Due on March 26th, 2018
40 points total

General Directions: If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time. There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice.

All the answers must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a pdf file will also be accepted. Before submitting the pdf file, please make sure that it can be opened using any standard pdf reader (such as Acrobat Reader) and your entire answer is readable. **Handwritten answers or pdf files that cannot be opened will not be graded and will not receive any credit.**

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Problem 1 (40 points)
Let $G = (V, E)$ be an directed graph where $u : E \to \mathbb{Z}_+$ and $\ell : E \to \mathbb{Z}_+$ are functions that give non-negative integer upper and lower capacities for each edge, where $\ell(x, y) \leq u(x, y)$ for all $(x, y) \in E$. In addition to the usual flow constraints (capacity constraints and flow balance constraints), a feasible flow on this graph must also satisfy the property that the flow on an edge is at least its lower capacity.

(a) (5 points) Give an example of a graph with no feasible s-t flow.

(b) (20 points) Give an algorithm for finding a feasible flow from s to t (if it exists).
Hint: First, send flow on each edge equal to its lower capacity. Note that this can violate balance constraints. Argue that the total flow deficit is equal to the total flow surplus, including the s, t imbalance. Finally, create a new network and show that finding a max flow of a certain value on this new network solves for a flow that is now feasible .

(c) (15 points) Show how to find a minimum value feasible flow from the feasible flow you found in part (b).
Hint: Slightly alter the definition of residual networks to solve this problem.