**Due on March 4, 2019**
**39 points total**

**General directions:** We will exclusively use Python 3 for our programming assignments, and allow only the use of modules in the Python 3 standard library unless explicitly specified otherwise on an individual assignment basis. This forbids the use of common third-party libraries such as Numpy, Sympy, etc., but not the use of math or io.

Unless specified otherwise, for the X-th homework, download the single "hwX_skeleton.py" file from the course website, and rename it to "hwX.py" on your machine. When you are done and ready to submit, upload your file named **exactly** "hwX.py" on Gradescope for assignment "HW X (Programming)." When you upload your file, the autograder will run a simple test for each function so that you can confirm it was properly uploaded and executed. Generally, if an assignment involves printing or writing a file in a specific format, there will be at least one simple test that checks your output is formatted as we expect. These tests are not worth any credit — once the due date is over, your submission will be graded by a collection of additional test cases.

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

**Point values:** Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a ”+1” in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

**Problem 1 (12+1 points)**
Prove the following statement using induction:

The sum of the first $n$ positive odd integers is equal to $n^2$, i.e.,

$$\forall n \in \mathbb{Z}^+ .\ 1 + 3 + \cdots + (2n-1) = n^2.$$

**Problem 2 (12+1 points)**
Recall that $F_n$ is the $n^{th}$ number of the Fibonacci sequence, defined as follows:

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_n &= F_{n-1} + F_{n-2} \quad \text{if } n \geq 2. \end{aligned}$$

Prove that every positive integer can be written as the sum of distinct non-consecutive Fibonacci numbers, i.e.,

$$\forall n \in \mathbb{Z}^+ .\ \exists i_1, \ldots, i_t \in \mathbb{Z}^+, \text{ such that } i_{k+1} > i_k + 1 \text{ for } k = 1, \ldots, (t-1) \text{ and}$$

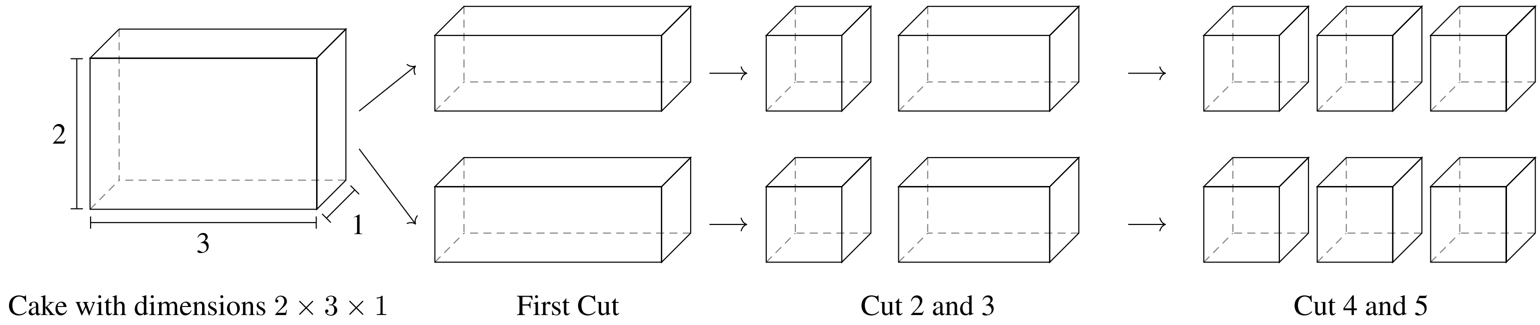
$$n = \sum_{k=1}^{t} F_{i_k}.$$

For example, $7 = 2 + 5 = F_3 + F_5$.

**Problem 3 (12+1 points)**
Suppose we have a cake. The cake is a rectangular prism with integral dimensions, i.e. $i, j, k \in \mathbb{Z}^+$ and the cake has dimensions $i \times j \times k$. We want to cut the cake into pieces such that each piece is a single unit cube (dimensions $1 \times 1 \times 1$). We are only allowed to make one cut at a time, and each cut must result in a smaller rectangular prism (e.g. each cut is a straight cut along one dimension). Only integral dimensions are allowed.

For example, suppose our cake has dimensions $2 \times 3 \times 1$. We can cut the cake into $6$ unit cubes using $5$ cuts, see Figure 1.

For a cake with dimensions $i \times j \times k$, we want to make a sequence of cuts so that we divide the cake into $n$ unit cube pieces where $n = i \cdot j \cdot k$. Prove that no matter which sequence of cuts you make, the minimum number of cuts needed is $n - 1$.



Fgure 1: Cutting a cake into unit cubes.