Due on January 18, 2017
20 points total

General Directions: If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time (for this assignment, this means arguing why your algorithm achieves the target running time specified by the question). There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice.

Please make sure that you **START EARLY**. Some of these questions might take you several hours to solve. Since this is a mathematical/theoretical class, the ratio between how much time you spend thinking and how long it takes you to write your solution will be high (at least higher than what you might be used to in coding assignments).

All the answers must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a pdf file will also be accepted. Before submitting the pdf file, please make sure that it can be opened using any standard pdf reader (such as Acrobat Reader) and your entire answer is readable. **Handwritten answers or pdf files that cannot be opened will not be graded and will not receive any credit.**

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Problem 1 (10 points)
Let $f_1(n), f_2(n), g_1(n)$, and $g_2(n)$ be functions of $n \in \mathbb{Z}^+$ such that $f_1(n) = \Theta(f_2(n))$ and $g_1(n) = \Theta(g_2(n))$.

(a) (5 points) Show that the following may not hold:

$$f_1(n) - g_1(n) = \Theta(f_2(n) - g_2(n)).$$

(Here, you need to explicitly construct a counterexample.)

(b) (5 points) Show that the following must hold:

$$f_1(n)/g_1(n) = \Theta(f_2(n)/g_2(n)).$$

Problem 2 (10 points)
Solve the following recurrences for their closed forms. Express your final answer as a tight asymptotic bound (i.e., give your answer as $\Theta(f(n))$).

You can solve these however you like, but it should be clear from your explanation how you obtained your answer. If you use the tree-expansion method, simply describe the structure of the tree in words — there is no need to include any kind of figure. You do not need to give the formal inductive proofs shown in class.

(a) (5 points) $T(n) = 2T(\sqrt{n}) + \Theta(1)$

(b) (5 points) $T(n) = T(n/2) + \Theta(\sqrt{n})$