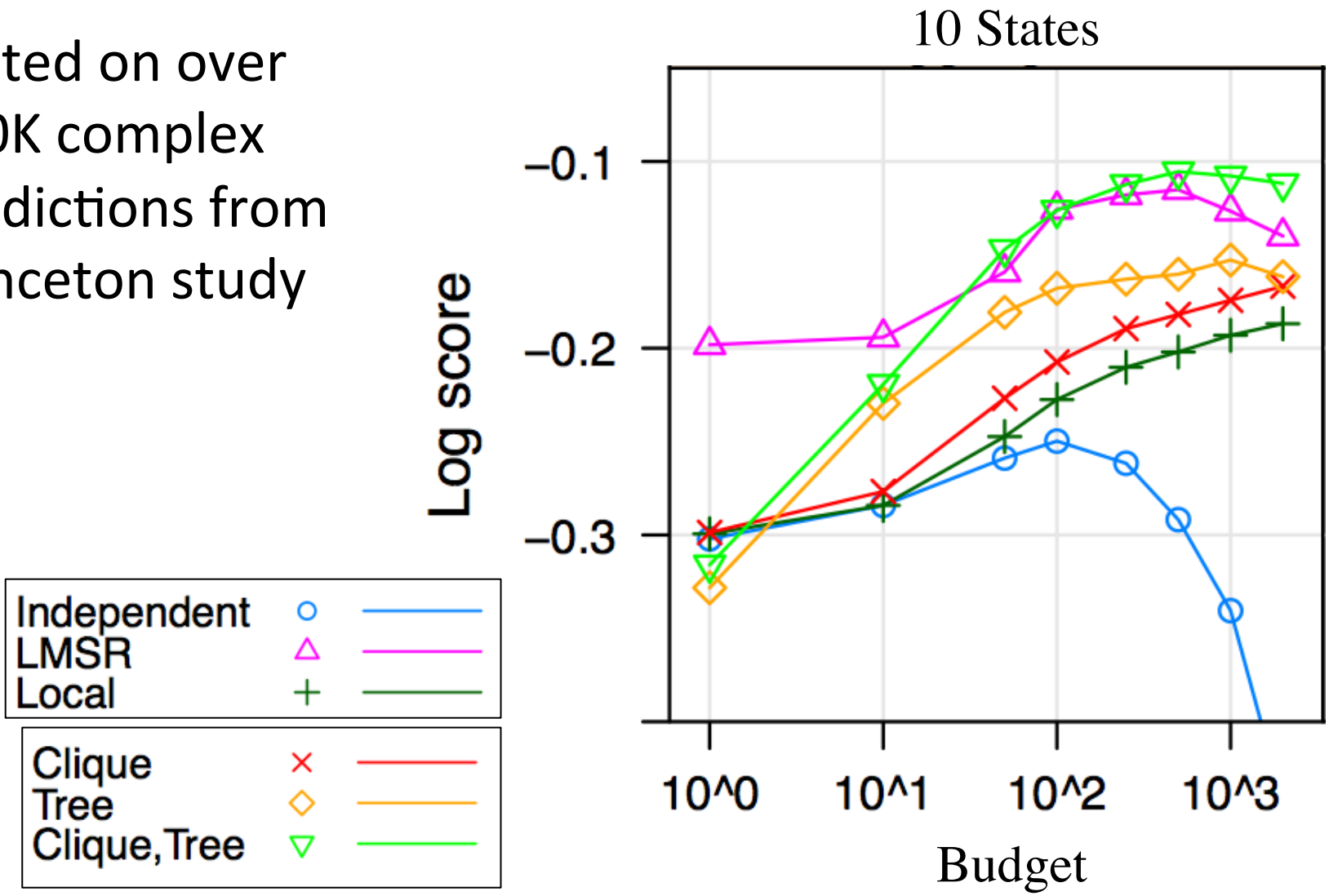


Approximate pricing



Does it work?

Tested on over 300K complex predictions from Princeton study



Does it work?

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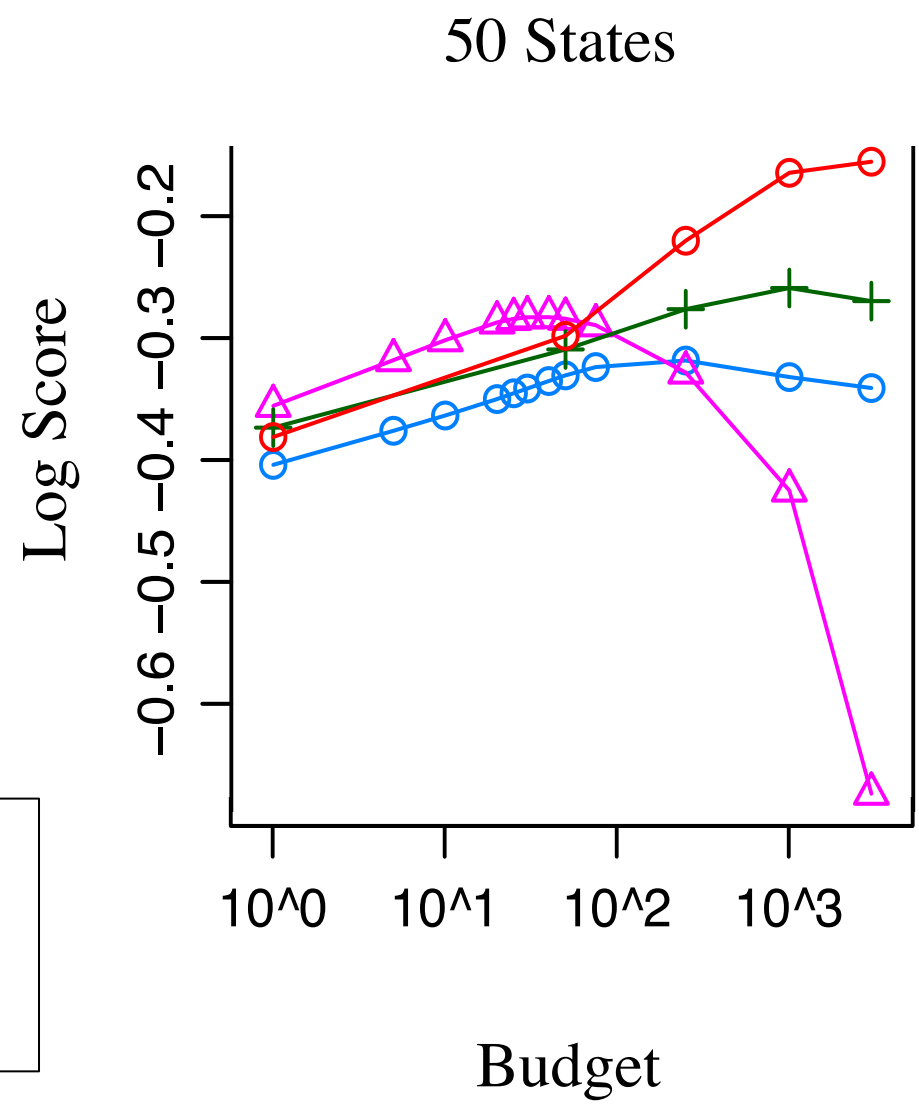
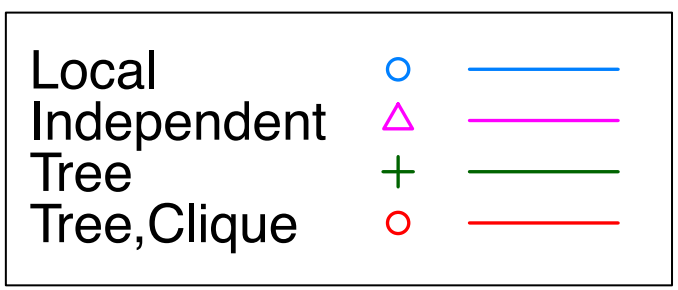
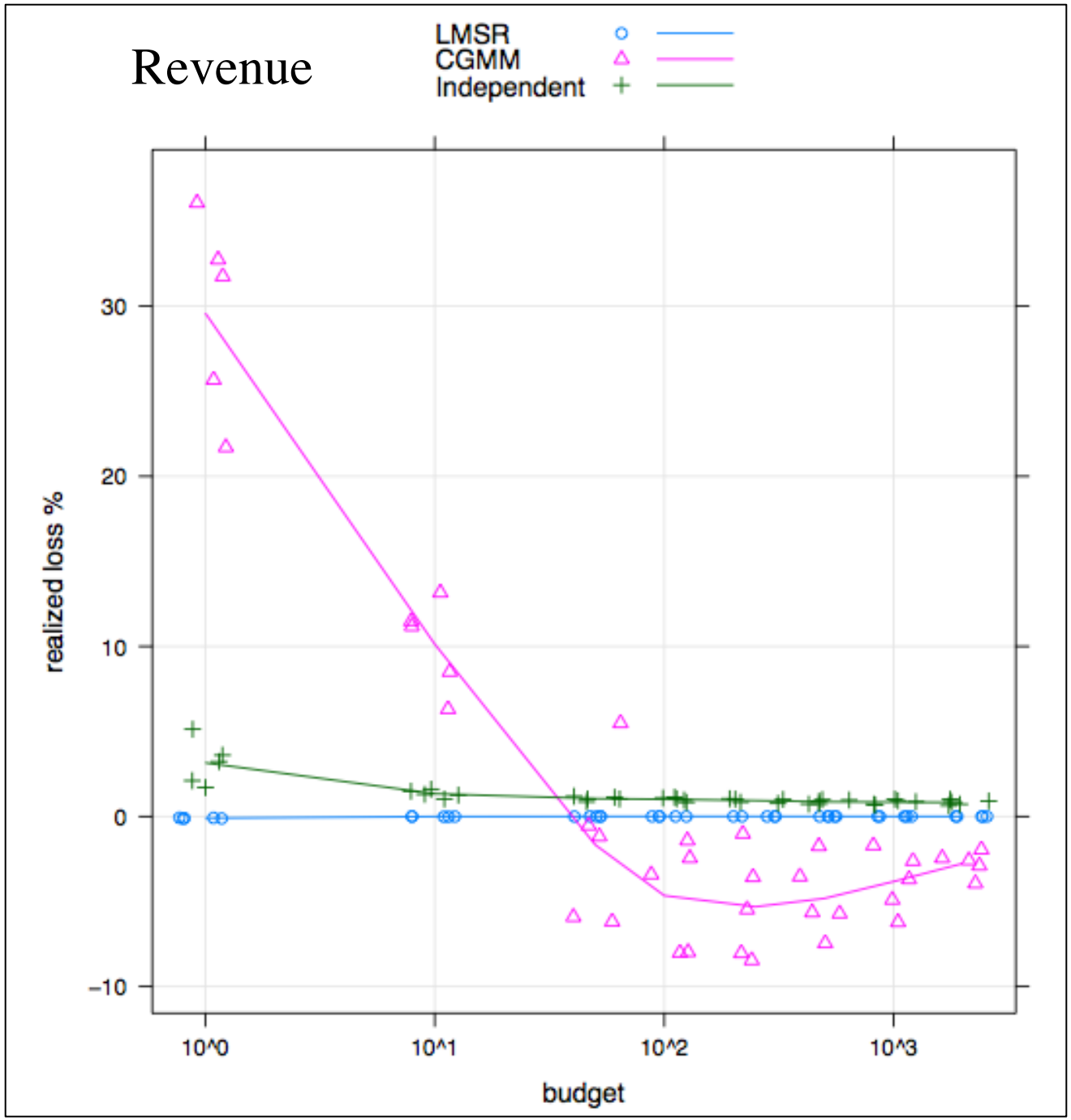


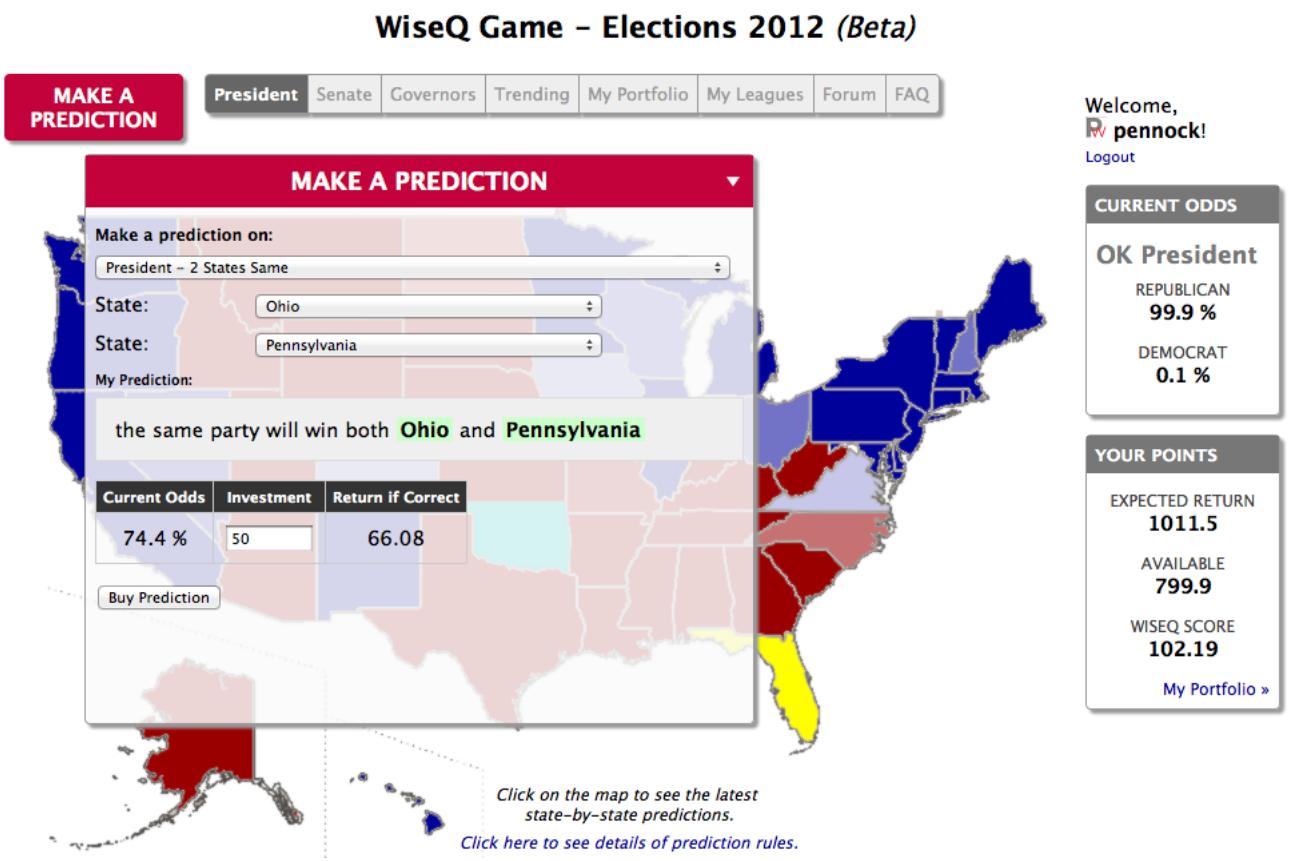


Tested on over 300K complex predictions from Princeton study

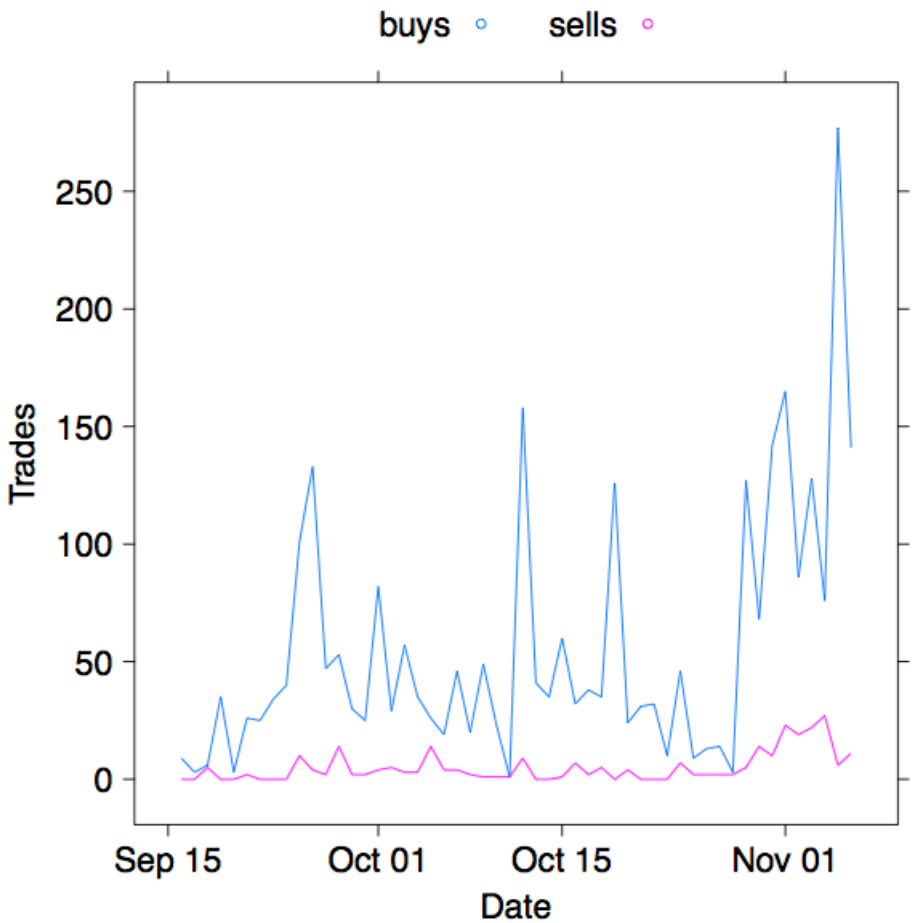


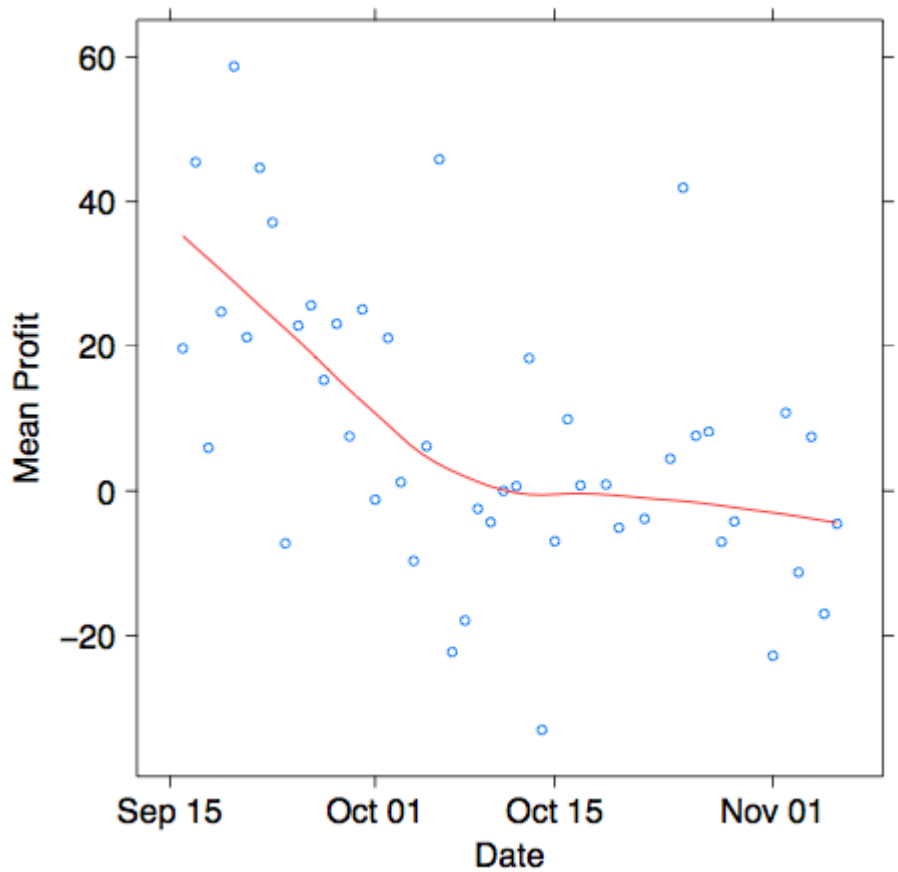
No really, does it work?

- http://PredictWiseQ.com



User activity, Mean daily profit





Focal predictions

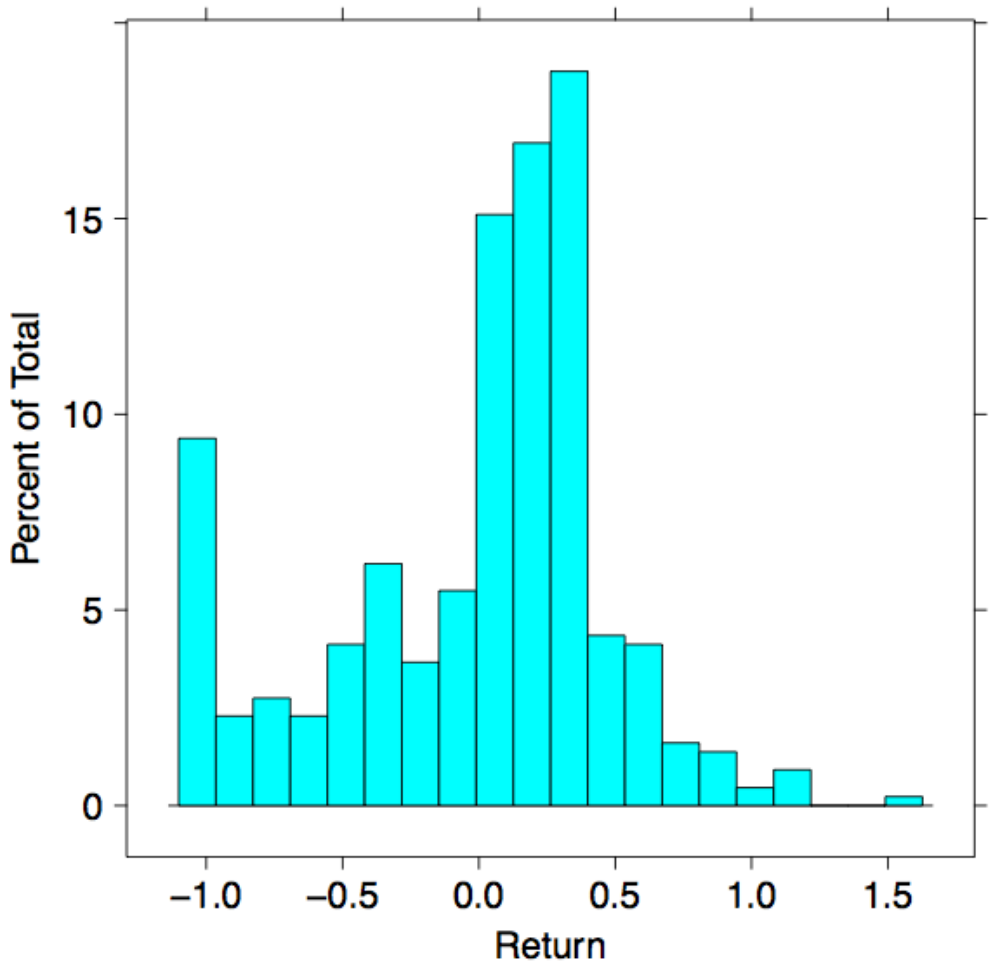
Top singleton securities

- Federal, 43K
- FL, 26K
- OH, 17K
- VA, 15K
- CO, 12K
- Electoral votes, 10K
- NC, 10K
- Senate, 8K

Top combinatorial securities

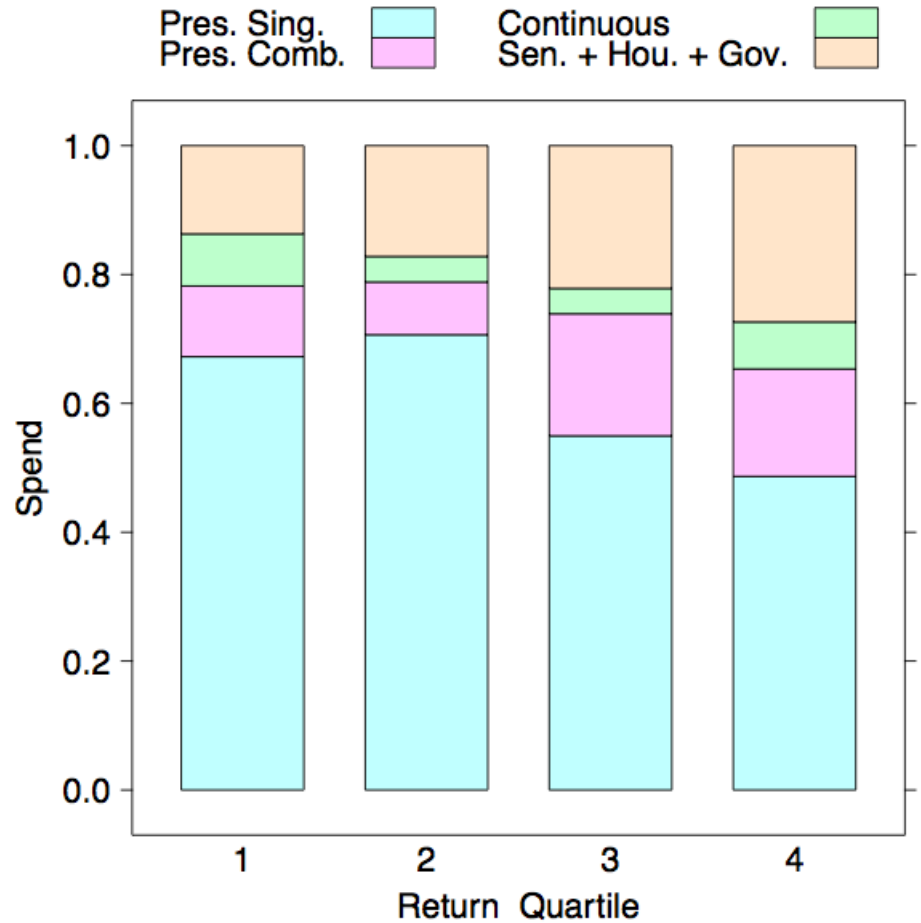
- VA, WI = Dem
- CO, IN, NH, OH, VA = Dem
- Fed, OH = Rep
- OH, VA = Dem
- FL=Rep, Fed=Dem
- FL, NC = Rep
- FL, Fed, OH = Dem
- NH, NV, OH, VA, WI = Dem

Return



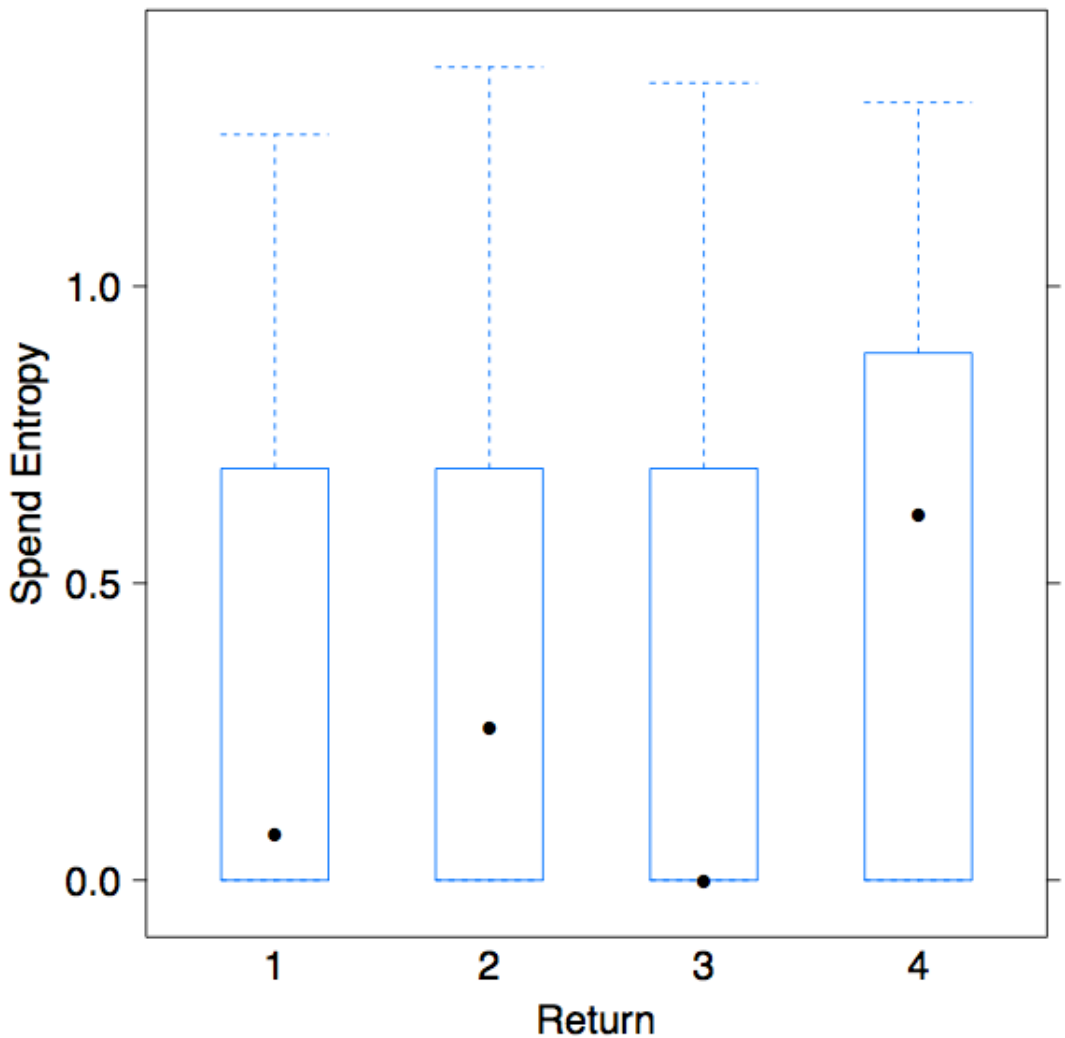
Percentile	Revenue	Spend	Profit	Return	Buys	Sells
0%	0.00	1.97	-1400.00	-1.00	1	0
25%	60.83	100.00	-50.07	-0.34	2	0
50%	238.47	350.02	9.93	0.12	4	0
75%	1006.14	1199.97	95.29	0.29	9	0
100%	5041.55	4505.96	1841.55	1.53	64	37

Types of securities by user return

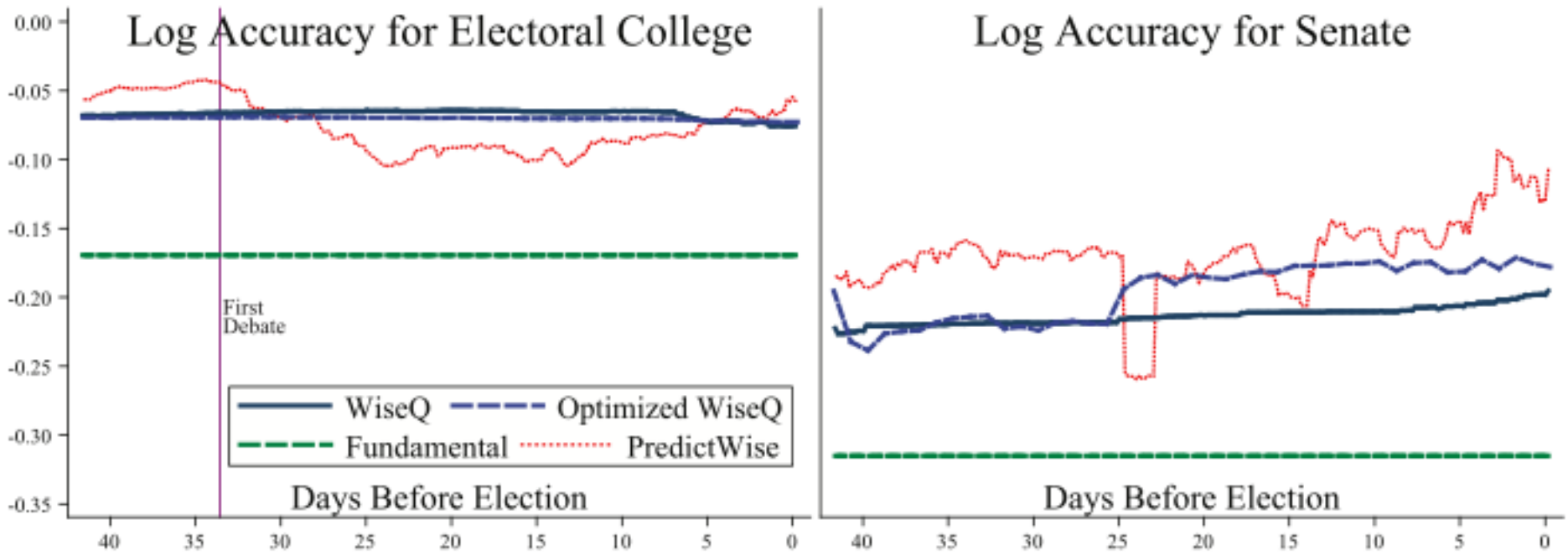


Prediction Type	Unique Users	Trades per User
Presidential—Singleton	413	3.87
Senate and House—Singleton	176	3.12
Presidential—Combinatorial	150	3.12
Electoral Votes	63	1.79
Governor	47	1.49
Economic Indicators	37	1.54
Senate and House—Combinatorial	12	1.50

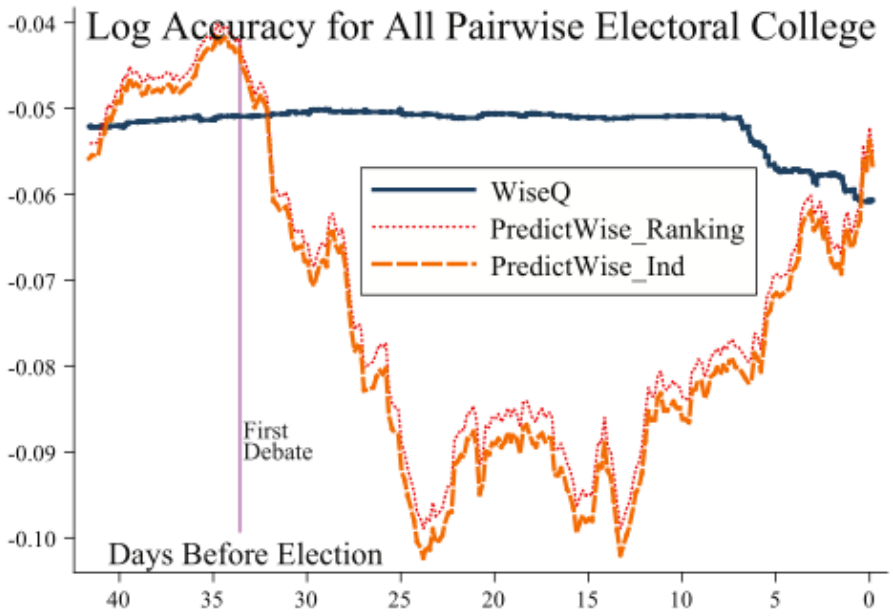
“Diversification” by user return

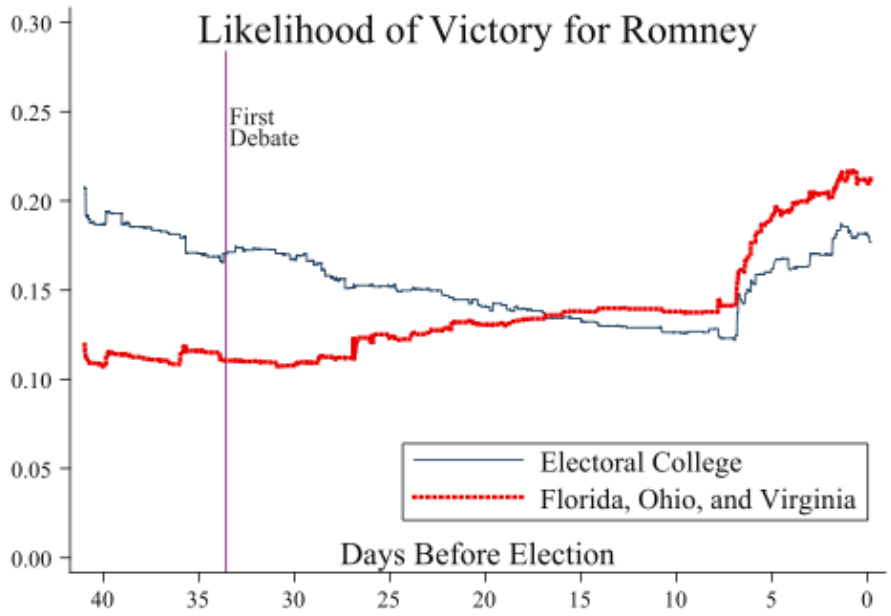


Accuracy

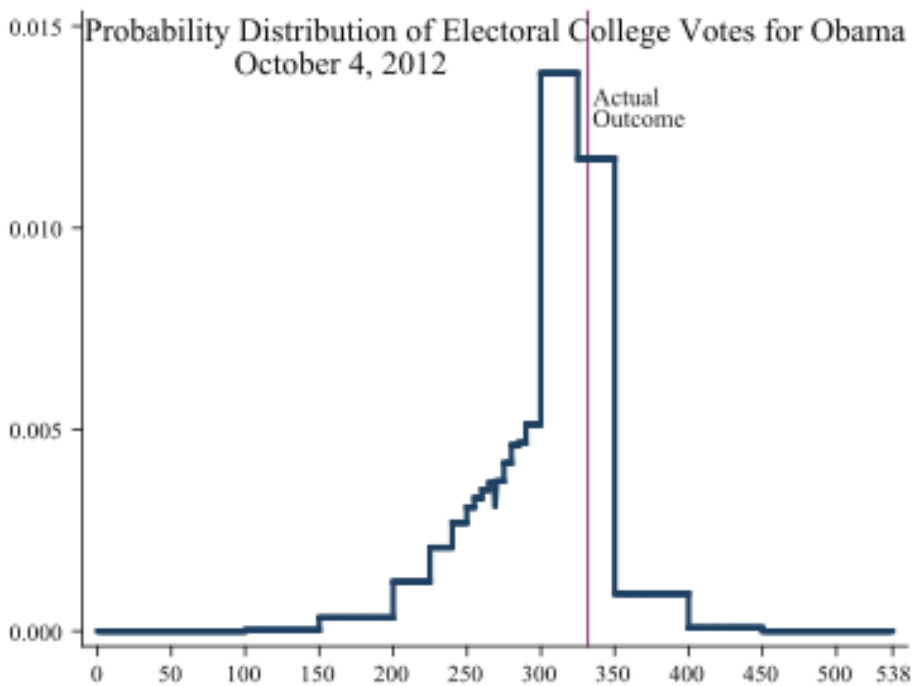

Log Accuracy for Electoral College
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
First Debate
WiseQ
Optimized WiseQ
Fundamental
PredictWise
Days Before Election
40
35
30
25
20
15
10
5
0
Log Accuracy for Senate
Days Before Election
40
35
30
25
20
15
10
5
0

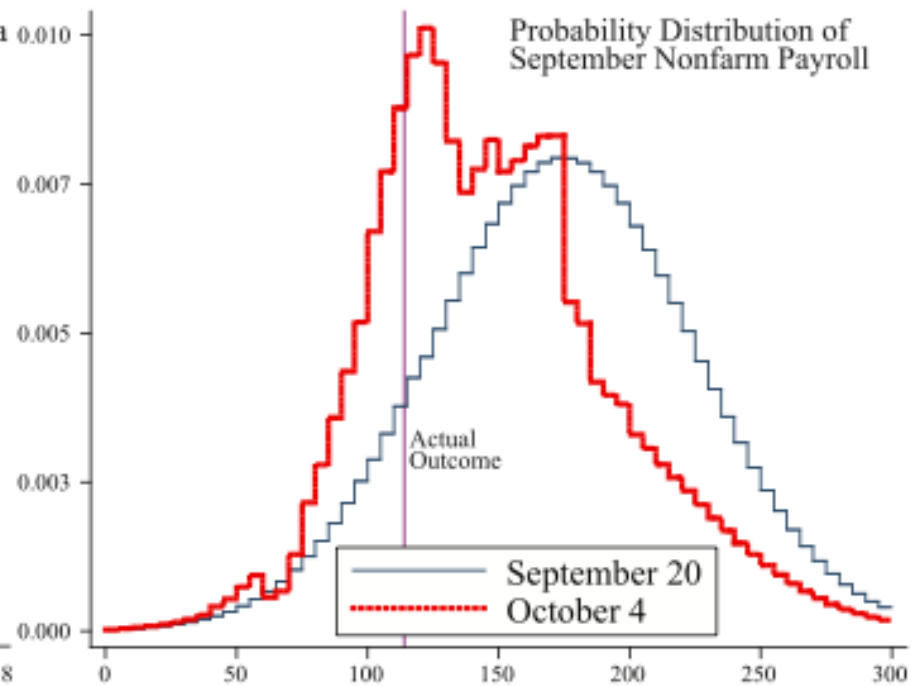
Accuracy


Log Accuracy for All Pairwise Electoral College
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.10
WiseQ
PredictWise_Ranking
PredictWise_Ind
First
Debate
Days Before Election
40
35
30
25
20
15
10
5
0

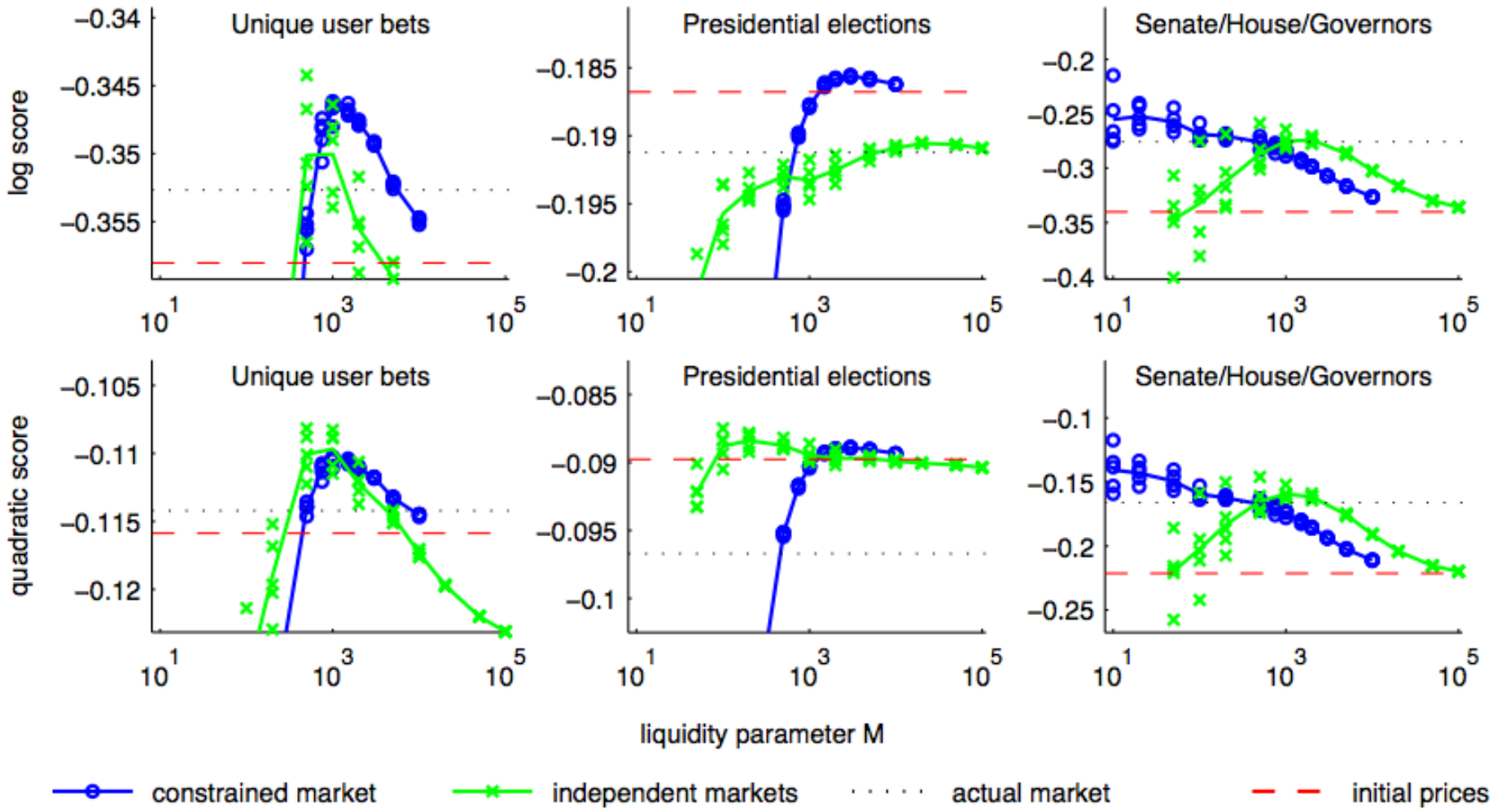

Likelihood of Victory for Romney
0.30
0.25
0.20
0.15
0.10
0.05
0.00
First
Debate
Electoral College
Florida, Ohio, and Virginia
Days Before Election
40
35
30
25
20
15
10
5
0

Full distributions





Counterfactuals



Further reading

Our paper in **ACM EC'13**
http://research.microsoft.com/apps/pubs/default.aspx?id=192965

Our paper in **ACM EC'12**
http://research.microsoft.com/apps/pubs/default.aspx?id=167977

Blog post on **PredictWiseQ**
http://blog.oddhead.com/2012/10/06/predictwiseq/

Blog on gory details: **What is (and what good is) a combinatorial prediction market?**
http://bit.ly/combopm

Guest post on **Freakonomics**
http://bit.ly/combopmfreak

NYSE 1926

http://online.wsj.com/article/SB10001424052748704858404576134372454343538.html

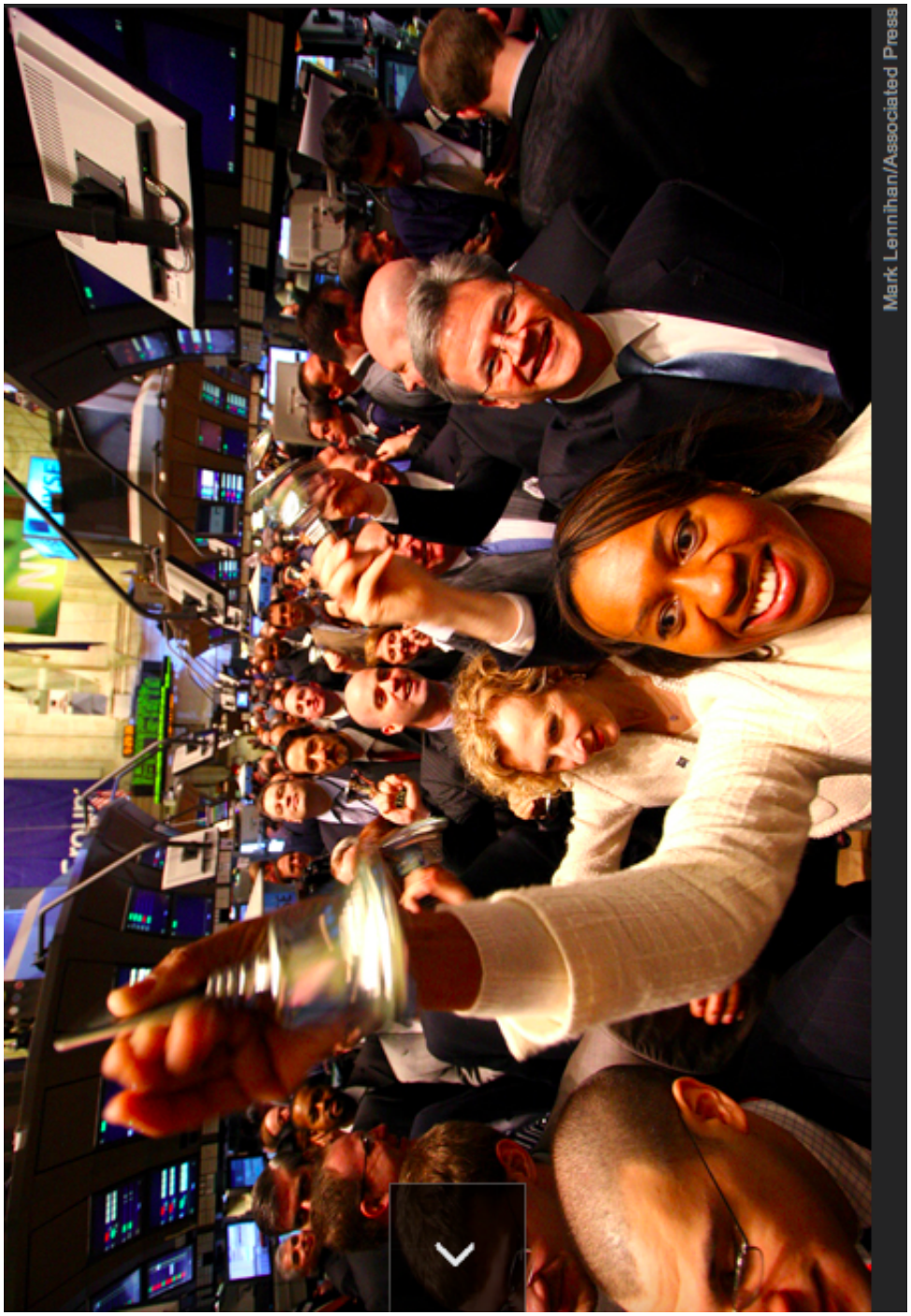
NYSE 1987

Maria Bastone/Agence France-Presse/Getty Images

http://online.wsj.com/article/SB10001424052748704858404576134372454343538.html

NYSE 2006-2011

- 2011 Deutsche Börse AG
- 2007 Euronext
- 2006 Archipelago, ipo



NYSE 7pm Sep 10, 2012

Exxon Mobil Corporation (XOM) - NYSE

89.48 ↓0.44(0.49%) 4:00PM EDT | After Hours

Order Book

Top of Book

Bid		Ask	
Price	Size	Price	Size
88.60	100	89.98	100
88.59	1,000	90.31	700
		94.41	50

New markets, same old CDA

Intrade

Order Book			
2012.PRES.OBAMA			
Bid		Offer	
Qty	Price	Price	Qty
1	56.2	56.9	12
51	56.1	57.0	494
156	56.0	57.1	21
132	55.9	57.4	6
987	55.8	57.5	523
36	55.7	57.9	45
348	55.6	58.0	104
1050	55.5	58.5	133
600	55.4	59.0	100
500	55.3	59.3	60
500	55.2	59.5	100
625	55.1	59.8	100
1124	55.0	59.9	215
100	54.9	60.0	200
104	54.8	60.2	207

Betfair

1.53	£130		£110,216
1.54	£992		£44,612
1.55	£6,573		£58,305
1.56	£3,499		£85,048
1.57			£122,367
1.58		£7,630	£63,837
1.59		£747	£64,479
1.60		£539	£70,414
1.61		£78	£63,195
1.62		£121	£55,921
1.63		£95	£58,138
1.64		£127	£86,808
1.65		£1,598	£92,735
1.66		£2,673	£29,193
1.67		£182	£21,888
1.68		£228	£30,840
1.69		£50	£41,203
1.70		£129	£25,480
1.71		£114	£16,005
1.72		£50	£17,674
1.73		£2	£54,606
1.74		£1,525	£71,804
1.75		£263	£62,116

Figure 7: The order books for Obama to win the presidency, Intrade left and Betfair right, at 1:45 PM ET on August 16, 2012

A Better Way
(Or,... Bringing trading into digital age)

- Expressiveness
  - Linear programming
  - Bossaerts, Fine, Ledyard: Combined Value Trading
    Fortnow et al.: Betting Boolean Style
  - `http://bit.ly/multipm`
- Expressiveness + Liquidity
  - Automated market maker
  - Always quote a price on *anything*
  - Downside: requires subsidy/risk

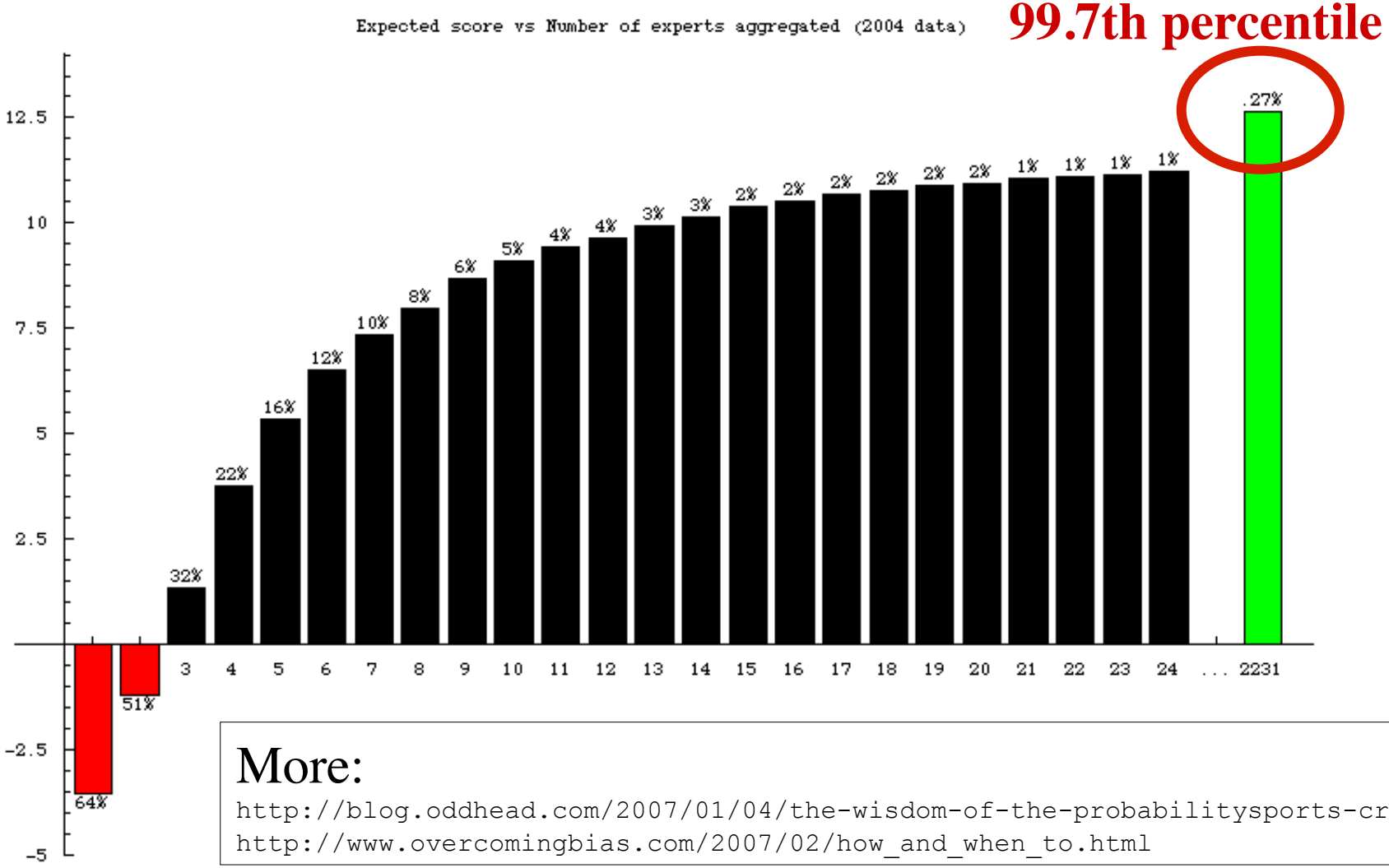
Performance of Prediction Markets with Kelly Bettors

David Pennock, Microsoft*
John Langford, Microsoft*
Alina Beygelzimer (IBM)

*work conducted at Yahoo!

Wisdom of crowds

When to guess: if you're in the 99.7th percentile



More:

http://blog.oddhead.com/2007/01/04/the-wisdom-of-the-probabilitysports-crowd/

http://www.overcomingbias.com/2007/02/how_and_when_to.html

Can a prediction market learn?

Typical

- Mkt >> avg expert
- Static

This paper

- Mkt >? best expert
- Dynamic/Learning (experts algos)
- Assumption: Agents optimize growth rate (Kelly)

Kelly betting

- You have $1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?

Kelly betting

- You have $1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Wrong A: $1000
  P(bankruptcy)$\rightarrow$1

Kelly betting

- You have $1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Wrong A: fixed $1
  No compounding magic

Kelly betting

- You have $1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Optimal A: **$52.63**
  or f* fraction of your wealth
  where f* = (p-mp)/(1-mp) "edge/odds"

Why?

- Kelly betting ≡ maximizing log utility
  - Maximizes compounding growth rate
    Maximizes geometric mean of wealth
  - Minimizes expected doubling time
    Minimizes exp time to reach, say, $1M
  - Does not maximize expected wealth
    ("All in" does, but ensures bankruptcy)

Wealth-weighted average

- Assume all agents optimize via Kelly and are "price takers"
- Then $mp = \sum w_i p_i$

Fractional Kelly betting

- Bet λf^* fraction of your wealth, $\lambda \in [0,1]$
- Why? Ad hoc reasons
  - Full Kelly is too risky (finite horizon)
  - I'm not confident of p (2nd-order belief)
- Our (new) reason
  - λ fraction Kelly behaviorally equiv to full Kelly with revised belief $\lambda p+(1-\lambda)mp$
  - Has Bayesian justification

Wealth-weighted average

- Assume all agents optimize via Kelly and are "price takers"
- Then $mp = \sum w_i p_i$
- Fractional Kelly: $mp = \sum \lambda_i w_i p_i$

Wealth dynamics

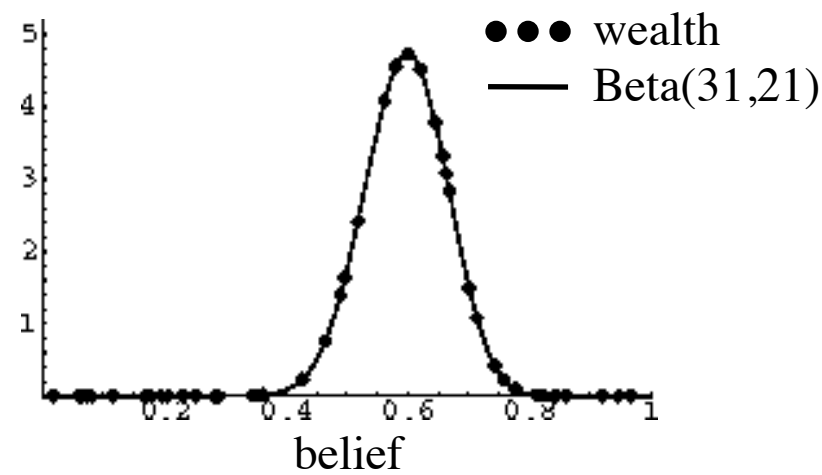
- Agents trade in a binary pred market
  Event outcome is revealed
  Wealth is redistrib. (losers pay winners)
  Repeat
- After each round

  If event happens: $w_i \leftarrow w_i\, p_i/pm$

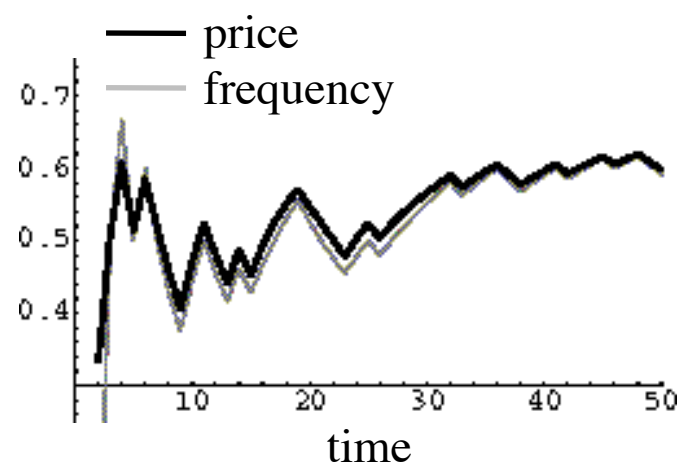
  If event doesn't: $w_i \leftarrow w_i(1-p_i)/(1-pm)$
- Wealth is redistributed like Bayes rule!

Wealth dynamics

- Single security
- Multiperiod market
- Agents with log util
- Fixed beliefs



successes:30 trials:50



Wealth dynamics

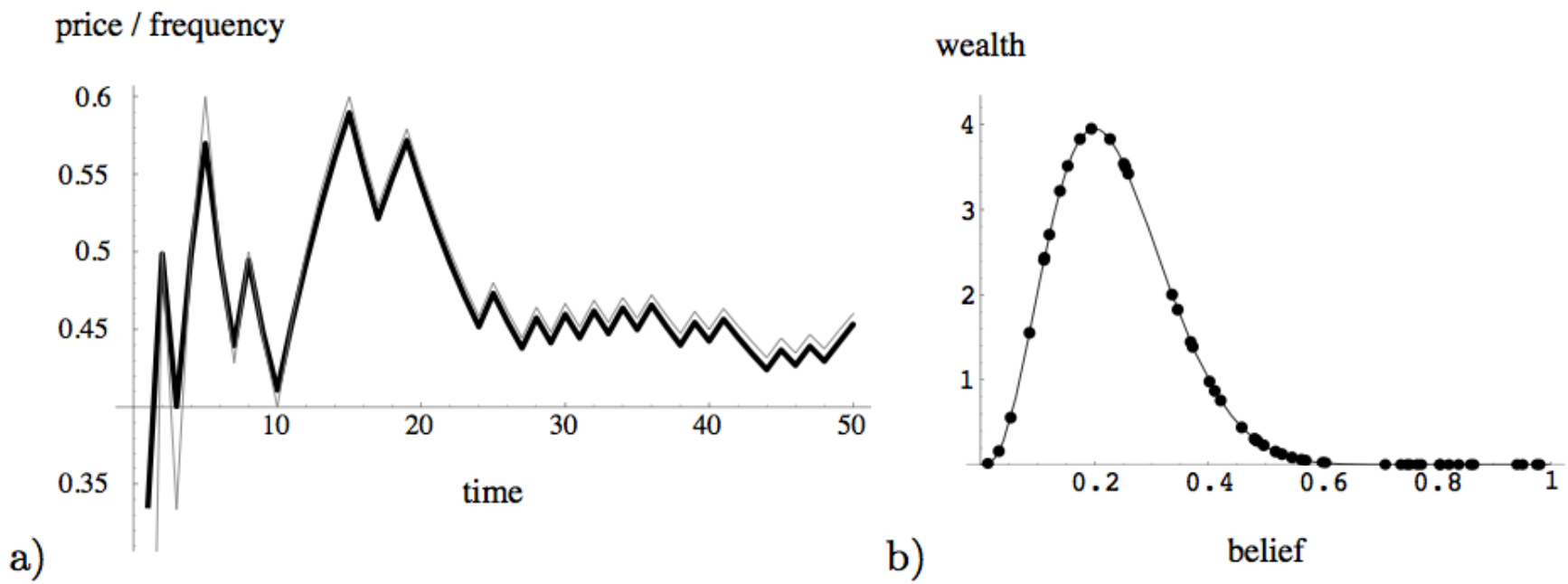


Fig. 1. a) Price (black line) versus the observed frequency (gray line) of the event over fifty time periods. The market consists of one hundred full-Kelly agents with initial wealth $w_i = 1$. b)Wealth after fifteen time periods versus belief for fifty Kelly agents. The event has occurred in three of the fifteen trials. The solid line is the posterior Beta distribution consistent with the observation of three successes in fifteen independent Bernoulli trials.

Wealth dynamics: Regret

- Theorem:

Mkt log loss < $\min_i$ agent i log loss - ln w_i

- Applies for all agents, all outcomes, even adversarial
- Same bound as experts algorithms
  $\Rightarrow$ No "price of anarchy"

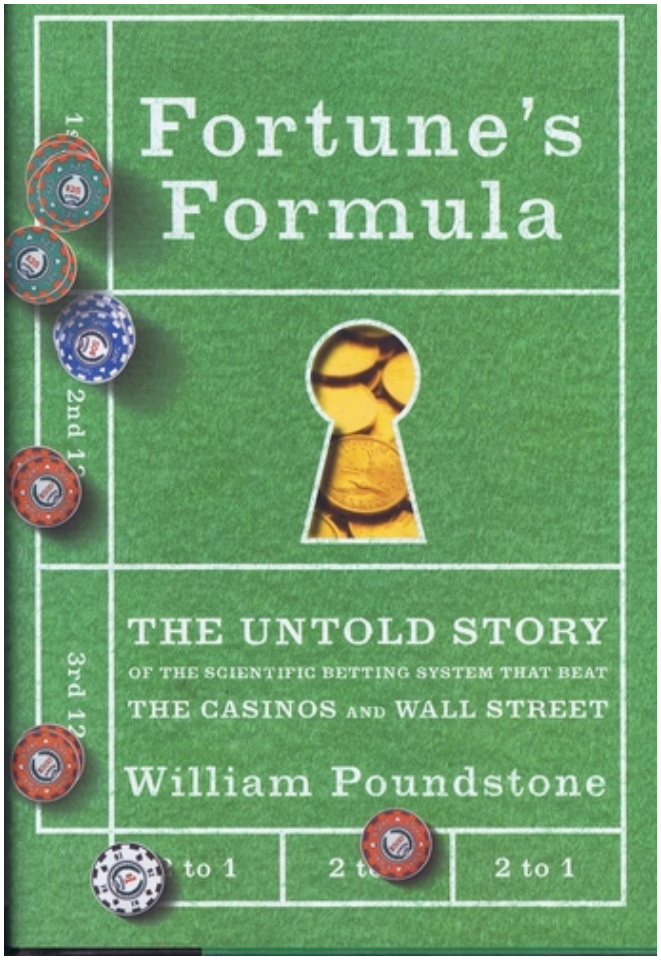
Learning the Kelly fraction

- Competitive equilibrium: $\lambda = 1$
- Rational expectations equil: $\lambda = 0$
- Practical (wisdom of crowds): $\lambda = \varepsilon$
- A proposal: Fractional Kelly as an experts algorithm btw yourself and the market
  - Start with $\lambda = 0.5$
  - If right, increase λ; If wrong, decrease
  - Won't do much worse than market (0)
    Won't do much worse than original prior p

Summary and Questions

- Self-interested Kelly bettors implement Bayes' rule, minimize market's log loss
- If mkt & agents care about log loss: win-win
- Questions
  - Can non-Kelly bettors be induced to minimize log loss?
  - Can Kelly bettors be induced to minimize squared loss, 0/1 loss, etc.?
  - Can/should we encourage Kelly betting?

Recommended Read



- Great science writing (Pulitzer prize winner)
- Fun read
- Accessible yet accurate
- Well researched
- Mob meets politics meets Time Warner meets Wall Street meets Vegas meets Bell Labs scientists