Due on March 22th, 2017
55 points total

General Directions: If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time. There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice.

All the answers must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a pdf file will also be accepted. Before submitting the pdf file, please make sure that it can be opened using any standard pdf reader (such as Acrobat Reader) and your entire answer is readable. **Handwritten answers or pdf files that cannot be opened will not be graded and will not receive any credit.**

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Problem 1 (10 points)
Use the potential method to show that the amortized running time of each union operation in the star data structure for disjoint sets is $O(\log n)$, where n is the total number of elements.

Problem 2 (5 points)
Show that the augmenting paths algorithm for finding a maximum flow will converge in finite time if all edge capacities are rational (recall that a rational number is one that can be written in the form a/b where a and b are integers).

Note: for the following two problems, you may use a max flow algorithm in your solution without describing it, and the running time analysis can involve a parameter F where the maxflow algorithm runs in time $O(F)$.

Problem 3 (15 points)
Duke is hosting a conference for which the organizers want to ensure that traffic moves smoothly from the hotel where the delegates will be staying to the conference venue. After taking into account existing traffic patterns, the city has set limits on the number of cars carrying delegates that can pass through each intersection (the roads themselves have no limit). Can you help the conference organizers decide the maximum number of delegates they can invite? (Assume that each delegate will use a separate car. Roads can be one-way or two-way.)

Problem 4 (25 points)
Since you are taking this course in February, everyone in class is camping out for the UNC game. After lecture, all of you need to get to your tents in Krzyzewskiville as fast as possible. Being the all-star student that you are, everyone turns to you to design an algorithm that will find the fastest way to route everyone in the lecture to the campsite. In other words, you goal is to minimize the latest arrival time over all students.

You bring up a map of Duke on your smartphone consisting of walkways and intersections between the walkways. For simplicity, assume that any segment of a walkway connecting two intersections takes unit time to traverse, and students can only leave the lecture hall at integer times (so if the lecture ends at time $t = 0$, then students can leave at times $t = 0, 1, 2,$, etc.). For each of these walkway segments, the map also specifies its pedestrian capacity, i.e., if the walkway segment that connects intersections a and b has pedestrian capacity u, then you can send up to u people from a to b in a unit amount of time (and vice versa).

Design an algorithm to figure out the minimum time in which all of you can reach Krzyzewskiville.

Hint: Use maximum flows. Let T be the set of times and V be the set of intersections. Consider a network with vertices $T \times V$ (this denotes the Cartesian product of T and V; look up this notation if it is unfamiliar). What should the edges of this network be, and how can you use it to solve the problem?