Due on April 26th, 2017
65 points total

General Directions: If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time. There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice.

All the answers must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a pdf file will also be accepted. Before submitting the pdf file, please make sure that it can be opened using any standard pdf reader (such as Acrobat Reader) and your entire answer is readable. **Handwritten answers or pdf files that cannot be opened will not be graded and will not receive any credit.**

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Problem 1 (35 points)
Consider the following problem: given an undirected graph $G = (V, E)$, find a smallest subset of vertices S with the property that for every vertex $v \in V$, $|N_v \cap S| \geq 1$, where $N_v = \{v\} \cup \{u : (u, v) \in E\}$.

(a) (5 points) Formulate the above problem using an integer linear program (ILP).

(b) (10 points) Give the fractional linear program corresponding to the above ILP and show that it is not integral, i.e., give an example demonstrating an integrality gap greater than 1 for this LP.

(c) (5 points) Write the dual of this LP.

(d) (5 points) Write the ILP corresponding to this dual LP, and state the problem it represents in words.

(e) (10 points) Show that this dual LP is also not integral, i.e., give an example demonstrating an integrality gap greater than 1 for the dual LP.

Problem 2 (20 points)
Consider the following decision problem: given an undirected graph $G = (V, E)$, a set of terminal vertices $T \subseteq V$, and a fixed parameter K, determine whether there exists a subgraph G' of G such G' connects all vertices in T (i.e., there exists a path between any $t_1, t_2 \in T$ in G') and includes at most K non-terminal vertices.

Prove that the above problem is NP-complete. Note that this involves showing:

(a) (5 points) The above problem is in NP.

(b) (15 points) The above problem is NP-hard.

(**Hint:** *You can use the fact that the vertex cover problem that we saw in class is NP-hard.*)

Problem 3 (10 points)
This question is about your experience in the course. You will get full credit for answering this question, irrespective of the actual answer.

(a) (5 points) What did you like the most about this class?

(b) (5 points) Give one suggestion for improvement in future offerings of this class.