

Homework 3: Decision and Game Theory (due April 11 before class)

Please read the rules for assignments on the course web page (<http://www.cs.duke.edu/courses/spring18/compsci223/>). Show all your work, but circle your final answers. Use Piazza (preferred) or directly contact Harsh (harsh.pariikh@duke.edu), Hanrui (hrzhang@cs.duke.edu), or Vince (conitzer@cs.duke.edu) with any questions.

1. Risk attitudes. Bob is making plans for Spring Break. He most prefers to go to Cancun, a trip that would cost him \$3000. Another good option is to go to Miami, which would cost him only \$1000. Bob is really excited about Spring Break and cares about nothing else in the world right now. As a result, Bob's utility u as a function of his budget b is given by:

- $u(b) = 0$ for $b < \$1000$;
- $u(b) = 1$ for $\$1000 \leq b < \3000 ;
- $u(b) = 2$ for $b \geq \$3000$.

Bob's budget right now is \$1100 (which would give him a utility of 1, for going to Miami).

Bob's wealthy friend Alice is aware of Bob's predicament and wants to offer him a "fair gamble." Define a *fair gamble* to be a random variable with expected value \$0. An example fair gamble (with two outcomes) is the following: \$-75 with probability $2/5$, and \$50 with probability $3/5$. Note that the expected value of this gamble is \$0, so it is indeed fair. If Bob were to accept this gamble, he would end up with \$1025 with probability $2/5$, and with \$1150 with probability $3/5$. In either case, Bob's utility is still 1, so Bob's expected utility for accepting this gamble is $(2/5) \cdot (1) + (3/5) \cdot (1) = 1$.

a (10 points). Find a fair gamble with two outcomes that would strictly decrease Bob's expected utility.

b (10 points). Find a fair gamble with two outcomes that would strictly increase Bob's expected utility.

2. Finding Nash equilibria of normal-form games. (50 points.)

Find *all* the Nash equilibria of each of the following five two-player normal-form games. Argue why the games have no other Nash equilibria. (Hint: for

some of these games, you may wish to use strict dominance or iterated strict dominance, because any strategy eliminated by (iterated) strict dominance cannot get positive probability in any Nash equilibrium. Also keep in mind that you may want to use strict dominance by a mixed strategy.)

4, 4	8, 2
2, 8	7, 7

0, 8	4, 0
2, 0	0, 1

7, 7	6, 8
9, 2	0, 1

3, 5	5, 4
1, 7	7, 6

4, 0	4, 0	1, 2
3, 5	3, 4	2, 4
4, 0	1, 1	5, 0

3. **Extensive-form games.** Consider the game below.

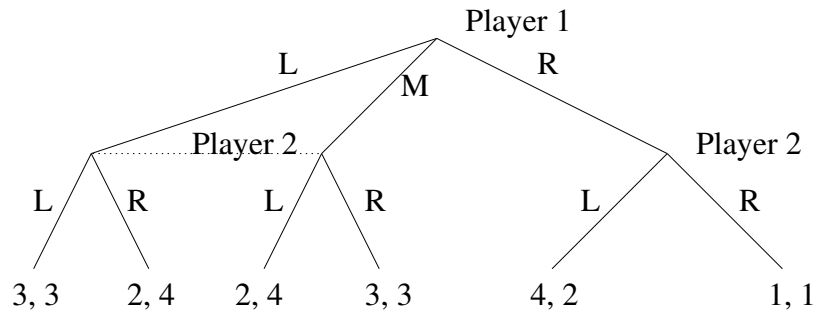


Figure 1: An extensive-form game with imperfect information.

- a (10 points). Give the normal-form representation of this game.
- b (10 points). Give a Nash equilibrium where player 1 sometimes plays Left. (Remember that you must specify each player's strategy at *every* information set.)
- c (10 points). What are the subgame perfect equilibria of the game? (Remember that you must specify each player's strategy at *every* information set.)