COMPSCI 270 : Artificial Intelligence

Homework 3 : First-order logic

Due: March $14^{th}, 11:59PM$

Please read the rules for assignments on the course web page. Please use Piazza for questions and Gradescope to turn in the assignment. If you want to use LaTeX that is great. If you write by hand and scan, please make sure that your handwriting is clear; illegible/ambiguous handwriting will receive no points. Please make sure that your answers are clear and be careful with parentheses and symbols. For problems 3 and 4, please include the steps you took.

In your gradescope submission, please submit each section as a separate page, making it a total of four pages. It is acceptable to have multiple pages for a problem.

1 Convert the following English sentences to first-order logic

(4 * 5 = 20 points)

1. Every thing that is an enemy of some thing that is an enemy of me is a friend of me ("the enemy of my enemy is my friend").
2. Every person who is smart and studies hard will get a higher score than every person who is not smart and does not study hard.
3. A silver medal is worth more than a bronze medal, if they are medals in the same event.
4. Every thing that walks like a duck and talks like a duck is either a duck or a human imitating a duck.

2 Choose all correct first-order logic options for given English sentences. Some questions have multiple correct options; choose all options that are correct (4 * 5 = 20 points)

1. No one who cheats wins.
   (a) $\neg\exists x\ (\text{cheats}(x) \wedge \text{wins}(x))$
   (b) $\forall x\ (\neg\text{cheats}(x) \Rightarrow \text{wins}(x))$
   (c) $\neg\exists x\ (\text{cheats}(x) \Rightarrow \text{wins}(x))$
   (d) $\forall x\ (\text{cheats}(x) \Rightarrow \neg\text{wins}(x))$

2. If anyone is noisy, everyone is annoyed.

   (a) $\forall y\ ((\forall x \text{ noisy}(x)) \Rightarrow \text{annoyed}(y))$

   (b) $\forall x \forall y\ (\text{noisy}(x) \Rightarrow \text{annoyed}(y))$

   (c) $(\exists x \text{ noisy}(x)) \Rightarrow \forall y \text{ annoyed}(y)$

   (d) $\forall y \exists x\ (\text{noisy}(x) \Rightarrow \text{annoyed}(y))$

3. Mary does not hate anyone.

   (a) $\neg \exists x \text{ hate}(\text{Mary}, x)$

   (b) $\exists x\ \neg \text{hate}(\text{Mary}, x)$

   (c) $\forall x\ \neg \text{hate}(\text{Mary}, x)$

   (d) $\neg \forall x \text{ hate}(\text{Mary}, x)$

4. Some boys in the class are taller than all the girls.

   (a) $(\exists x)\ (\text{boy}(x) \Rightarrow (\forall y)\ (\text{girl}(y) \wedge \text{taller}(x, y)))$

   (b) $(\exists x)\ (\text{boy}(x) \wedge (\forall y)\ (\text{girl}(y) \wedge \text{taller}(x, y)))$

   (c) $(\exists x)\ (\text{boy}(x) \Rightarrow (\forall y)\ (\text{girl}(y) \Rightarrow \text{taller}(x, y)))$

   (d) $(\exists x)\ (\text{boy}(x) \wedge (\forall y)\ (\text{girl}(y) \Rightarrow \text{taller}(x, y)))$

3 Apply resolution to obtain the most general conclusion possible (20 points)

Write the conclusion both in first-order logic and in English.

- $\forall x, y$: LovesTheCombinationOf(John, x, y) $\vee$ MakesSick(x,John) $\vee$ RuinsTasteOf(y, x)
- $\forall v, w$: $\neg$ LovesTheCombinationOf(v, Rice, w) $\vee$ Flavorful(w)

For brevity, use: LTCO = LovesTheCombinationOf, MS = MakesSick, RTO = RuinsTasteOf, F = Flavorful
Hint: Set up the knowledge base. Use substitutions and resolution to arrive at the conclusion.

4 Enemies and Friends (40 points)

Suppose you know the following.

1. For any x, any enemy of any enemy of x is a friend of x. (stated more naturally in English, in 1.1)
2. If x is an enemy of y, then y is an enemy of x.
   $\forall x, y : \text{enemyOf}(x, y) \Rightarrow \text{enemyOf}(y, x)$
3. Every x has at least two enemies.
   $\forall x : \exists y, z : \neg(y = z) \wedge \text{enemyOf}(x, y) \wedge \text{enemyOf}(x, z)$

Formally prove that Alice has at least one friend that is not equal to herself.
Hint: An informal proof outline goes as follows: Alice has an enemy; that enemy in turn has two enemies; because there are two of them, one of them is not equal to Alice; and that one must be Alice's friend. You can use Skolemization and transitivity of equality ($a = b$ and $b = c$ implies $a = c$) as needed.