HW2 OUT!

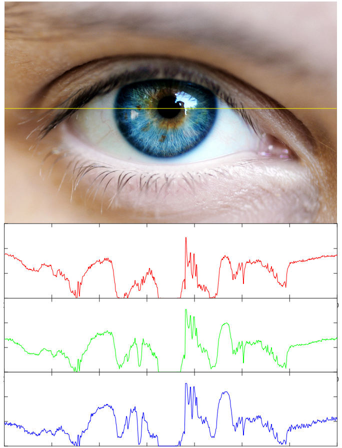
Image Differentiation

COMPSCI 527 — Computer Vision

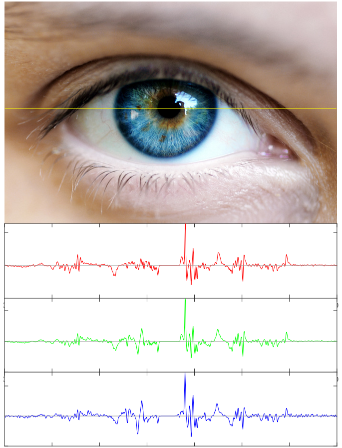
Outline

What Does Differentiating an Image Mean?

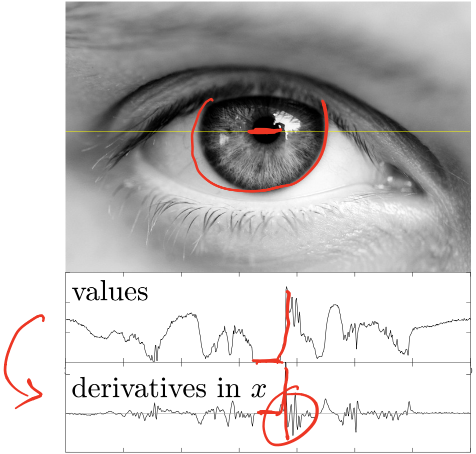
Values

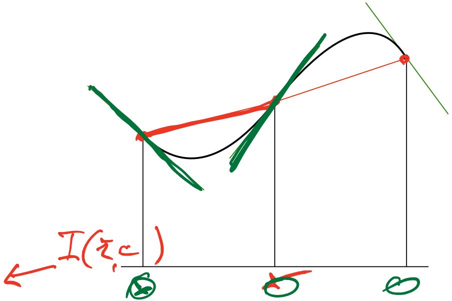


Derivatives in x



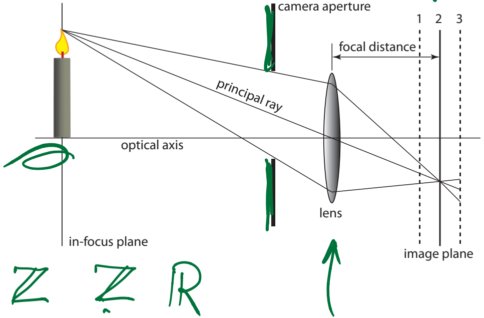
What Does Differentiating an Image Mean?

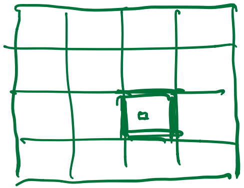




Can we reconstruct the black curve?

Cameras

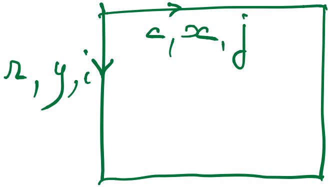




$C(x, y)$

$x, y \in \mathbb{R}$

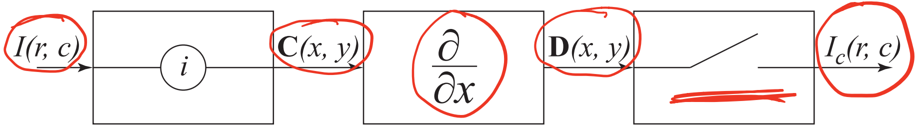
Can I recover $C(x, y)$ from $I(r, c)$ $\in \mathbb{Z}$



$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{R}$
r	i	y
c	j	x

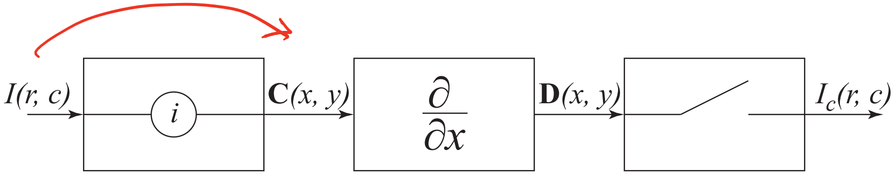
A Conceptual Pipeline

$I_x(r,c)$

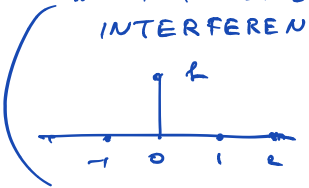


- Somehow reconstruct the continuous sensor irradiance C from the discrete image array I
- Differentiate C to obtain D
- Sample the derivatives D back to the pixel grid
- Each would be hard to implement
- Surprisingly, *the cascade turns out to be easy!*

From Discrete Array to Sensor Irradiance



NON INTER-SYMBOL INTERFERENCE

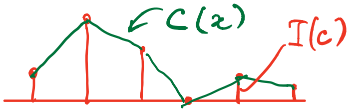


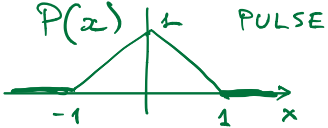
$$P(j) = \delta(j)$$

$\Downarrow$

INTERPOLATION

What would the transformation from *I* to *C* look like formally, if we could find one? Example: Linear interpolation

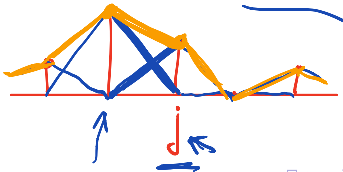


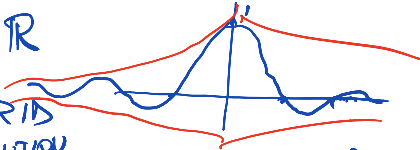


$$C(x) = \sum_{j=-\infty}^{\infty} I(j)\,P(x-j)$$

$\in \mathbb{R}$

HYBRID CONVOLUTION





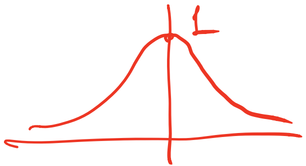
$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

Linear Interpolation as a Hybrid Convolution

$$\mathbf{C}(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i, j) P(x - j, y - i)$$

in 2D

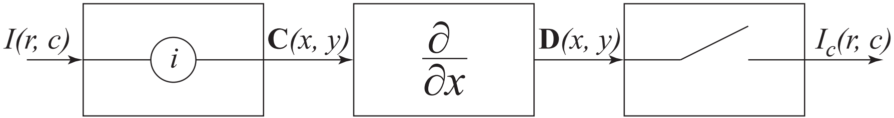
Gaussian Instead of Triangle



- Noise $\Rightarrow$: fit rather than interpolating
- Noise $\Rightarrow$: filter with a truncated Gaussian
- $P(x, y) = G(x, y) \propto e^{-\frac{1}{2}\frac{x^2+y^2}{\sigma^2}}$

$$C(x,y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i,j)\, G(x-j, y-i)$$

Differentiating



$\mathbf{C}(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i, j) G(x - j, y - i)$

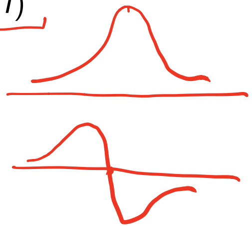
(still don't know how to do this, just plow ahead)

$\mathbf{D}(x, y) = \frac{\partial \mathbf{C}}{\partial x}(x, y) = \frac{\partial}{\partial x} \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i, j) G(x - j, y - i)$

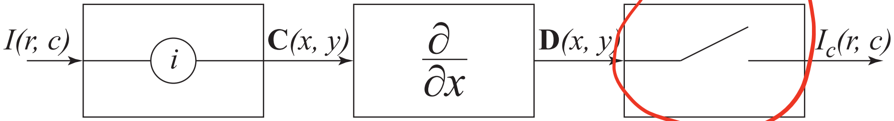
$\mathbf{D}(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i, j) G_x(x - j, y - i)$

- We transferred the differentiation to G, and we know how to do *that*!

(still don't know how to implement a hybrid convolution)



Sampling



$$\mathbf{D}(x,y) = \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} I(i,j)G_x(x-j, y-i)$$

- We are interested in the values of $\mathbf{D}(x,y)$ *on the integer grid*: $x \to c$ and $y \to r$

$$I_c(r,c) = \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} I(i,j)G_x(c-j, r-i)$$

Wait! This is a standard, discrete convolution

We know how to do *that*!

To differentiate an image array, convolve it (discretely) with the (sampled, truncated) derivative of a Gaussian

The Derivatives of a 2D Gaussian

- The Gaussian function is separable:

  $$G(x, y) \propto e^{-\frac{1}{2}\frac{x^2+y^2}{\sigma^2}} = g(x)\,g(y) \text{ where}$$

  $$g(x) = e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

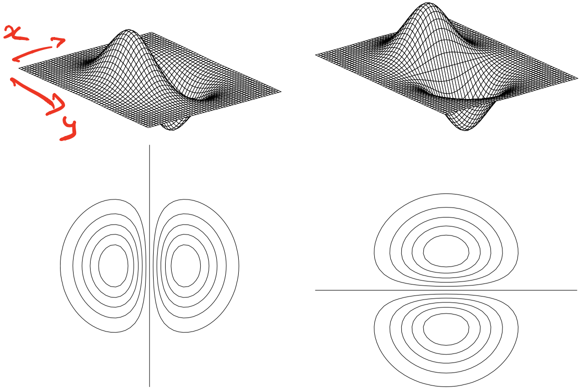
  $$G_x(x, y) = \frac{\partial G}{\partial x} = \frac{\partial g}{\partial x}\,g(y) = d(x)g(y)$$

  $$d(x) = \frac{dg}{dx} = -\frac{x}{\sigma^2}g(x)$$

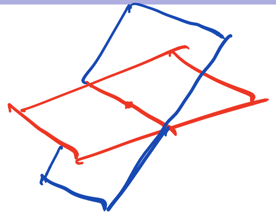
- Similarly, $G_y(x, y) = g(x)d(y)$
- Differentiate (smoothly) in one direction, smooth in the other
- $G_x(x, y)$ and $G_y(x, y)$ are separable as well

The Derivatives of a 2D Gaussian

$$G_x(x, y) = d(x)g(y) \text{ and } G_y(x, y) = g(x)d(y)$$



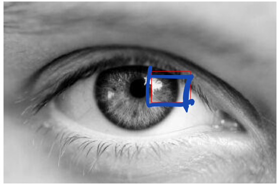
Normalization



- Can normalize $d(c)$ and $g(r)$ separately
- For smoothing, constants should not change:
- We want $k * g = k$ (we saw this before)
- For differentiation, a unit ramp should not change:
  $u(r, c) = c$ is a ramp
- We want $u * d = u$ (see notes for math)

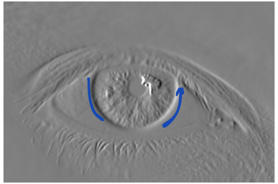
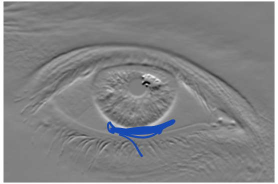
The Image Gradient

- Image *gradient*: $\nabla I(r, c) = \frac{\partial I}{\partial \mathbf{x}} = \mathbf{g}(r, c) = \begin{bmatrix} I_x(r, c) \\ I_y(r, c) \end{bmatrix}$
- View 1: Two scalar images $I_x(r, c)$, $I_y(r, c)$

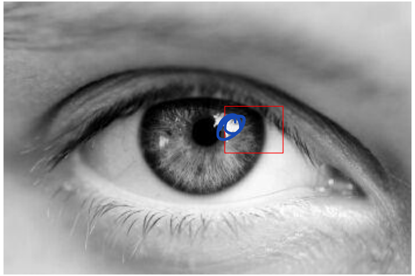
$\|\nabla I\|$

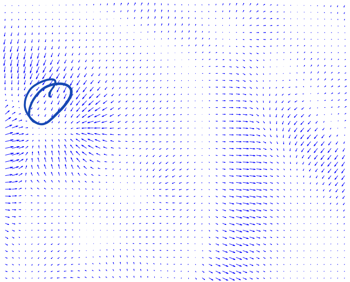
I_x I_y

The Image Gradient

- View 2: One vector image $\mathbf{g}(r, c)$





- We can now measure changes of image brightness
- *Edges* are of particular interest