# Illumination Models

## *Z*-buffer methods:

- ☆ Compute only direct lighting.
- ☆ Ignore secondary light sources.

## Ray-tracing methods:

- ☆ Model specular reflection and refraction well.
- ☆ Still uses directionless ambient lighting.
- ☆ Not good for global lighting.

## Radiosity methods:

- ☆ Introduced in 1984 by (Goral, Torrance, Greenberg, & Battaile).
- ☆ Use thermal radiation models to calculate global lighting.
- ☆ Good for ideal *diffuse* environments.
- ☆ Assumes conservation of light energy in a closed environment.
- ☆ Determines all light interactions in a view-independent way.

# Radiosity

**Radiosity:** Power (Energy/unit time) leaving a surface at a given point, per unit area.

- ☆ Allows any surface to radiate power.
  - If light source, emits energy.
  - Otherwise, radiates a portion of the incident energy.
- ☆ Surfaces have finite area.
- ☆ Power leaving a surface:

  *emitted power + reflected power*.
- ☆ Need to *mesh* original surfaces to capture fine-scale illumination.

# Radiosity

- $S_1, \ldots, S_n$: Set of surface patches.
- $A_i$: Area of $S_i$.
- $\rho_i$: Reflectance of $S_i$.
- $B_i$: Radiosity of $S_i$.
- $E_i$: Rate at which $S_i$ emits power.
  (Energy per unit time per unit area.)

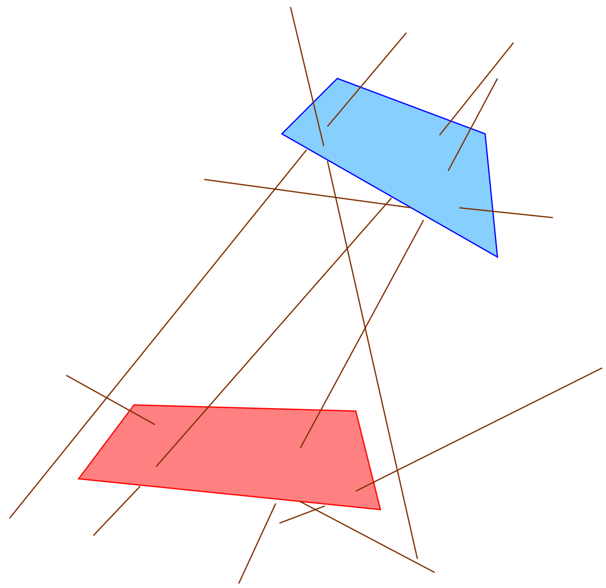
$$\begin{aligned} B_i A_i &= E_i A_i + \rho_i \cdot \text{total power incident to } S_i \\ &= E_i A_i + \rho_i \sum_{j=1}^{n} \text{Incident power from } S_j \\ &= E_i A_i + \rho_i \sum_{j=1}^{n} B_j A_j F_{ji} \end{aligned}$$

$F_{ji}$: Fraction of light leaving $S_j$ that reaches $S_i$.

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ji} \frac{A_j}{A_i}.$$

## Form Factors

- ⋆ $F_{ji}$: Fraction of light leaving $S_j$ that arrives at $S_i$
  - Depends on shape, orientation, & occlusion.
  - $F_{ii} \neq 0$ (e.g., concave surfaces).



- ⋆ $MA_i$: Number of lines through $S_i$.
- ⋆ $MA_j$: Number of lines through $S_j$.
- ⋆ $MA_jF_{ji}$: # lines leaving $S_j$ & reaching $S_i$.
- ⋆ $MA_iF_{ij}$: # lines leaving $S_i$ & reaching $S_j$.

$$A_iF_{ij} = A_jF_{ji}$$

$$F_{ij} = \frac{A_j}{A_i}F_{ji}.$$

# Radiosity Equation

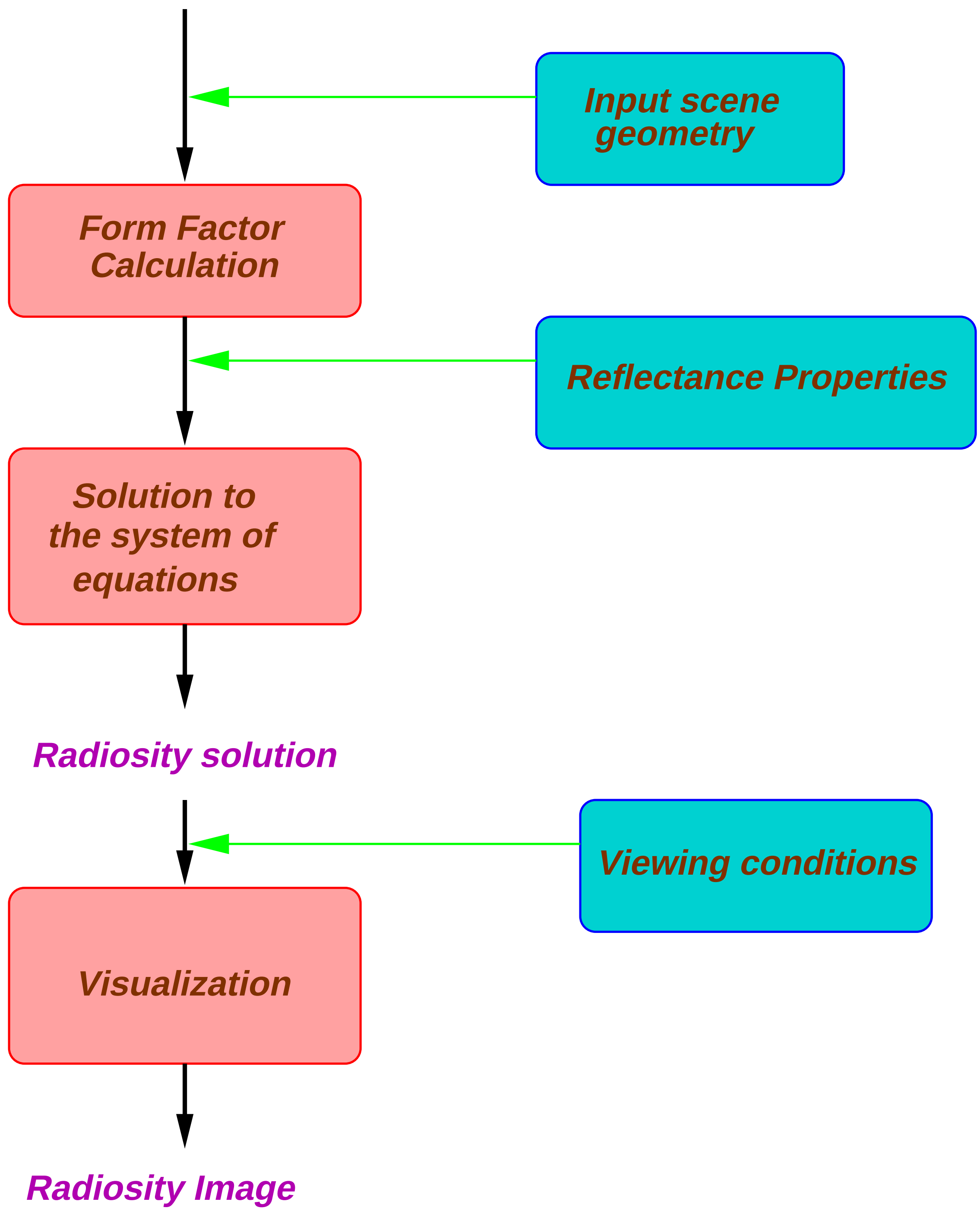
$$B_i = E_i + \rho_i \sum_{i=1}^{n} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{n} B_j F_{ij} = E_i$$

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_1 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_2 F_{nn} \end{bmatrix}}_{\mathbf{M}} \cdot \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}}_{\mathbf{B}} = \underbrace{\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}}_{\mathbf{E}}$$

$$\mathbf{M} \cdot \mathbf{B} = \mathbf{E}$$

# Radiosity Methods



# Iterative Methods

- ⋆ No closed form for the radiosity equation.
- ⋆ Use numerical methods.
- ⋆ Compute form factors $F_{ij}$, $1 \leq i, j, \leq n$.
- ⋆ Set up initial conditions
  - $E_i > 0$ for light sources.
  - $E_i = 0$ for other surfaces.
  - Guess initial values of $B_i$, $1 \leq i \leq n$.

# Iterative Methods

- ☆ Iterate the system until convergence.
- ☆ Computes a better approximation of $B_i$ at each step.

$$\mathbf{M} \cdot \mathbf{B} = \mathbf{E} \qquad \mathbf{M} = [M_{ij}] \quad M_{ii} > 0$$

$$B_i = - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{M_{ij}}{M_{ii}} B_j + \frac{E_i}{M_{ii}}.$$

Use any of the relaxation methods to compute the new value of $B_i$.

- ☆ Jacobian relaxation
- ☆ Gauss-Seidel relaxation

## Iterative Methods

*How do we compute $B_i^{(m)}$, value of $B_i$ in the $m$-th iteration?*

***Jacobian relaxation:***

Use values from the previous iteration for all $B_j$.

$$B_i^{(m)} = -\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{M_{ij}}{M_{ii}} B_j^{(m-1)} + \frac{E_i}{M_{ii}}.$$

***Gauss-Seidel relaxation:***

Use values from the previous iteration for $j < i$ and from the current iteration for $j > i$.

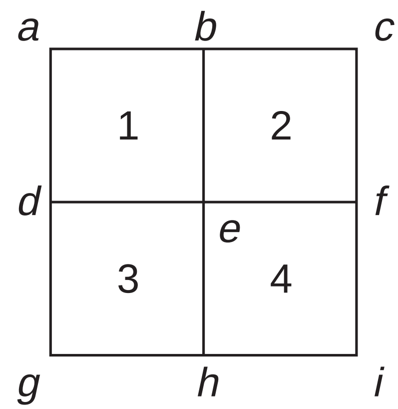
$$B_i^{(m)} = -\sum_{j=1}^{i-1} \frac{M_{ij}}{M_{ii}} B_j^{(m)} - \sum_{j=i+1}^{n} \frac{M_{ij}}{M_{ii}} B_j^{(m-1)} + \frac{E_i}{M_{ii}}.$$

- ⋆ In-place update of $B_i$'s.
- ⋆ Convergence rate is better.
- ⋆ *Strictly diagonal dominant* matrices converge.

## Continuous Shading

Decompose each surface into smaller pacthes
Radiosity within each patch is the same.

### Interpolated Shading:




- ☆ Convert patch radiosity to vertex radiosity.
- ☆ Interpolate patch radiosity.

### Vertex radiosity:

- ☆ *Interior vertex $v$:* Average of radiosity over adjacent patches

$$B_e = (B_1 + B_2 + B_3 + B_4)/4$$

*Boundary vertex $v_b$:* More complex procedure.

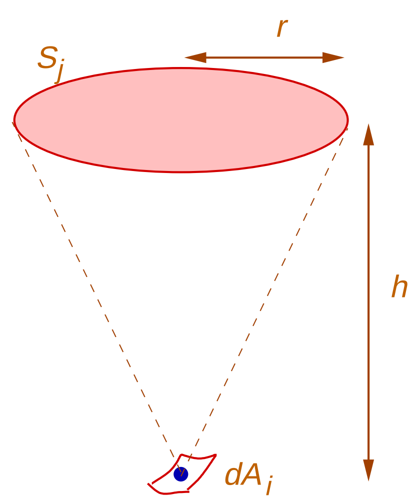
  - Find a nearest interior vertex $v_I$.
  - $f_1, \ldots, f_k$: faces adjacent to $v_b$.
  - $(B_b + B_I)/2 = \sum_{i=1}^{k} B_i/k$.
  - $(B_b + B_e)/2 = (B_1 + B_2)/2 \Rightarrow$ $B_b = (3B_1 + 3B_2 - B_3 - B_4)/4$.

# Form Factors

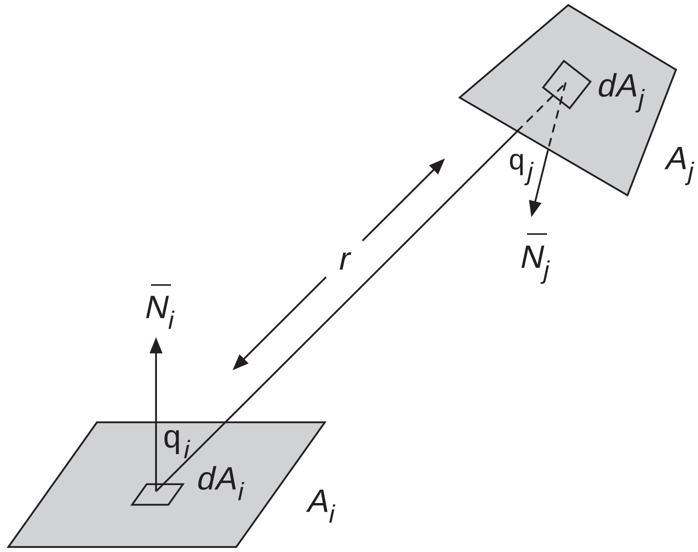
$F_{ij}$: What is the average number of lines leaving a point from $S_i$ and reaching $S_j$?

**Example:**

- Small patch $dS_i$ with area $dA_i$.
- Parallel disk of radius $r$ at distance $h$.
- $F_{ij}$: Solid angle from a point in $dS_i$ to $S_j$.
- $F_{ij} = \dfrac{r^2}{h^2 + r^2}$

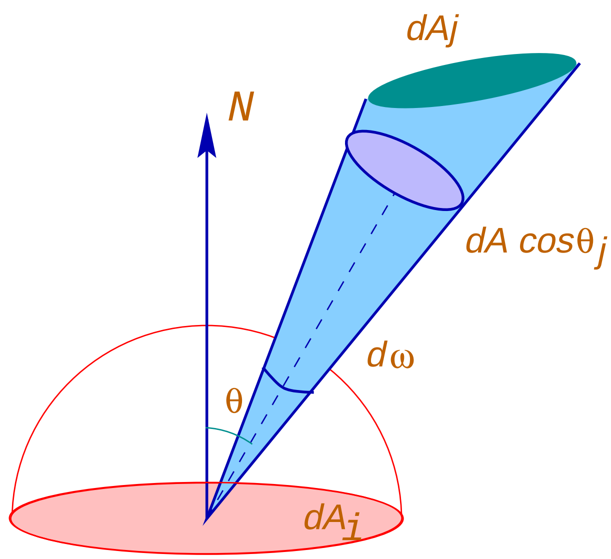


# Form Factors




- ☆ $dS_i, dS_j$: Differential surfaces $S_i, S_j$
- ☆ $dA_i, dA_j$: Areas of differential surfaces $dS_i, dS_j$
- ☆ $F_{di,dj}$: Differential form factor from $dS_i$ to $dS_j$.
- ☆ $H_{ij}$: 1 if $dS_i$ visible from $dS_j$.

$$F_{di,dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j.$$



# Form Factors

- ☆ $F_{di,j}$: Form factor from $dS_i$ to $S_j$.
- ☆ $F_{ij}$: Form factor from $S_i$ to $S_j$.

$$
\begin{aligned}
F_{di,j} &= \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j \\
F_{i,j} &= \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j dA_i
\end{aligned}
$$

# Form Factors