# Logical Equivalences

**Order of operations by precedence**: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

$$A \wedge B \leftrightarrow B \wedge A \qquad \text{(commutativity of } \wedge\text{)}$$
$$A \vee B \leftrightarrow B \vee A \qquad \text{(commutativity of } \vee\text{)}$$
$$(A \wedge B) \wedge C \leftrightarrow A \wedge (B \wedge C) \qquad \text{(associativity of } \wedge\text{)}$$
$$(A \vee B) \vee C \leftrightarrow A \vee (B \vee C) \qquad \text{(associativity of } \vee\text{)}$$
$$A \wedge A \leftrightarrow A \qquad \text{(idempotence for } \wedge\text{)}$$
$$A \vee A \leftrightarrow A \qquad \text{(idempotence for } \vee\text{)}$$
$$A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C) \qquad \text{(distributivity of } \wedge \text{ over } \vee\text{)}$$
$$A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C) \qquad \text{(distributivity of } \vee \text{ over } \wedge\text{)}$$
$$\neg\neg A \leftrightarrow A \qquad \text{(double negation)}$$
$$A \rightarrow B \leftrightarrow \neg A \vee B \qquad \text{(implication in terms of } \vee\text{)}$$
$$\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B \qquad \text{(De Morgan for } \wedge\text{)}$$
$$\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B \qquad \text{(De Morgan for } \vee\text{)}$$
$$A \vee \neg A \leftrightarrow \mathbf{T} \qquad \text{(tautology)}$$
$$A \wedge \neg A \leftrightarrow \mathbf{F} \qquad \text{(contradiction)}$$

## Universal/Existential Quantifiers

$$\neg\forall x.\, P(x) \leftrightarrow \exists x.\, \neg P(x)$$
$$\neg\exists x.\, P(x) \leftrightarrow \forall x.\, \neg P(x)$$
$$\exists x \exists y.\, P(x,y) \leftrightarrow \exists y \exists x.\, P(x,y)$$
$$\forall x \forall y.\, P(x,y) \leftrightarrow \forall y \forall x.\, P(x,y)$$

## Set Notation

$$x \in A \leftrightarrow x \text{ is an element of } A$$
$$x \notin A \leftrightarrow \neg(x \in A)$$
$$\emptyset \leftrightarrow \{\}, \text{the empty set}$$
$$A \cup B = \{x \mid x \in A \cup B \leftrightarrow x \in A \vee x \in B\}$$
$$A \cap B = \{x \mid x \in A \cap B \leftrightarrow x \in A \wedge x \in B\}$$
$$\overline{A} = \{x \mid x \in \overline{A} \leftrightarrow x \notin A\}$$
$$A \setminus B = \{x \mid x \in A \setminus B \leftrightarrow x \in A \wedge x \notin B\}$$
$$A \subseteq B \leftrightarrow \forall x.\, x \in A \rightarrow x \in B$$

## Set Equivalences

$$A \cap B = B \cap A \qquad \text{(commutativity of } \cap\text{)}$$
$$A \cup B = B \cup A \qquad \text{(commutativity of } \cup\text{)}$$
$$(A \cap B) \cap C = A \cap (B \cap C) \qquad \text{(associativity of } \cap\text{)}$$
$$(A \cup B) \cup C = A \cup (B \cup C) \qquad \text{(associativity of } \cup\text{)}$$
$$A \cap A = A \qquad \text{(idempotence for } \cap\text{)}$$
$$A \cup A = A \qquad \text{(idempotence for } \cup\text{)}$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \qquad \text{(distributivity of } \cap \text{ over } \cup\text{)}$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \qquad \text{(distributivity of } \cup \text{ over } \cap\text{)}$$
$$\overline{\overline{A}} = A \qquad \text{(double complement)}$$
$$A \setminus B = A \cap \overline{B} \qquad \text{(subtraction in terms of } \cap\text{)}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B} \qquad \text{(De Morgan for } \cap\text{)}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \text{(De Morgan for } \cup\text{)}$$
$$A \cap \emptyset = \emptyset$$
$$A \cup \emptyset = A$$
$$A \cap \overline{A} = \emptyset$$