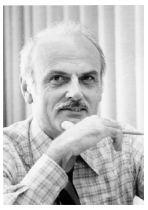
Relational Model and Algebra

Introduction to Databases

CompSci 316 Fall 2016

DUKE COMPUTER SCIENCE

Edgar (Ted) F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- The "theoretical foundation" of database management
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - Values are "atomic"
  - Can be strings, integers, reals, characters,..
  - Cannot be a struct, set, list, array, …
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Recall: Simplicity is a virtue!

Example

Relation?
Attributes?
Tuples?

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

Ordering of rows doesn't matter (even though output is always in some order)

"set semantic"

Schema vs. instance

- Schema (metadata)
  - Specifies how the logical structure of data
  - Is defined at setup time
  - Rarely changes
  - But columns can be added/deleted
- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

☞Compare to types vs. collections of objects of these types in a programming language

Example

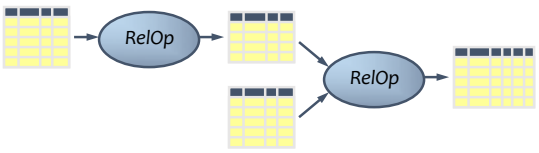
list of attributes is a set too but a standard order is assumed

- Schema
  - *User* (*uid* int, *name* string, *age* int, *pop* float)
  - *Group* (*gid* string, *name* string)
  - *Member* (*uid* int, *gid* string)
- Instance
  - *User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, … }
  - *Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
  - *Member*: {⟨142, dps⟩, ⟨123, gov⟩, … }

Relational algebra (RA)

- An algebraic query language for querying relational data based on "operators"
- Not used in commercial DBMSs
  - use "real" language SQL
- But SQL basically implements (extended) RA
- SQL query gets translated into RA for optimizations
- RA has well-defined meaning – builds the foundation of database queries in SQL

Relational algebra Operators



- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection

- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows or R that satisfy p

Selection example

- Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\sigma_{pop>0.5}$

uid	name	age	pop
142	Bart	10	0.9
[redacted]			
857	Lisa	8	0.7
[redacted]			
…	…	…	…

More on selection

- Selection condition can include any column of R, constants, comparison ($=, \leq$, etc.) and Boolean connectives ($\wedge$: and, $\vee$: or, $\neg$: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{(?????)} User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

$$\sigma_{pop \geq every\ pop\ in\ User} User$$

WRONG!

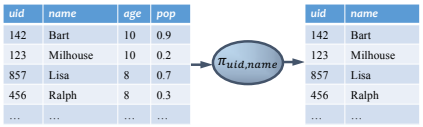
Projection

- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: same rows, but only the columns in L

Projection example

- IDs and names of all users

$$\pi_{uid,name}\ User$$



uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

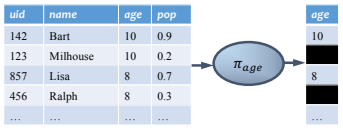
$\pi_{uid,name}$

uid	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\ User$$



uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

π_{age}

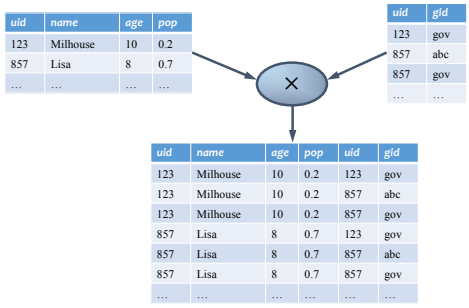
age
10
8
…

Cross product

- Input: two tables R and S
- Natation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S, output a row rs (concatenation of r and s)

Cross product example

$$User \times Member$$



uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

×

uid	gid
123	gov
857	abc
857	gov
…	…

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

A note a column ordering

- Recall: Ordering of columns is unimportant as far as contents are concerned

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

=

uid	gid	uid	name	age	pop
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

Derived operator: join

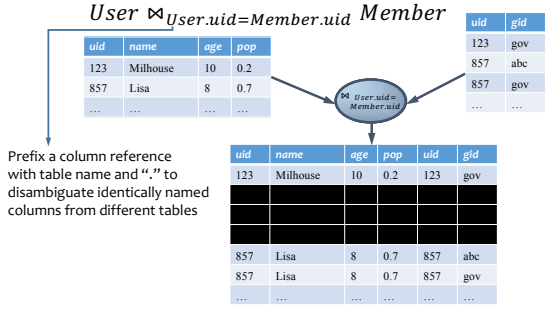
(A.k.a. "theta-join")

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$
- An important operator with various scope for optimization!

Join example

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

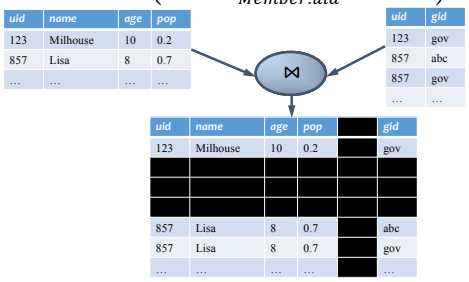


Derived operator: natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L(R \bowtie_p S)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)

Natural join example

$$User \bowtie Member = \pi_?(User \bowtie_? Member)$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{User.uid=Member.uid} Member\right)$$



Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Q. How do you write intersection using the previous operators?

Renaming

- Input: a table R and S
- Notation: $\rho_S\ R$, $\rho_{(A_1,A_2,\dots)}R$, or $\rho_{S(A_1,A_2,\dots)}R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as R, but called differently
- Used to
  - Avoid confusion caused by identical column names
  - Create identical column names for natural joins
- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

Renaming example

- IDs of users who belong to at least two groups

$$Member \bowtie_? Member$$

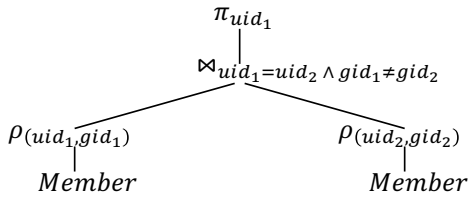
$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \wedge \\ Member.gid \neq Member.gid}} Member\right)$$

WRONG!

$$\pi_{uid_1}\left(\begin{array}{l}\rho_{(uid_1,gid_1)}Member \\ \bowtie_{uid_1=uid_2 \wedge gid_1 \neq gid_2} \\ \rho_{(uid_2,gid_2)}Member\end{array}\right)$$

"Expression tree" or "Logical Query Plan Tree" notation

User (uid, name, age, pop)
Member (uid, gid)



Q. What does this RA expression output?

Summary of core operators

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{S(A_1,A_2,\dots)}R$
  - Does not really add "processing" power

Summary of derived operators

- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$

- Many more
  - Semijoin, anti-semijoin, quotient, …

An exercise

User (uid, name, age, pop)
Member (uid, gid)

- Names of users in Lisa's groups

Writing a query bottom-up:

Another exercise

31

User (uid, name, age, pop)
Member (uid, gid)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:

A trickier exercise

32

User (uid, name, age, pop)
Member (uid, gid)

- Who are the most popular?

A trickier exercise

33

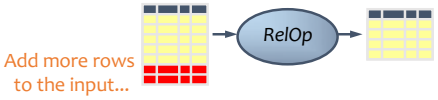
User (uid, name, age, pop)
Member (uid, gid)

- Who are the most popular?

A deeper question:
When (and why) is "–" needed?

Monotone operators

34



- If some old output rows may need to be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows always remain "correct" when more rows are added to the input
- Formally, for a monotone operator op:
  $R \subseteq R'$ implies $op(R) \subseteq op(R')$ for any R, R'

Classification of relational operators

35

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Union: $R \cup S$
- Difference: $R - S$
- Intersection: $R \cap S$

Monotone or not?

Why is "–" needed for "highest"?

36

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Is the "highest" query monotone?
  - No!
  - Current highest *pop* is 0.9
  - Add another row with *pop* 0.91
  - Old answer is invalidated

☞So it must use difference!

37

Why do we need core operator X?

- Difference
  - The only non-monotone operator
- Projection
  - The only operator that removes columns
- Cross product
  - The only operator that adds columns
- Union
  - The only operator that allows you to add rows?
  - A more rigorous argument?
- Selection?
  - Homework problem

38

Extensions to relational algebra

- Duplicate handling ("bag algebra")
- Grouping and aggregation
- "Extension" (or "extended projection") to allow new column values to be computed

☞All these will come up when we talk about SQL

☞But for now we will stick to standard relational algebra without these extensions

39

Why is RA a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat "procedural"
- Complete?
  - With respect to what?

40

Relational calculus

- $\{u.uid \mid u \in User \wedge \neg(\exists u' \in User: u.pop < u'.pop)\}$, or
- $\{u.uid \mid u \in User \wedge (\forall u' \in User: u.pop \geq u'.pop)\}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an "unsafe" relational calculus query
  - $\{u.name \mid \neg(u \in User)\}$
  - Cannot evaluate it just by looking at the database

41

Turing machine

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …


Alan Turing (1912-1954)

☞So how does relational algebra compare with a Turing machine?

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

42

Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Bart reach in his social network with any number of hops?
    - Writing this query in RA is impossible!
  - So RA is not as powerful as general-purpose languages
- But why not?
  - Optimization becomes undecidable
  - ☞Simplicity is empowering
  - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!

Announcements : 1/3 (Wed. Jan 18)

43

- Sign up for Piazza & Gradiance
- Change in midterm date: 02/22 (W) (from 02/20 (M))
  - To have enough time to go through solutions of HW2
- Homework #1 assigned
  - Due on 02/06
  - Start solving problems soon after the topics are covered
- Set up VM (instructions on course website)
- Next Wednesday: Yuhao and Junyang will walk through and help with VM setup for those who need it
  - Start solving the problems on paper and later try on VM
- The gradiance assignments will be posted "after" the corresponding lectures (by 8 pm)

Announcements : 2/3 (Wed. Jan 18)

44

- Reminder: Homework Policy

- You need to solve the problems on your own
- You can discuss ideas with your classmates, but mention names and acknowledge all helps you have received
- You cannot copy solution from another student
- Solutions must come from "your head"
  - i.e. you have to "own" the solution
- You cannot "search for" solutions from online material, forums, previous years' assignments, or any other sources
- Any violation will have serious consequences
- If in doubt whether something is allowed, send me (the instructor) an email and ask

Announcements : 3/3 (Wed. Jan 18)

45

- Project ideas and requirements to be posted by next class
- You do not have to start forming groups right away
- There will be a "project mixer" class (after two weeks)
  - If you have an idea, give a pitch preparing a few slides
  - If you like an idea, join a group with space
  - There will be a few rounds of random reshuffling of your seats after the presentations, so you will meet your classmates and discuss ideas
  - Look for project partners with diverse expertise and ideas
- Expected group size = 4
  - If 3, still have to do the same work
  - Do not go below 3
  - Only if there is no exact division by 4, may go to group size of 5 (at the end, not before)