Practice Questions for 590.2 Midterm Exam

1. Individual rationality of proper scoring rules. We have seen that proper scoring rules are, in a sense, incentive compatible. What about individual rationality? Assume that the score is actually a payment to the agent, and it does not cost the agent anything to report truthfully. Also, assume the scoring rule is proper.

a. Say that a proper scoring rule is *ex-post individually rational* if, no matter what the true probability distribution is and no matter what outcome happens, after the fact, the forecaster does not regret participating. **Give** a simple condition on either the score function $S(\cdot, \cdot)$ or the associated $G(\cdot)$ for when the proper scoring rule is ex-post IR.

b. Say that a proper scoring rule is *ex-interim individually rational* if, no matter what the true probability distribution is, the forecaster *in expectation* is better off participating (reporting truthfully) than staying home. **Give** a simple condition on either the score function $S(\cdot, \cdot)$ or the associated $G(\cdot)$ for when the proper scoring rule is ex-interim IR.

c. How do the answers to these questions change if we now use the rule $S(\cdot, \cdot)$ in a market scoring rule, so that i's reward is now $S(\vec{p}_i, \omega) - S(\vec{p}_{i-1}, \omega)$ where $\vec{p}_i$ is i's reported distribution.

2. Expressive financial markets.

Consider a market where you can make the following type of offer:

If the Democratic candidate wins North Carolina, and does not win Michigan, and does not win Ohio, then I want to be paid \$10; I am willing to pay \$3 for such a security.

Note that for the security to pay out, *all* of the conditions (in this case, there are 3) need to be true. In this market, a security can use "and" and "not," but not "or." I.e., you may *not* ask for a security that pays out \$10 if the Democratic candidate wins North Carolina OR the Democratic candidate wins Michigan.

Suppose the auctioneer has accepted offers for the following securities:

1. If A and B both happen, the auctioneer needs to pay out 1 to bidder 1.

2. If A happens, the auctioneer needs to pay out 1 to bidder 2.

3. If A does not happen and B does not happen, the auctioneer needs to pay out 1 to bidder 3.

In this case, the worst-case outcome for the auctioneer is that A and B both happen, in which case she has to pay out 2.

a. Suppose the auctioneer has accepted a certain set of offers S. **Give** an integer program for determining the worst-case (maximum payout) outcome for the auctioneer.

Now, consider the MINSAT problem, which is known to be NP-hard. In the MINSAT problem, we are given a Boolean formula in *conjunctive normal form*, which looks something like $(\neg x_A \vee \neg x_B) \wedge (\neg x_A) \wedge (x_A \vee x_B)$. That is, it is an "AND" of "ORs." The parts of the formula in parentheses are called *clauses*, so this example formula has 3 clauses. If we label each variable as "true" or "false" (or 1 or 0) then some clauses will be true and some will be false. For example, if we set $x_A = 1, x_B = 1$, then only the last clause is true. The goal in MINSAT is to come up with an assignment of truth values to the variables to minimize the number of satisfied clauses (i.e., clauses that are true).

b. What is the relationship between the MINSAT problem and the problem of finding out the worst outcome for the auctioneer in the securities market above? (Hint: look at the two examples.) Does this imply anything about the hardness of the latter problem?

3. Auction Theory.

Suppose that an auctioneer is selling a single item, and there are two bidders with valuations drawn independently from [0,1] with CDF of $F(x) = x^2$.

a. What is the expected revenue from a first price auction?

b. What is the maximum expected revenue that an auctioneer could achieve?

4. Modified Rock-Paper-Scissors.

Consider the following modified version of Rock-Paper-Scissors, where losing with Paper to Scissors is considered doubly humiliating:

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-2,2
Scissors	-1,1	2,-2	0,0

a. Wright argues that in every equilibrium of this game, every pure strategy must receive positive probability from both players. Is Wright right or wrong? Explain why.

b. Based on your answer in **a**, compute a Nash equilibrium of this game. Is it the unique equilibrium? Why (not)?

5. Planning to go to one or more restaurants.

We have some set of people who want to go to some set of restaurants. For each person i and each restaurant r, i has a value v_{ir} for going to that restaurant. Also, for every two people i and j, person i has a value of w_{ij} for going to the same restaurant as j (note w_{ij} is not necessarily equal to w_{ji}). An agent i's valuation is the sum of that agent's applicable v_{ir} and w_{ij} (you can get only one of your v_{ir} but potentially multiple of your w_{ij}). We wish to determine who should go to which restaurants, so as to maximize the sum of the agents' valuations. Every agent must go to a single restaurant. Note that not everyone needs to go to the same restaurant (though they can).

For example, consider three agents Alice, Bob, and Carol, who are considering whether to go to a French, Indian, or Mexican restaurant. Alice likes French ($v_{AF} = 10$) and to be with Bob ($w_{AB} = 8$). Bob likes Indian ($v_{BI} = 12$) and to be with Carol ($w_{BC} = 7$). Carol likes Mexican ($v_{CM} = 11$) and to be with Alice ($w_{CA} = 9$). Nobody likes anything or anyone else, i.e., all the other v_{ir} and w_{ij} are zero.

a. What is the optimal solution for this example?

b. Compute the Clarke mechanism (GVA) payments of all the agents in the example. (Here, some payments may be negative because some agents may contribute to the welfare of others by being present.)

c. Give an integer program for computing the optimal solution in general (for arbitrary v_{ir} and w_{ij}; your integer program doesn't have to compute the Clarke payments, just the optimal solution). You can write it either mathematically or in the modeling language (but if write it mathematically, be very precise in your use of $\forall$ and be clear about which variables you are summing over—the modeling language of course forces you to do so). You don't need to enter the data from the above example.