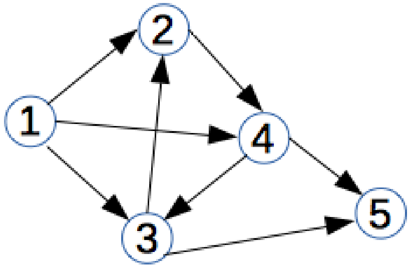
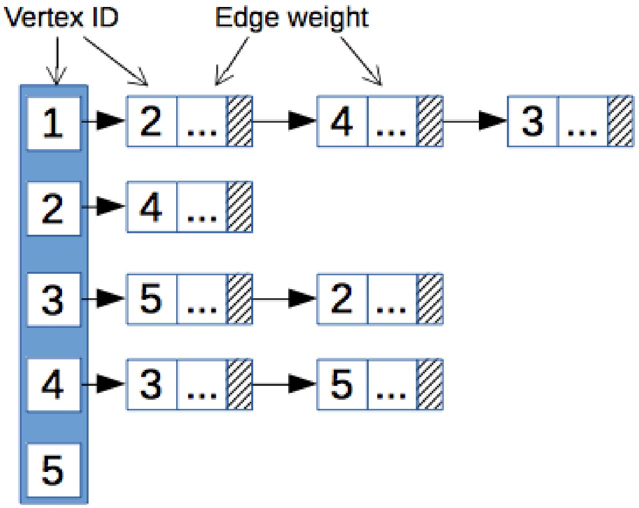
Compsci 201
Graphs, APTS, and More
Part 1 of 5





Susan Rodger

April 17, 2020

W is for …

- **World Wide Web**
  - Invented in 1989 – Sir Tim Berners Lee

Sir Tim Berners-Lee invented the World Wide Web in 1989.

- **Wifi**
  - We need this everyday

Announcements

- APT-7 due Thursday, April 16
- APT-8 due Tuesday, April 21
- Assignment P6 Huffman due April 22
  - All late work turned in by April 22 (APTs and Asgns)
  - Except Huffman grace through April 23
- Assignment P7 Optional out – Extra Credit!

- Exam 2 will focus on grading that the next few days
- APT Quiz 2 is April 12-18 – Your own work!
- Final Exam will be on April 30 – any time on this day

- Discussion 13, Monday, April 20, Last one!

Plan for the Day

- Logistics to Wrap up the Course
    - Extensions must end, Final Exam
- APTs and Algorithmic Concepts
    - Reminder about greedy
    - Memoizing

- Graphs and Graph Algorithms
    - Concepts, terminology, APTs
    - Toward Dijkstra's Algorithm

Last Chance Last Chance

- Extensions on APTs and Assignments
  - You must fill out the extension form
  - We can't take anything late after **April 22**!
    - Except P6 Huffman
      - only one grace day April 23!
      - NO LATE SUBMISSIONS!!!!!!
    - Except P7 Create
      - til Sunday April 26 with no penalty

Calculating your grade

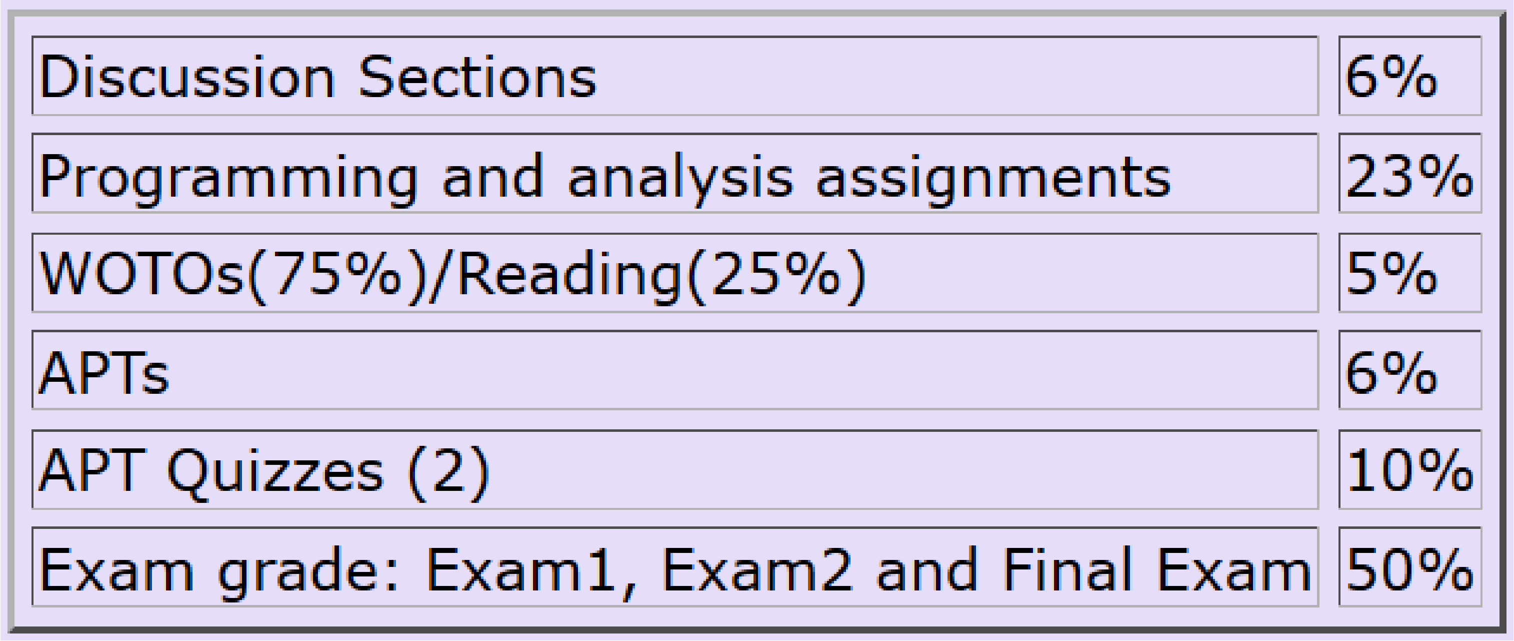
- Discussion sections (6% of your grade)
  - Add up your total points, max points are 13 disc*4pt =52
  - We drop 8 points
  - Divide your total points by 44.
  - Examples:
    - 42pts/44 = 95%
    - 50pts/44 = 100% (can't get > 100)

Calculating your grade (part 2)

- Programming and Analysis (23% of your grade)
  - Max points are: 178pts
  - Divide your total by 178 for your score
- WOTOS (3.75% of your grade)
  - If you have 40% of the points – you get 100%
- Reading Zybooks or 6 extra APTs (1.25% of grade)
- APTs (6%)
  - 38 APTs, 10 pts each 380+ points is 100%

Calculating your grade (part 3)

- 90 A-, 94 A

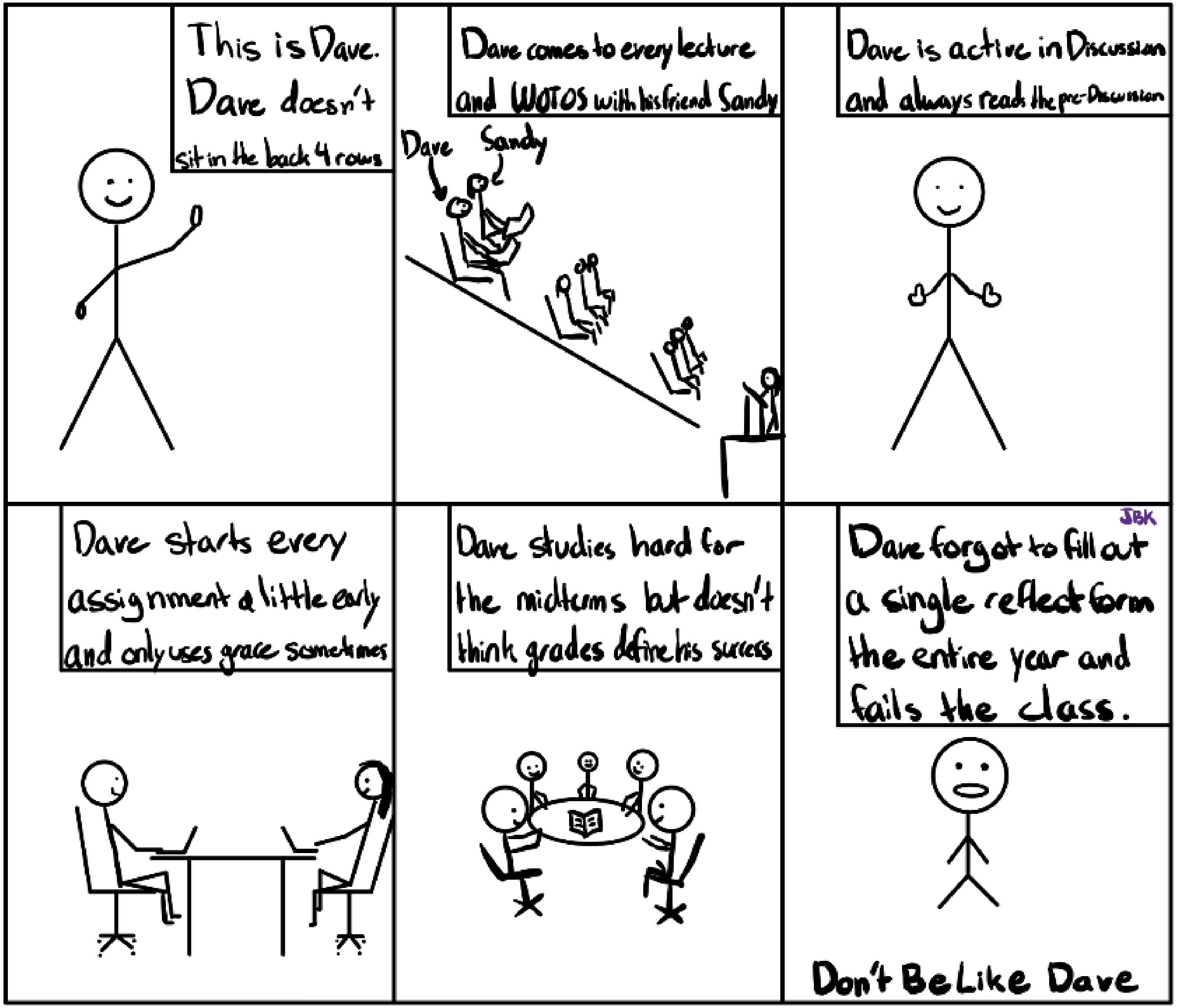


Discussion Sections	6%
Programming and analysis assignments	23%
WOTOs(75%)/Reading(25%)	5%
APTs	6%
APT Quizzes (2)	10%
Exam grade: Exam1, Exam2 and Final Exam	50%

Final Exam

- Final exam – must take on Thursday, April 30
- Must take in a 24 hour window.
- Once you start - have 3 hours plus 1 hour
  - That is 4 hours total
- About a 2 hour exam.
- Similar format to Exam 2
- If you don’t take it get a 0.
- Exam grade is MAX of (exam1, exam2, final exam)

P7 Create assignments coming in …


This is Dave. Dave doesn't sit in the back 4 rows
Dave comes to every lecture and WOTOS with his friend Sandy
Dave
Sandy
Dave is active in Discussion and always reads the pre-Discussion
Dave starts every assignment a little early and only uses grace sometimes
Dave studies hard for the midterms but doesn't think grades define his success
JBK
Dave forgot to fill out a single reflect form the entire year and fails the class.
Don't Be Like Dave

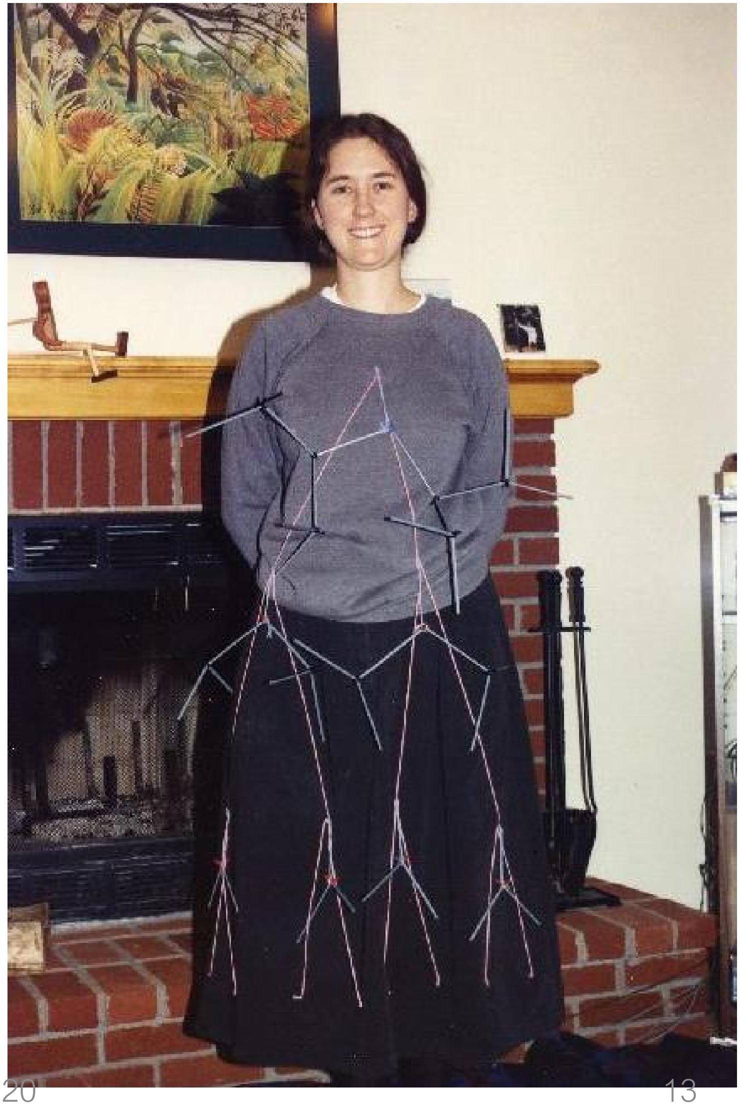
P7 Create assignments coming in …

Be A UTA
https://www.cs.duke.edu/undergrad/uta

- SIGN UP NOW by April 29 or before…
- CompSci 201
- CompSci 101
  - Python
- CompSci 94
  - Programming with Alice, so easy to learn

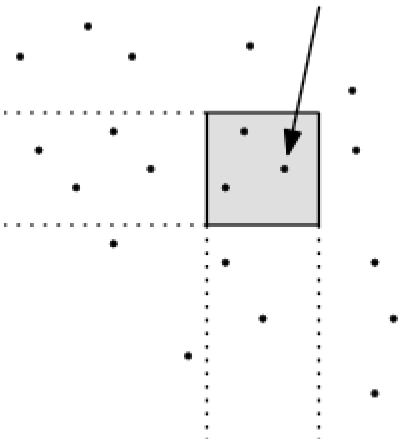
CS Concepts Coming Alive

- What data structure is this?

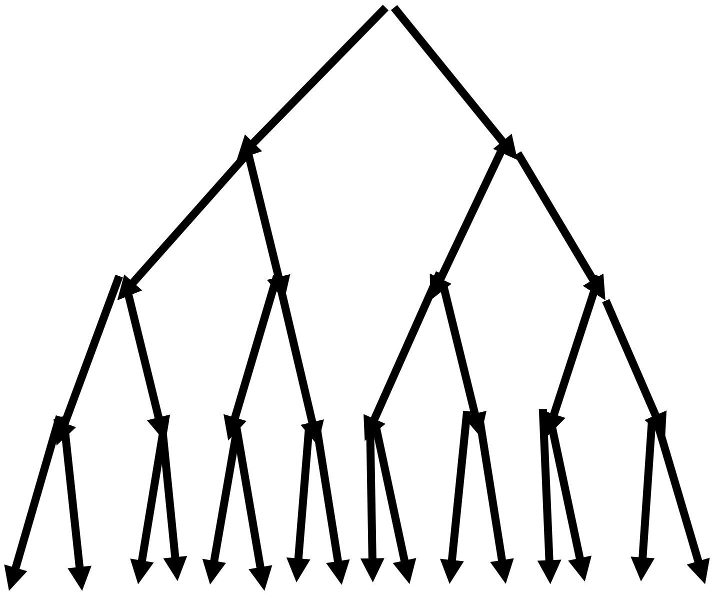


2D-range tree

- Search in x-y plane
- Main tree organized by x-values
- Subtree organized by y values



Binary Search tree of points in the plane – sorted by X-value



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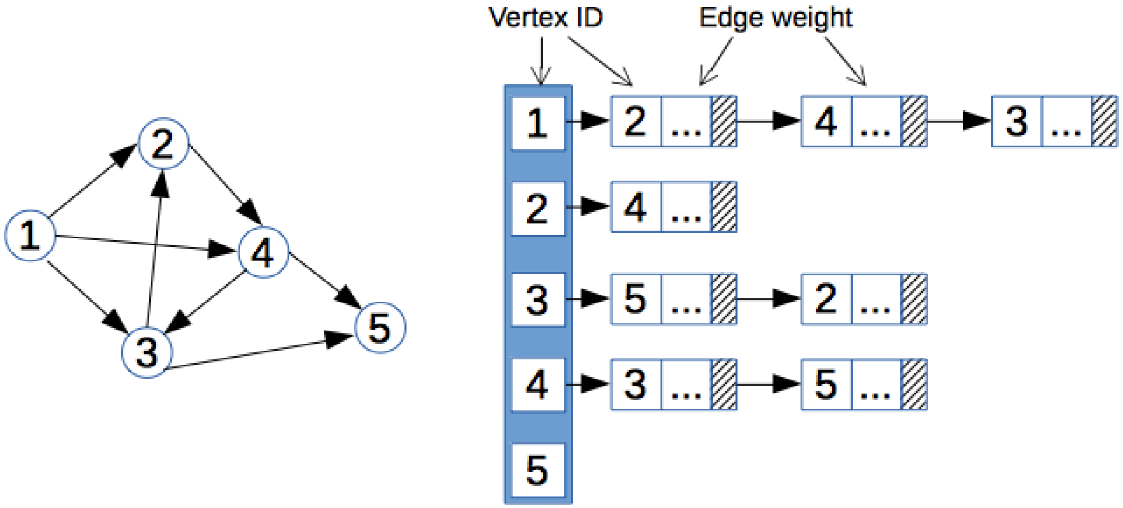
Part 2 of 5

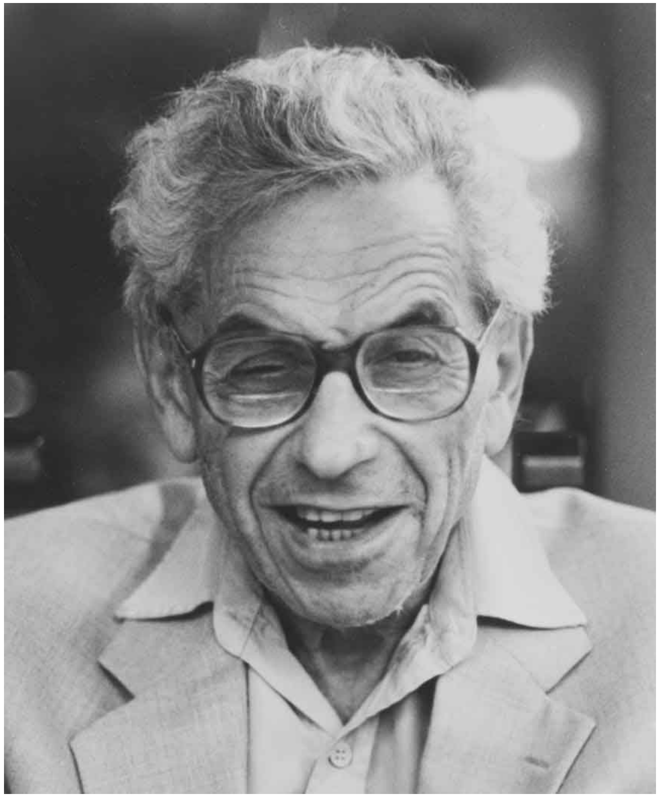


Susan Rodger

April 17, 2020

On to Graphs: From Bacon to Erdös

- https://mathscinet.ams.org/mathscinet/freeTools.html?version=2



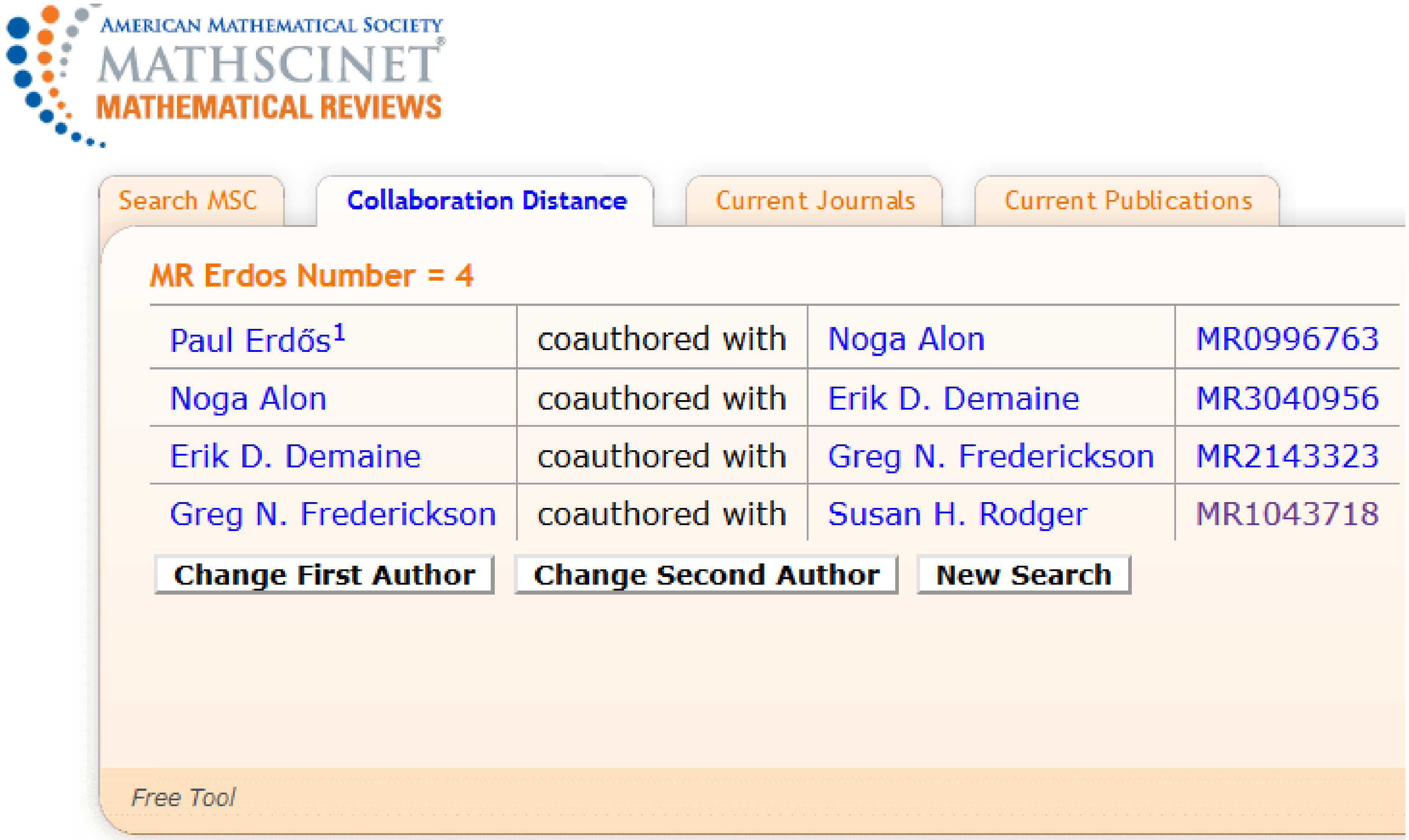
Bacon Number and Erdös Number

- Some actors are prolific: lots of movies
  - Chris in movie with Sam in movie with K. Bacon
    - Chris has a Bacon number of two
- Some authors are prolific: lots of papers/articles
  - Tina wrote paper with Tom wrote with P. Erdös
    - Tina has an Erdös number of two
- Graph terminology: connecting nodes with edges
  - *In-movie-with* or *wrote-paper-with* is *edge*

Erdös Numbers

- Authors connected by authorship/paper-writing

https://mathscinet.ams.org/mathscinet/freeTools.html?version=2

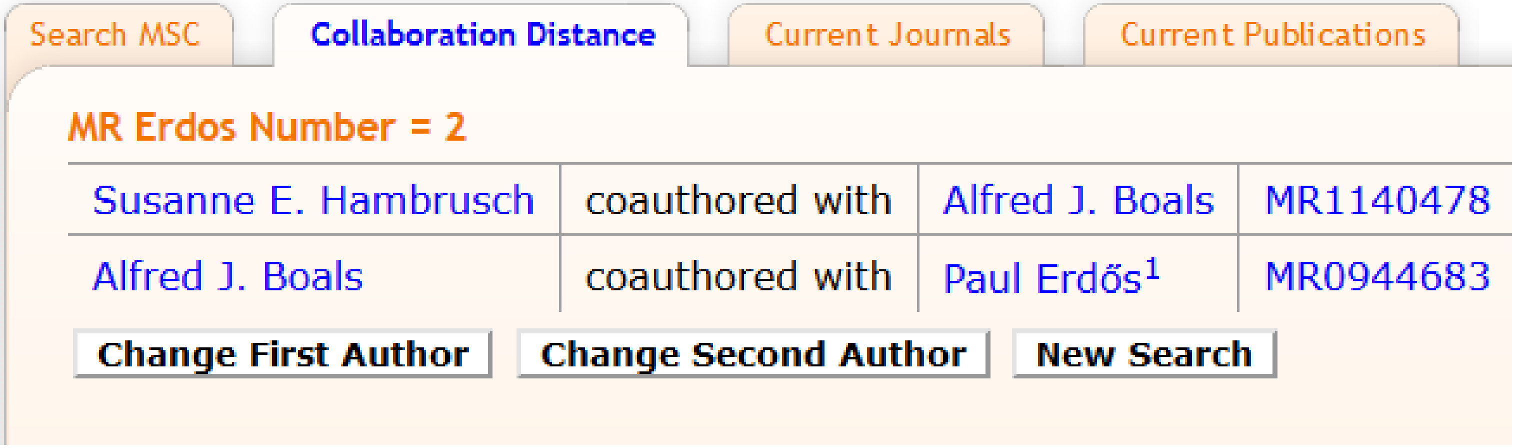


MR Erdos Number = 4

Paul Erdős[1]	coauthored with	Noga Alon	MR0996763
Noga Alon	coauthored with	Erik D. Demaine	MR3040956
Erik D. Demaine	coauthored with	Greg N. Frederickson	MR2143323
Greg N. Frederickson	coauthored with	Susan H. Rodger	MR1043718

Erdös Numbers

- Authors connected by authorship/paper-writing



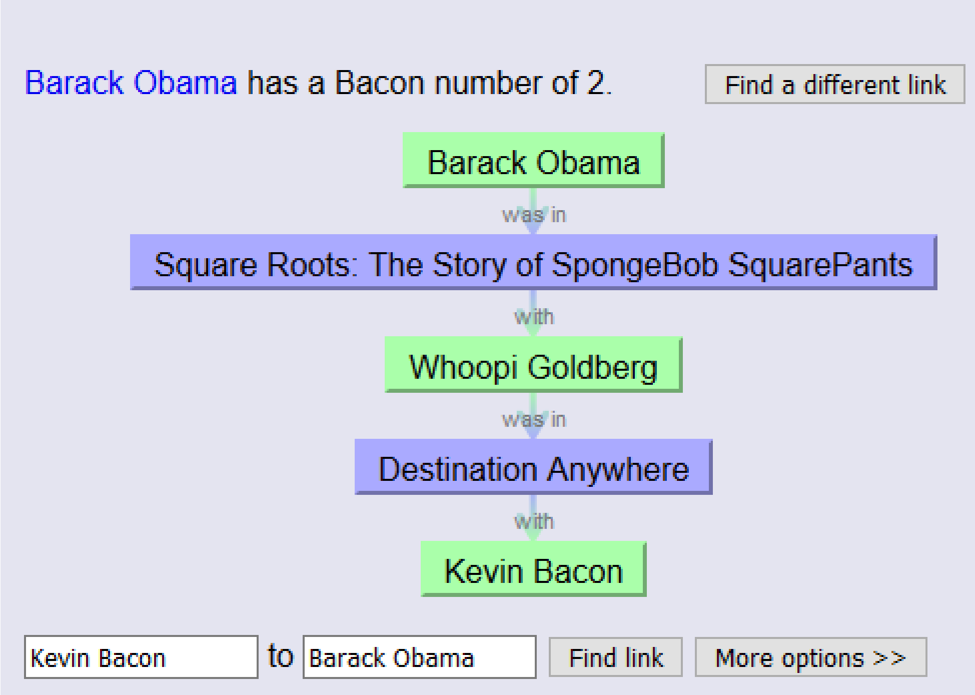
MR Erdos Number = 2

Susanne E. Hambrusch	coauthored with	Alfred J. Boals	MR1140478
Alfred J. Boals	coauthored with	Paul Erdős[1]	MR0944683

Bacon Number

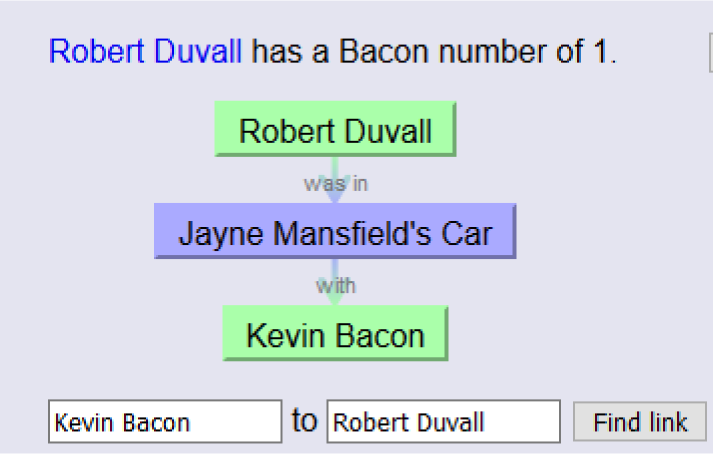
- Actors connected by acting/movie-roles

https://oracleofbacon.org/



Barack Obama has a Bacon number of 2.

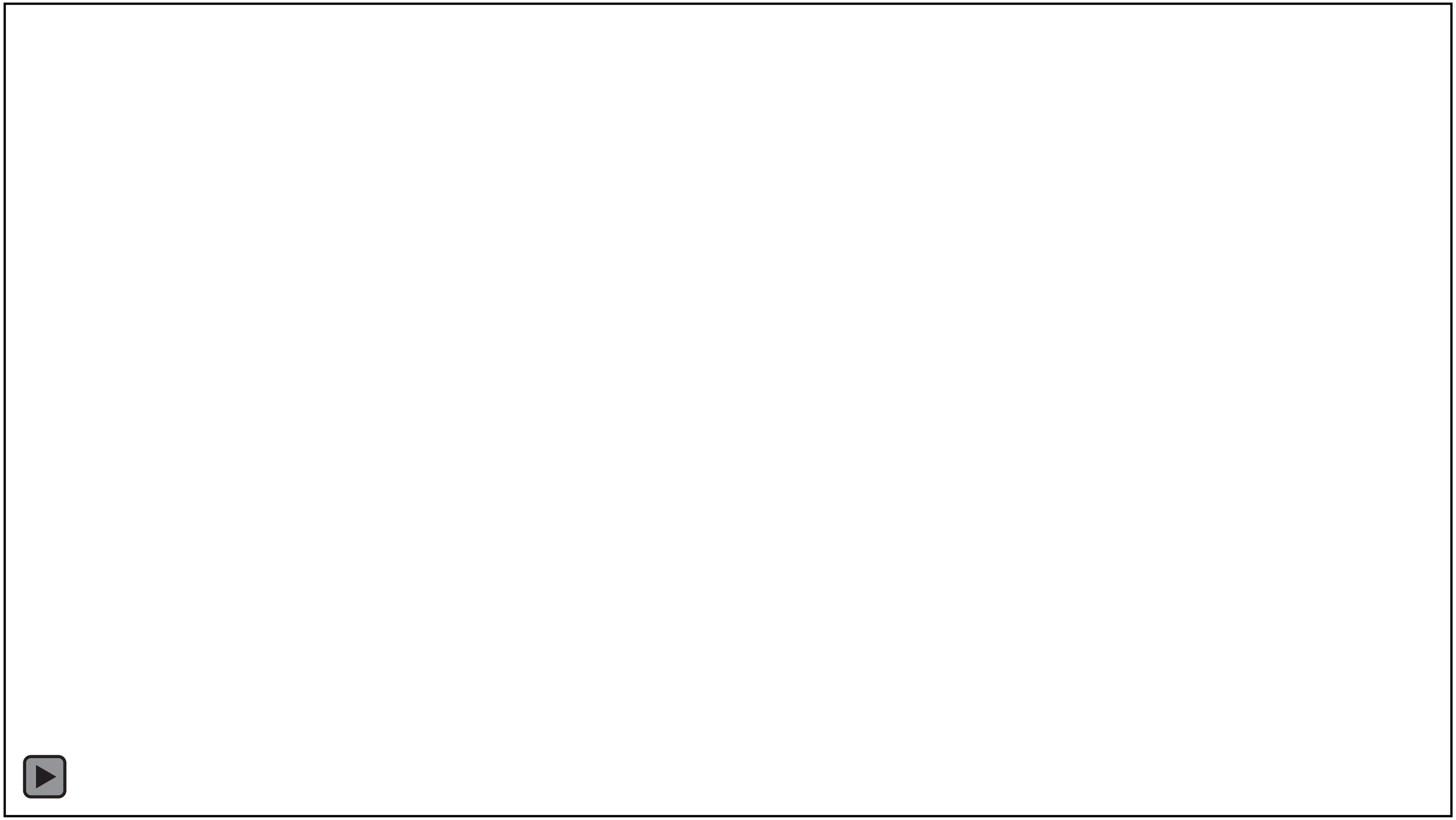
Kevin Bacon to Barack Obama



Robert Duvall has a Bacon number of 1.

Kevin Bacon to Robert Duvall

Obama in Movie

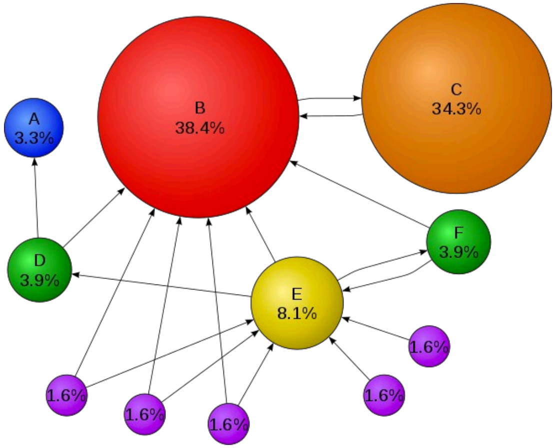


Widely Used Mobile Apps

- Google Maps, Uber, Lyft
  - Why is UI important for drivers
  - Why shortest path algorithms important
  - Which use graphs?

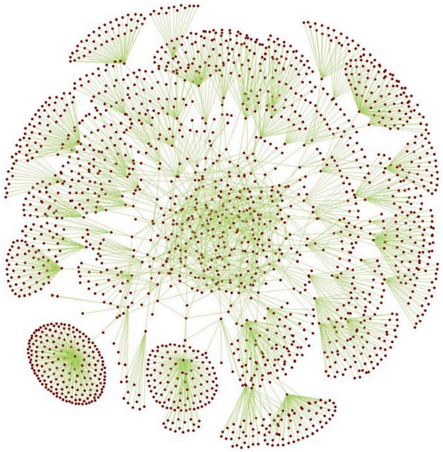
Why Graphs are Important

- **Google/Pagerank models webpages as a graph**
  - Nodes are webpages
  - Links are hyperlinks between pages
  - Weights based on "importance" of link/page
  - https://en.wikipedia.org/wiki/PageRank



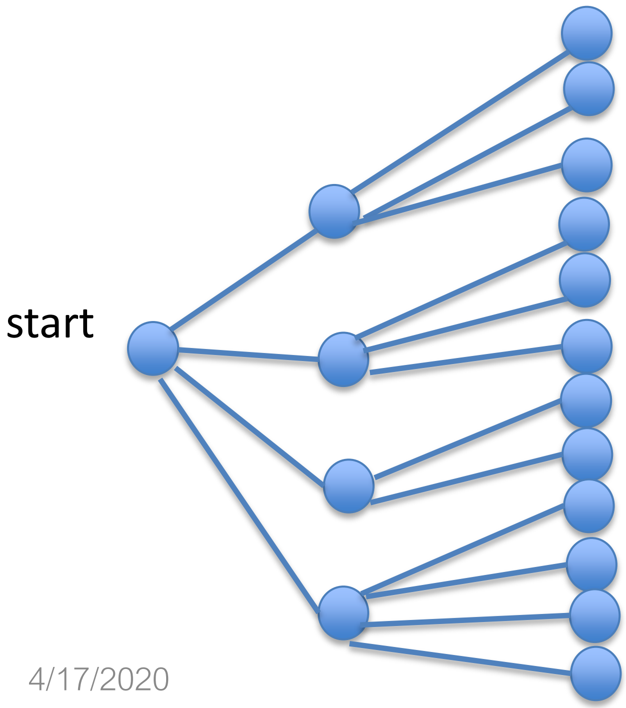
Is the Internet a Graph? It depends …

- Internet as graph
  - Nodes are anything with an IP address (IP)
  - Nodes are Autonomous Systems (AS)
  - Edges connect Thermostat to Website or …



The Coronavirus graph you don't see

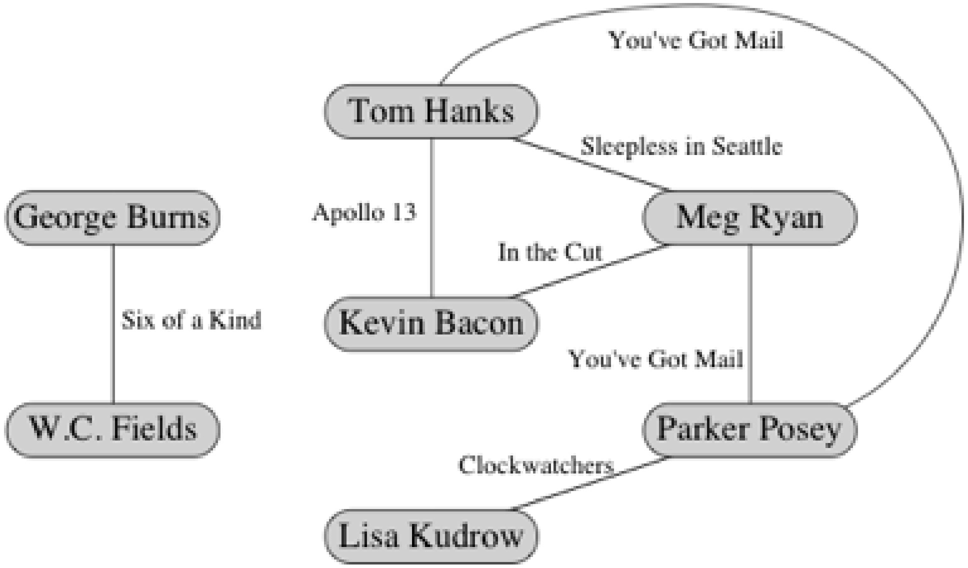
- Who infected who
  - Nodes – people
  - Edges – person A infected person B
  - Need a lot of testing to make this graph



You can see the growth is exponential, HUGE!

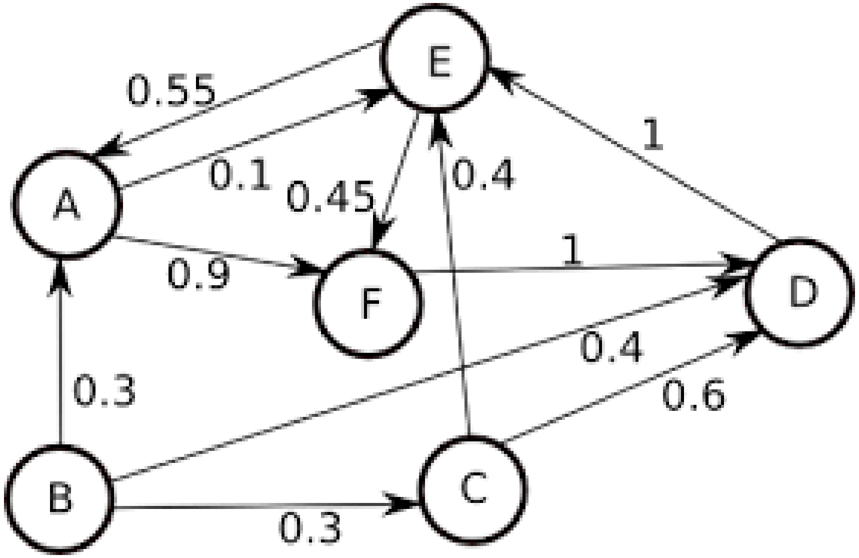
Graphs

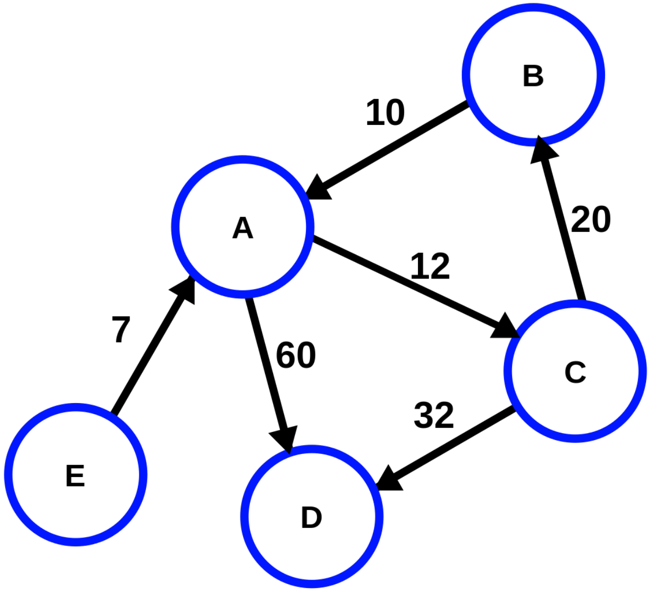
- **Graphs are collections of vertices and edges**
  - Vertices or nodes, edges or links
  - Undirected graph Tom-Kevin and Meg-Kevin
  - Sometimes edges have weights



Directed (weighted) Graph

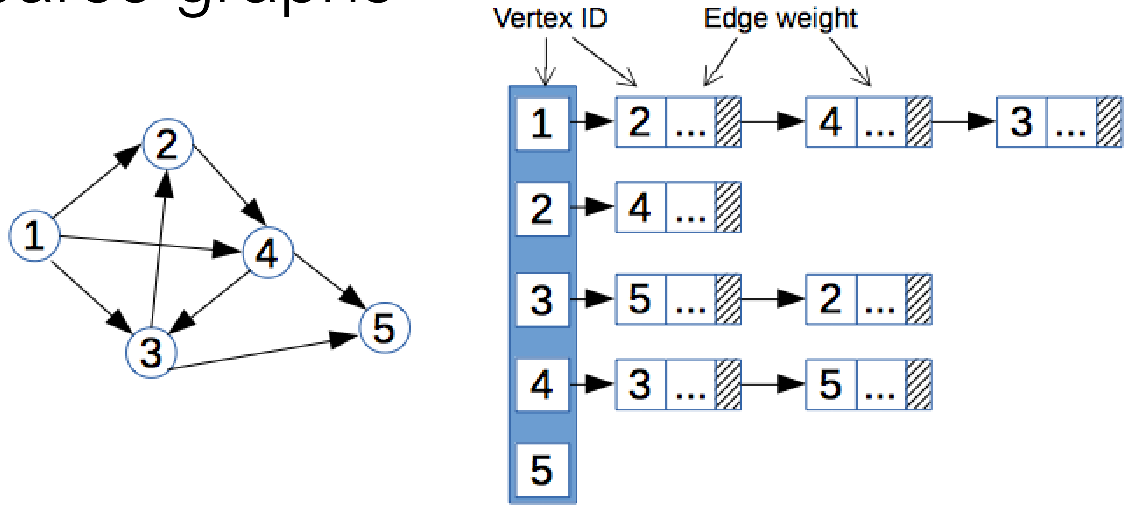
- **Edges can have direction: directed graph**
  - Not Facebook. Yes Tinder?





Data Structures for Graphs

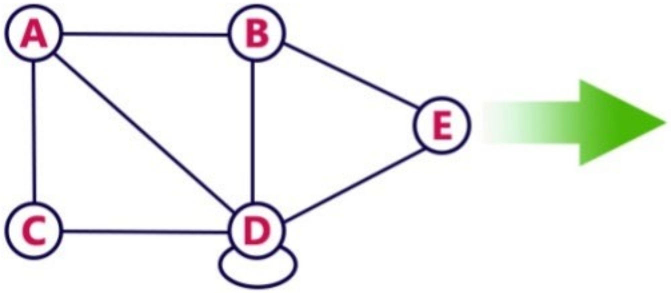
- **Use number for vertex, index into array**
  - Can use string and map as well
- **Adjacency List Representation**
  - Good for sparse graphs



http://lagodiuk.github.io/computer_science/2016/12/19/efficient_adjacency_lists_in_c.html

Adjacency Matrix

- **Good for dense graphs, vertices still numbers**
  - Symmetric matrix if undirected
  - Can have weights instead of 0,1



$$\begin{array}{c|ccccc|} & A & B & C & D & E \\ A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 1 & 0 \\ D & 1 & 1 & 1 & 1 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{array}$$

https://www.oreilly.com/library/view/php-7-data/9781786463890/32fd15e8-423f-49aa-84c2-db1518023299.xhtml

Theory and Practice

- Code is often simpler with Adjacency "list"
  - **`Map<String,Set<String>>`** for "list"
  - Vertex identified by String
  - Connected-by-edge? set of vertices
  - Need something more for weighted graphs

- For APTs, this is a good approach as we'll see
  - Simple to make, simple to use, scaling? meh

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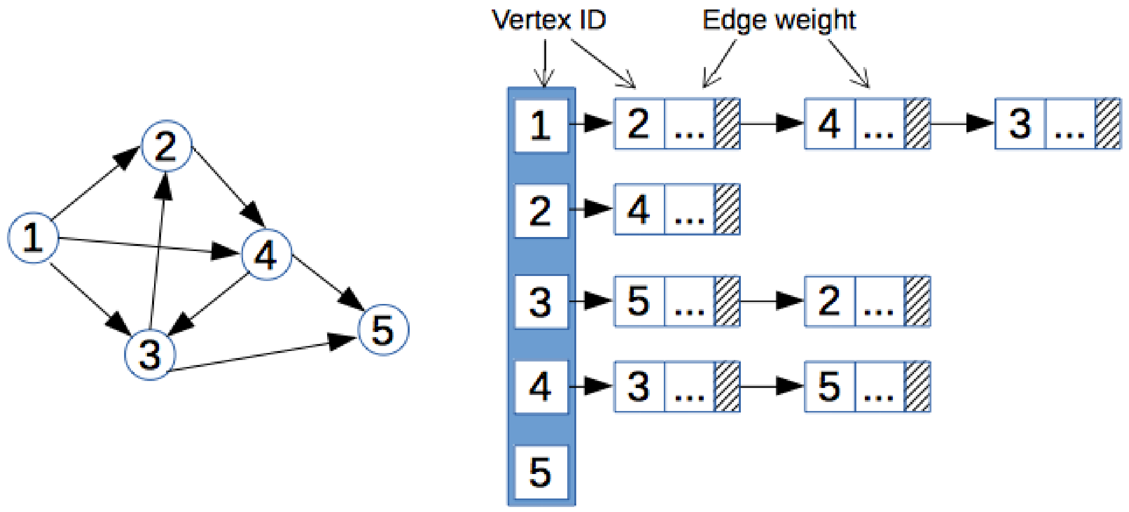

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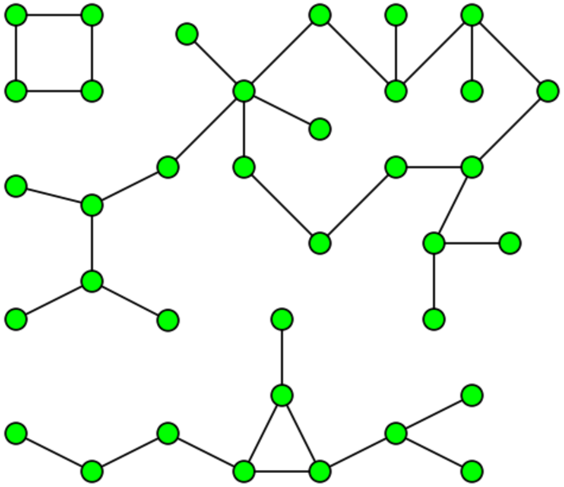
April 17, 2020

Simple Graph Algorithms

- What vertices are reachable from starting vertex?
  - Can use DFS or BFS to find connected vertices
  - Must avoid visiting same vertex more than once
- Find connected components
  - Many applications



https://en.wikipedia.org/wiki/Connected_component_(graph_theory)

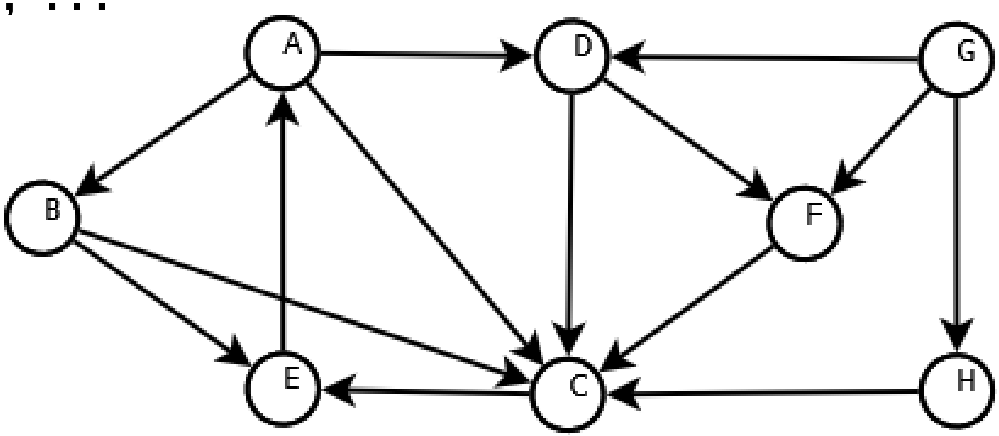
Breadth First Search

```
public Set<String> bfs(String start){
    Set<String> visited = new TreeSet<>();
    Queue<String> qu = new LinkedList<>();
    visited.add(start);
    qu.add(start);

    while (qu.size() > 0){
      String v = qu.remove();
      for(String adj : myGraph.getAdjacent(v)){
        if (! visited.contains(adj)) {
            visited.add(adj)'
            qu.add(adj);
        }
      }
    }
    return visited;
}
```

BFS Example

- BFS at A: B, C, D
    - from B: E, from C: …, from D: F
    - from E: …, from F, …



- BFS: all one-away
    - then all two, then all three, …

BFS becomes DFS

```
public Set<String> dfs(String start){
    Set<String> visited = new TreeSet<>();
    Queue<String> qu = new LinkedList<>();
    visited.add(start);
    qu.add(start);

    while (qu.size() > 0){
      String v = qu.remove();
      for(String adj : myGraph.getAdjacent(v)){
        if (! visited.contains(adj)) {
              visited.add(adj);
              qu.add(adj);
        }
      }
    }
    return visited;
}
```

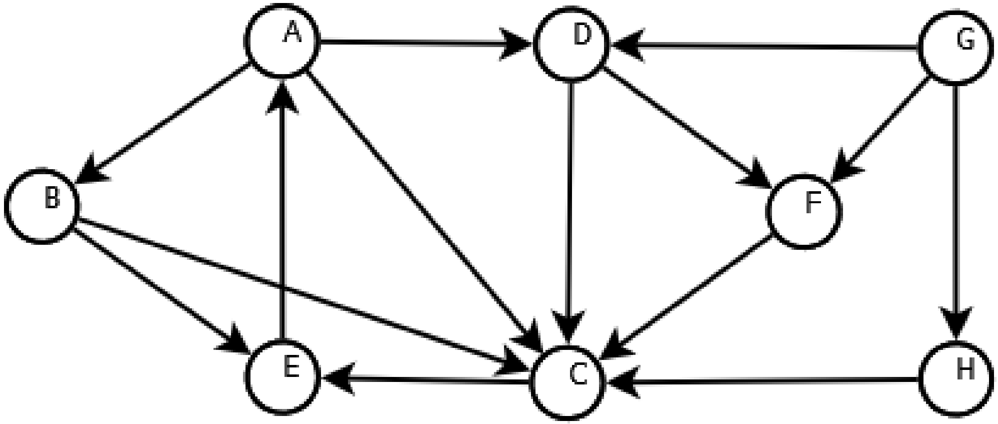
DFS arrives

```
public Set<String> dfs(String start){
    Set<String> visited = new TreeSet<>();
    Stack<String> qu = new Stack<>();
    visited.add(start);
    qu.push(start);

    while (qu.size() > 0){
      String v = qu.pop();
      for(String adj : myGraph.getAdjacent(v)){
        if (! visited.contains(adj)) {
            visited.add(adj);
            qu.push(adj);
        }
      }
    }
    return visited;
}
```

DFS Example

- DFS at A: B, C, D
  - then F, E

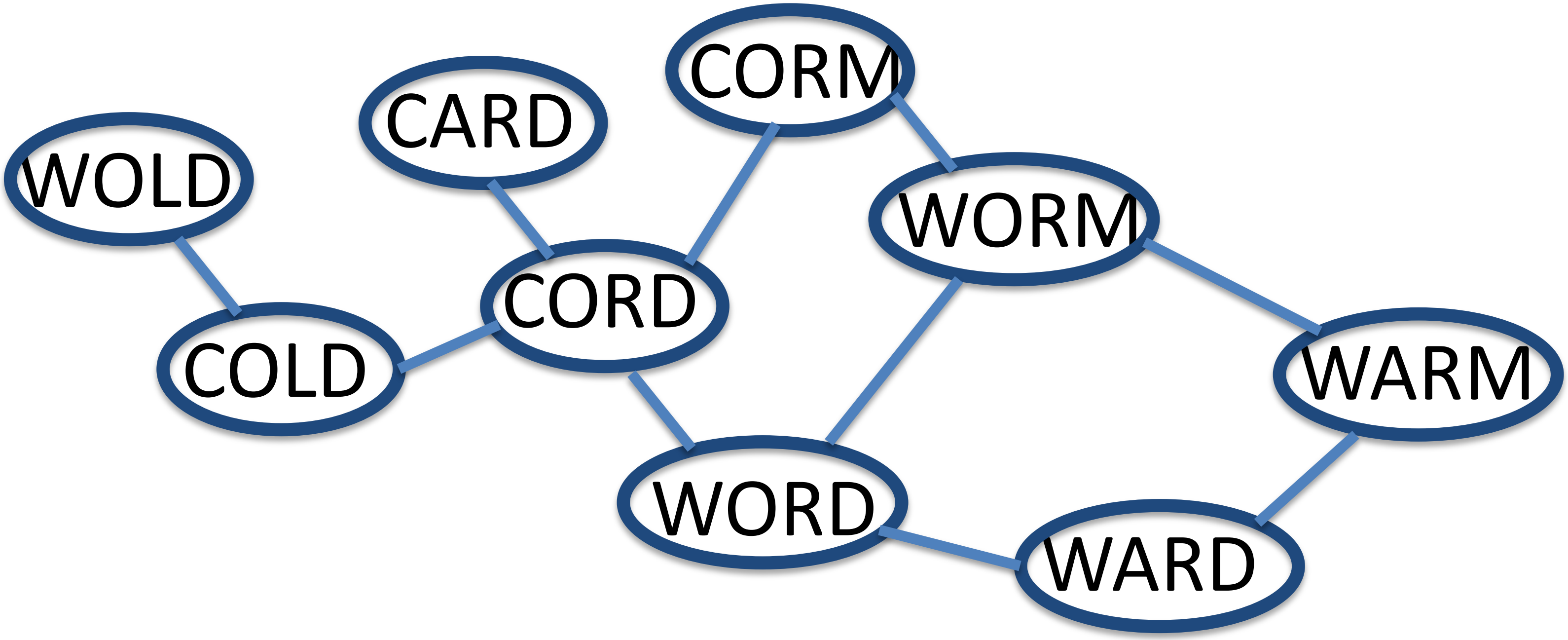


- DFS: goes deep one at a time

Example: Word Ladder Problem

- **Change a word into another word**
  - Change one letter at a time
  - Change COLD to WARM

  COLD -> CO**R**D -> **W**ORD -> WOR**M** -> W**A**RM



Algorithms + Data Structures

- BFS + Graphs = Word Ladder or Bacon Number
  - Getting from “above” to “zeros” in 17 steps!
  - above abode anode anole anile anise arise prise prime prims prams prats peats heats heads herds heros zeros
  - These edge weights are 1, so BFS works
- We can find the shortest path efficiently
  - Dijkstra's algorithm used in Internet today
  - Heuristics augment, absolute shortest needed?

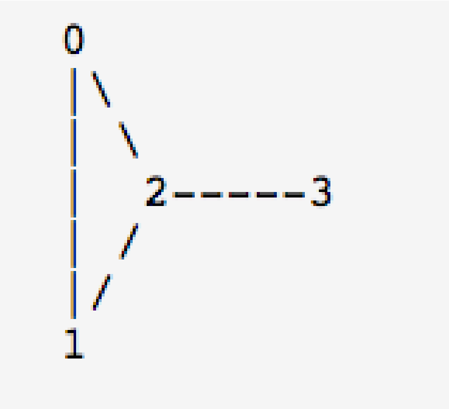
Shortest Path and Longest Path

- **We use breadth first search to find shortest path**
  - Same code we saw in word-ladder problem
    - White, While, Whale, Shale, … House
    - Efficient and polynomial time: edge-weight == 1
    - Need Dijkstra for positive edge-weight, still good

- **No efficient algorithm for longest path, it's hard**
  - If one found, every hard problem becomes easy
    - Most computer scientists don't think we'll find one

Connected Components: APT

- https://www2.cs.duke.edu/csed/newapt/internet.html
- Given a graph, a set of connected vertices
  - Which are important aka articulation points
  - Remove one? disconnect graph
- In example, removing 2 means …
  - Disconnect 3 from 0 and 1



```
0
| \
|  \
|   2-----3
|  /
| /
1
```

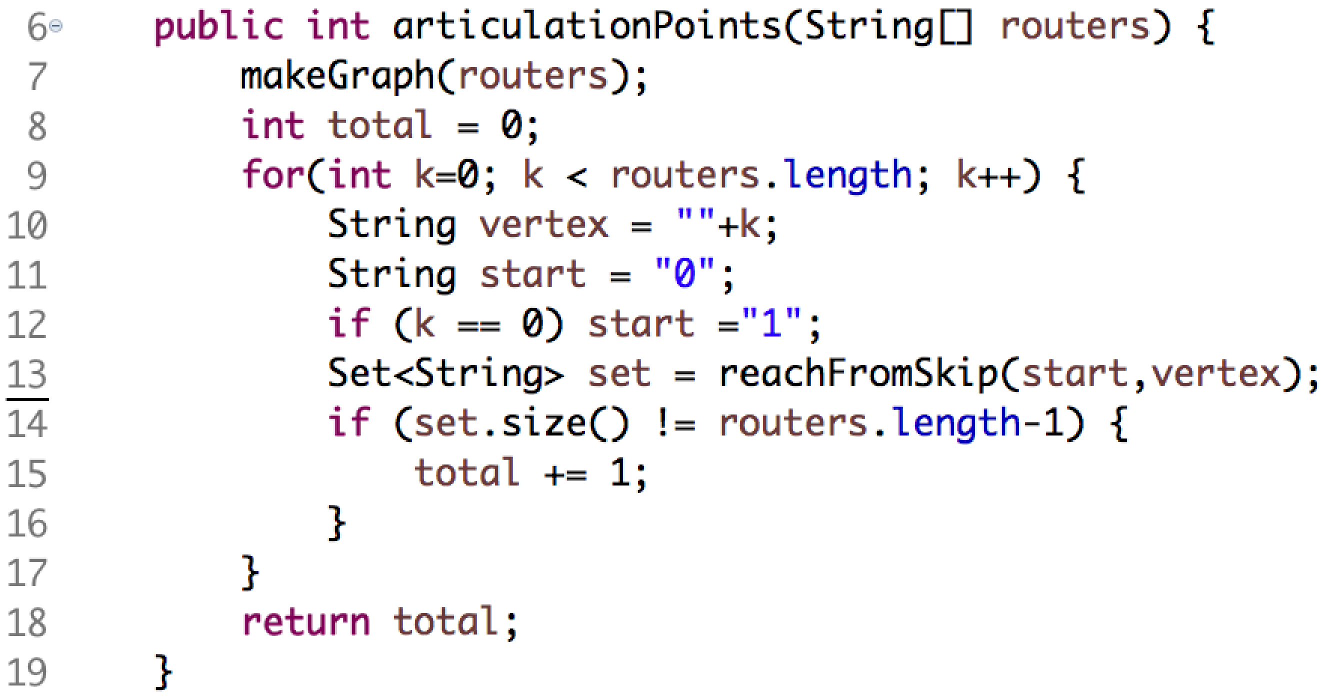
Connected Components: APT

- https://www2.cs.duke.edu/csed/newapt/internet.html
- What is this problem asking you to do?
  - What router, if removed, disconnects others?
- This is a graph problem! Vertices and edges?
  - Parse input, build graph, traverse graph

- Adjacency List: **`Map<String,Set<String>>`**
  - **`map.get("2") --`** set of connected vertices

Toward All Green

- **What part of this haven't you seen?**
  - How is DFS or BFS used? Modify based on …

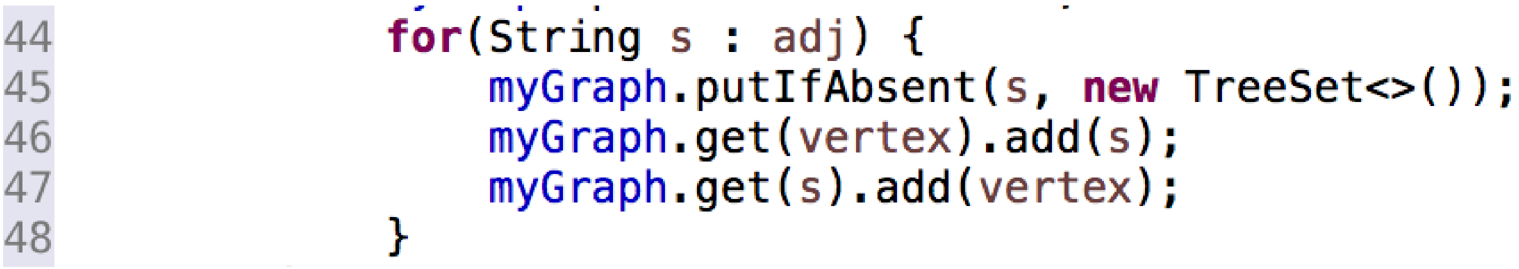


```java
public int articulationPoints(String[] routers) {
    makeGraph(routers);
    int total = 0;
    for(int k=0; k < routers.length; k++) {
        String vertex = ""+k;
        String start = "0";
        if (k == 0) start ="1";
        Set<String> set = reachFromSkip(start,vertex);
        if (set.size() != routers.length-1) {
            total += 1;
        }
    }
    return total;
}
```

What is reachFromSkip method?

- Use BFS or DFS as provided, but …
  - Do not push or enqueue skippable vertex/node
  - Can we reach everything from start? good!
    - Start from "0" unless skipping "0", …

- Must create graph from input



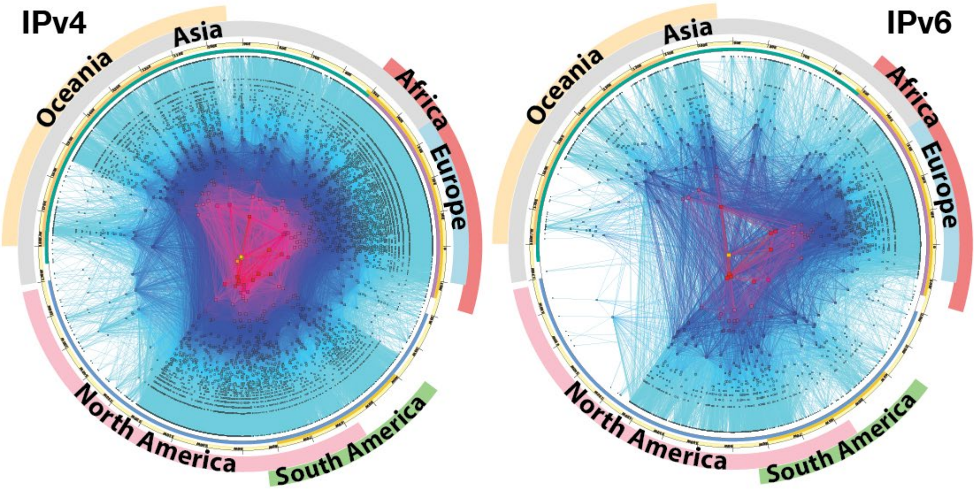
```
for(String s : adj) {
    myGraph.putIfAbsent(s, new TreeSet<>());
    myGraph.get(vertex).add(s);
    myGraph.get(s).add(vertex);
}
```

WOTO (4 minutes)

http://bit.ly/201spring20-0417-1

CAIDA's IPv4 vs IPv6 AS Core AS-level Internet Graph

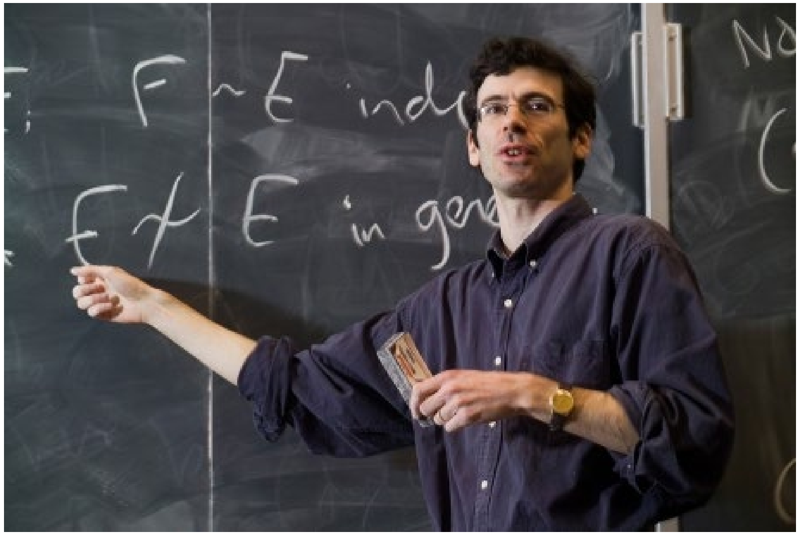
Archipelago July 2015



Jon Kleinberg

- Developed HITS, same time-frame as PageRank
- Professor at Cornell University
- MacArthur Genius award, Nevanlinna Prize, more

□ ***"It's much easier to make progress on a problem when you are enjoying what you are doing. In addition to finding work that is important, find work that has some personal interest for you.... ACM Infosys Interview***



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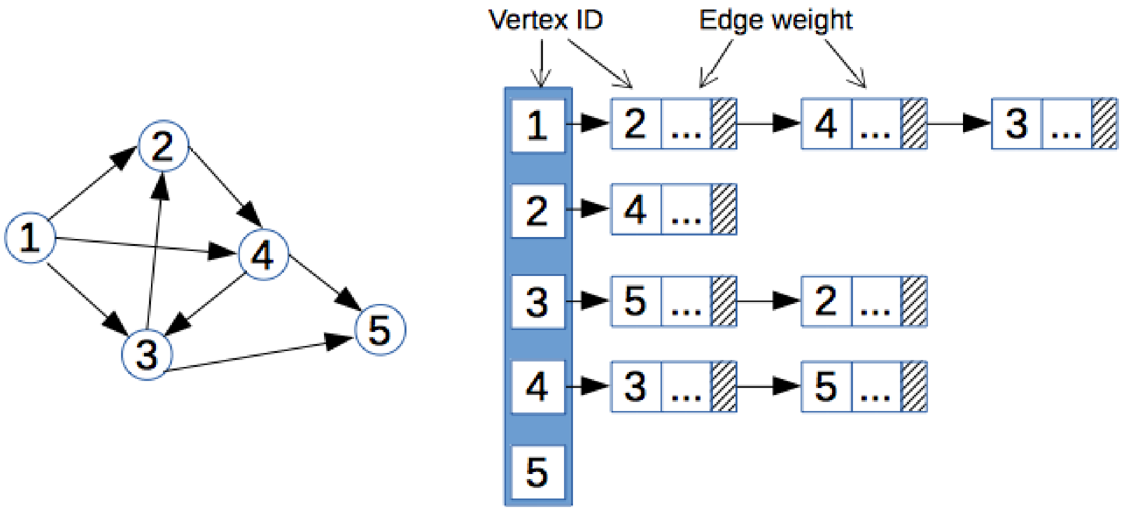


Susan Rodger

April 17, 2020

Greedy Algorithms

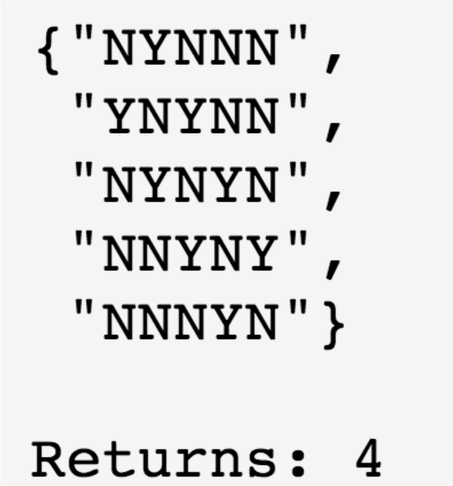
- **Which candles to burn?**
  - The tallest ones: leads to more burning days

- **Which votes to steal?**
  - Opponent with the most: fewer "steals" to win

- **Which weighted nodes to join in Huffman coding?**
  - Smallest weights first: save bits, optimal!

A friend of a friend: APT

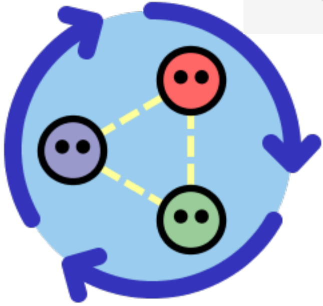
- https://www2.cs.duke.edu/csed/newapt/friendscore.html
- Model as a graph? Vertex: number, Edge? == 'Y'
  - 0: has one friend: 1
  - 1: 0 and 2
  - 2: 1 and 3
  - 3: 2 and 4
  - 4: 3



```
{"NYNNN",
 "YNYNN",
 "NYNYN",
 "NNYNY",
 "NNNYN"}

Returns: 4
```

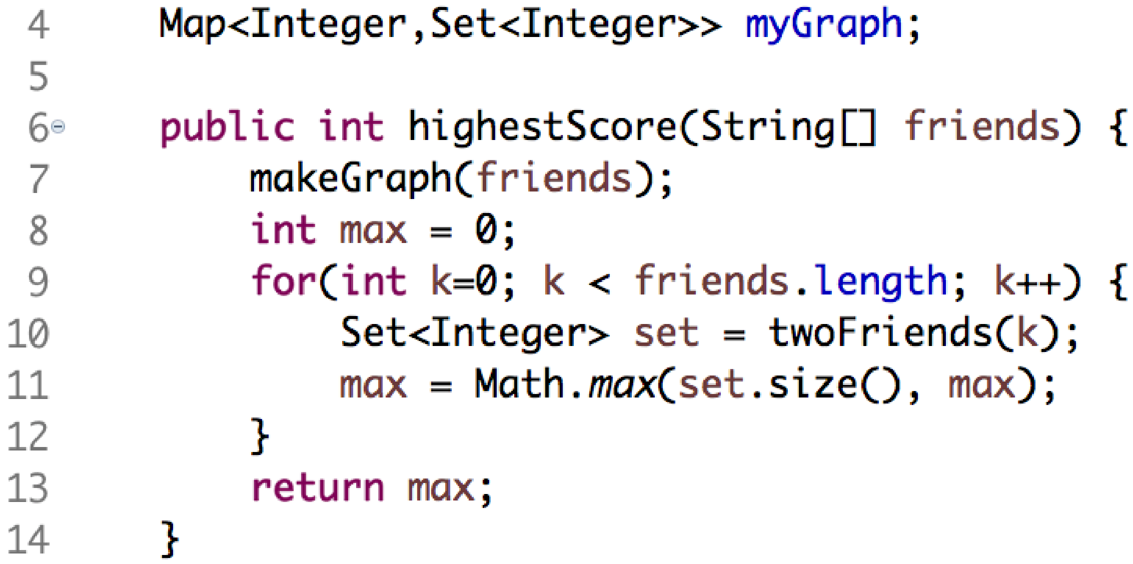
- So 2 has four two-friends
  - 1 has three two-friends
  - 0 has two two-friends



FRIEND
OF A
FRIEND

General Framework to Solve

- How to write **`twoFriends`**?
  - Make graph, find two-friends via …
  - Find 1 friends? index of each 'Y'. Repeat

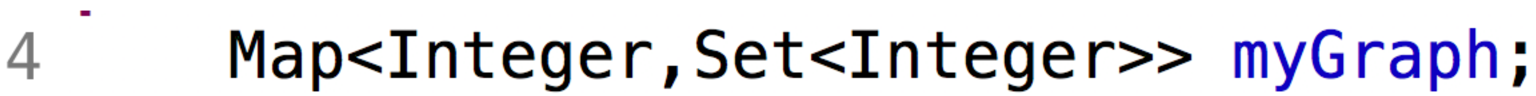


```
    Map<Integer,Set<Integer>> myGraph;

    public int highestScore(String[] friends) {
        makeGraph(friends);
        int max = 0;
        for(int k=0; k < friends.length; k++) {
            Set<Integer> set = twoFriends(k);
            max = Math.max(set.size(), max);
        }
        return max;
    }
```

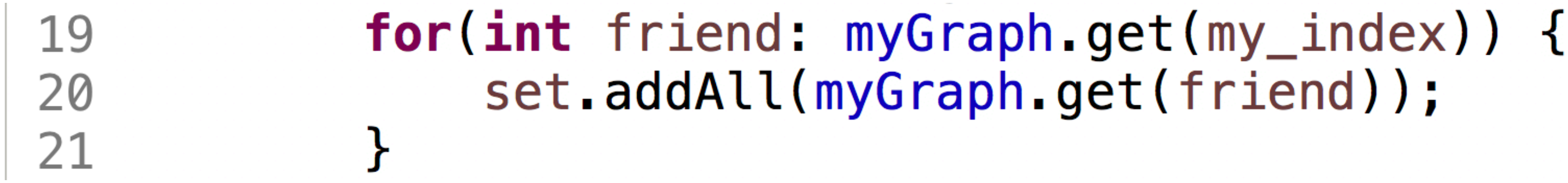
Set.addAll --- all my friends

- Model data using graph: parse via makeGraph



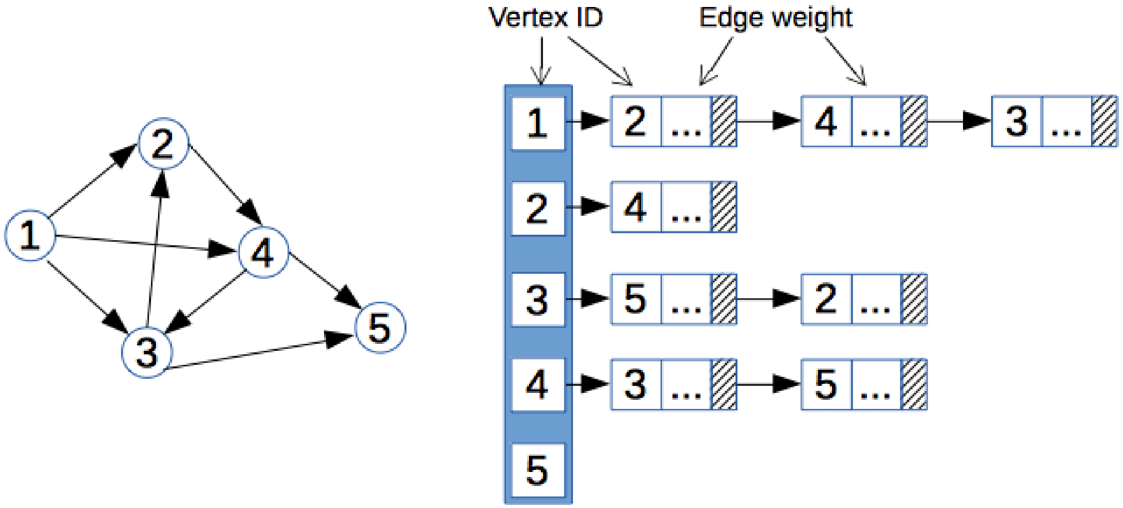
```java
    Map<Integer,Set<Integer>> myGraph;
```

- My friends: **`myGraph.get(my_number))`**
  - Friend of a friend? for each of my friends …



```java
        for(int friend: myGraph.get(my_index)) {
            set.addAll(myGraph.get(friend));
        }
```

Compsci 201
Graphs, APTS, and More
Part 5 of 5



Susan Rodger

April 17, 2020

Mathematics and Computer Science

- How do we solve differential equations?
  - It depends
- How do we estimate percolation threshold?
  - It depends
- How do we model cardiac behavior? …

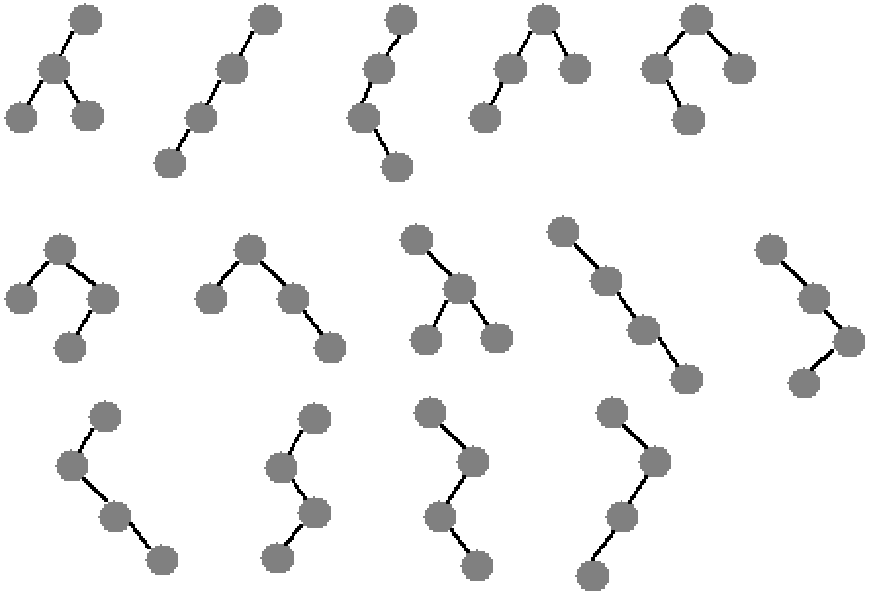
- Use simulation when no analytic solutions
  - Monte Carlo simulation for many problems
  - https://en.wikipedia.org/wiki/Monte_Carlo_method

Thinking about math+compsci

https://www2.cs.duke.edu/csed/newapt/bstcount.html

- How many different binary search trees are there?
  - Size = 4, Size = 5 … Size = N?
  - What about N = 6?

N	# trees
0	1
1	1
2	2
3	5
4	14
5	42



Combinatorics and Catalan

- Binary search trees with 6 nodes
  - Left subtree has: 0,1,2,3,4,5 nodes
    - What will right subtree have?
  - For each left, there is a right…
    - Count how many ways this happens

N	# trees
0	1
1	1
2	2
3	5
4	14
5	42

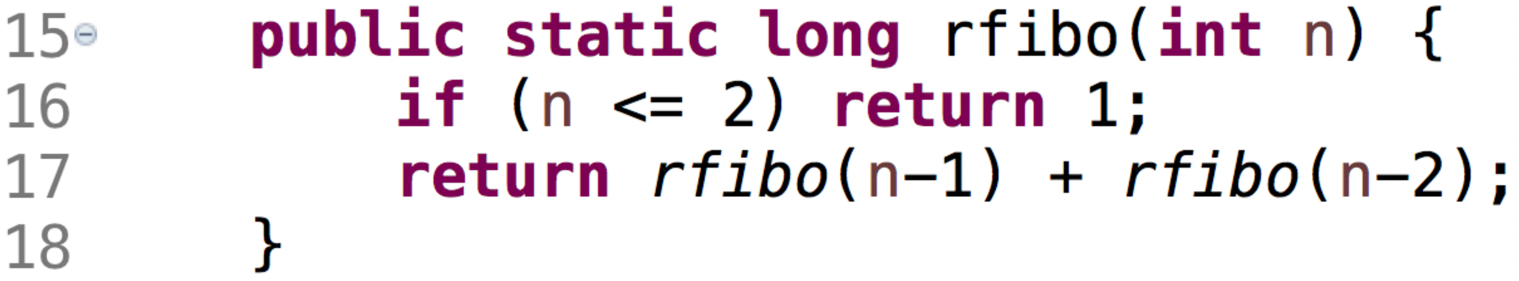
```
(1*42)+(1*14)+(2*5)+(5*2)+(14*1)+(42*1)= 132
```

Verify via: https://en.wikipedia.org/wiki/Catalan_number

Aside: From Catalan to Fibonacci

- Read about the Golden Ratio and Fibonacci #'s
  - **`1,1,2,3,5,8,13,21`**, … it's about rabbits?
    - Inevitable we discuss this, factorial, Bubble sort
- Do not do this at home, see classwork on Git

https://coursework.cs.duke.edu/201spring20/classcode/

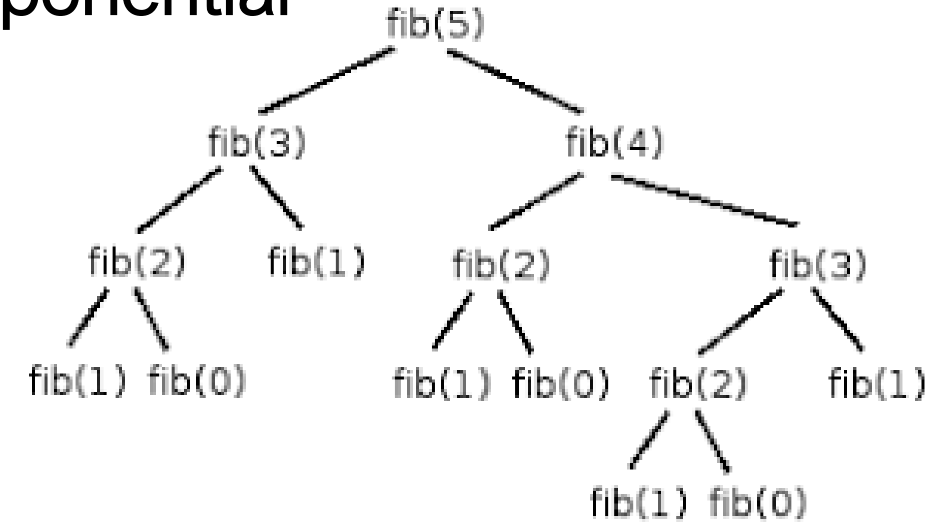


```java
    public static long rfibo(int n) {
        if (n <= 2) return 1;
        return rfibo(n-1) + rfibo(n-2);
    }
```

Exponential number of calls

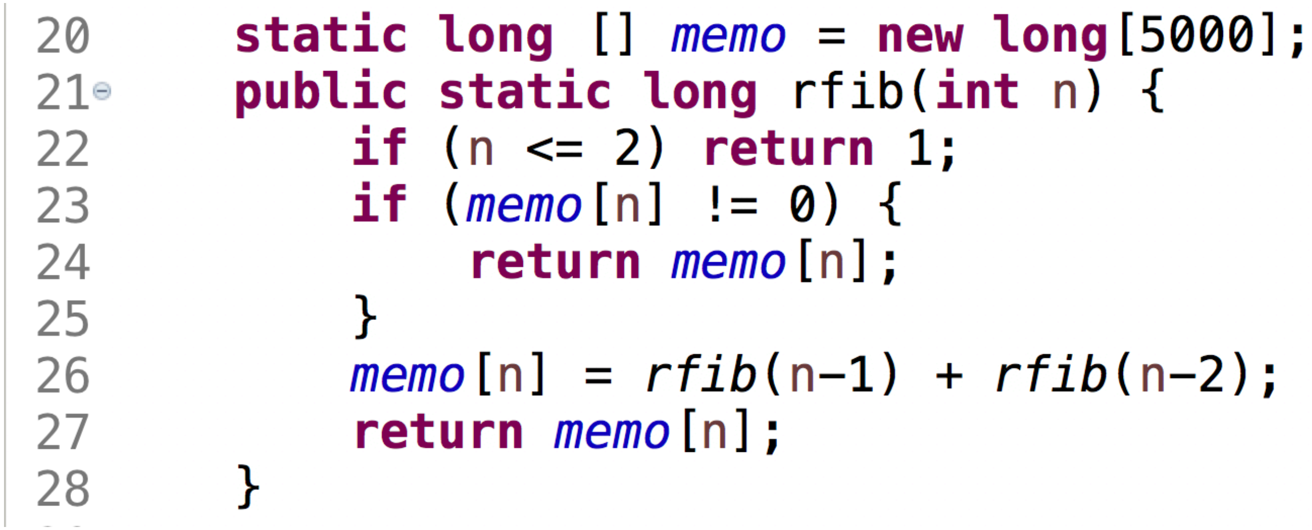
- Since fib(8) calls fib(7) and fib(6)
  - And fib(6) calls … which calls … which …
  - What is the recurrence? ~ T(n) = 2T(n-1) + O(1)
  - Solution to this is O(2^n)
- Actual fib isn't 2^n, is exponential
  - Golden ratio: φ^n

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$$



Memoize aka Caching

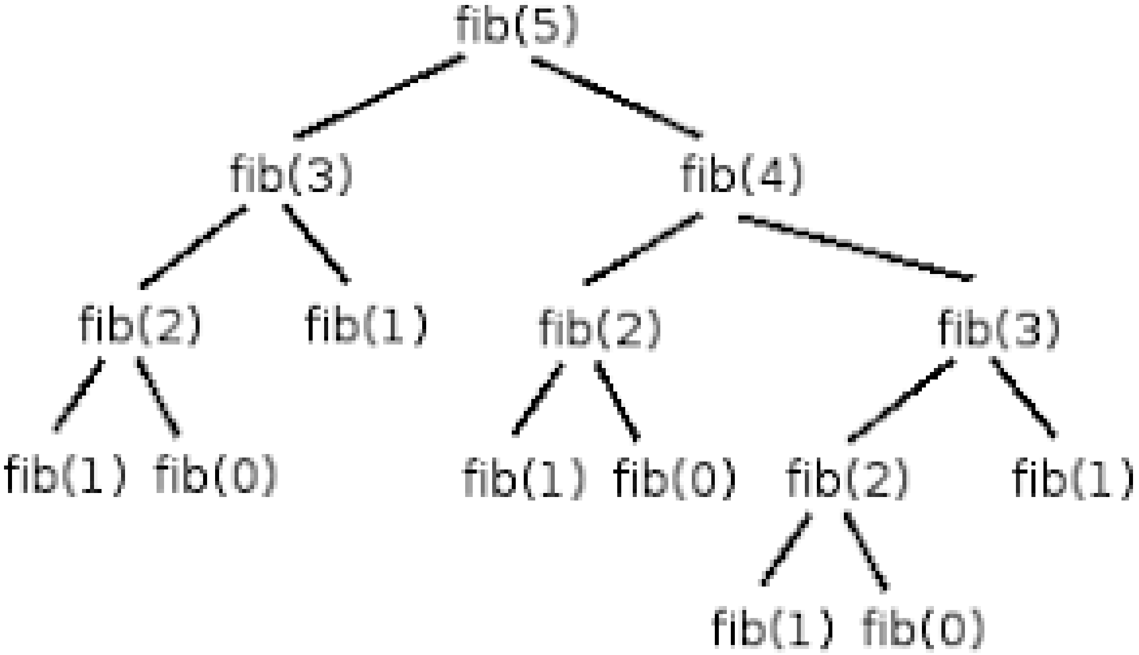
- Caching in computer science is … store to re-use
  - Similar to dynamic programming, but top-down
- If already seen? use that result, no recursion
  - Otherwise, recurse, store, return



```
    static long [] memo = new long[5000];
    public static long rfib(int n) {
        if (n <= 2) return 1;
        if (memo[n] != 0) {
            return memo[n];
        }
        memo[n] = rfib(n-1) + rfib(n-2);
        return memo[n];
    }
```

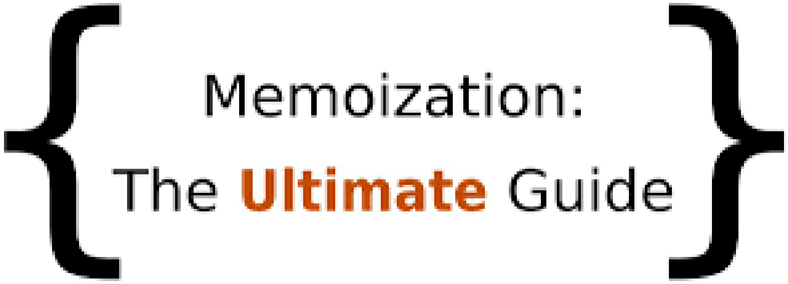
Look at this tree again

- Instead of doing this…..



Avoid repeated recursion …

- Store calculated values in a map
  - Look up first, re-use what's already done
  - Use **`Map<Integer,Long>`** or **`long[]`**
  - An array is a map of index to value

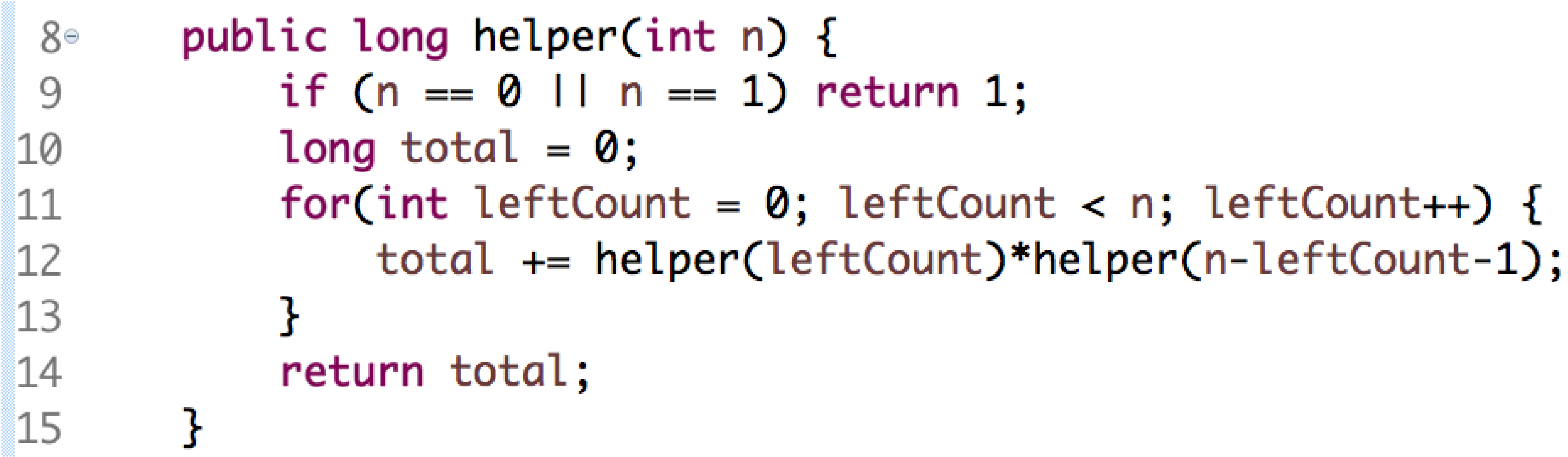


Memoization:

The **Ultimate** Guide

All Green? Which one …

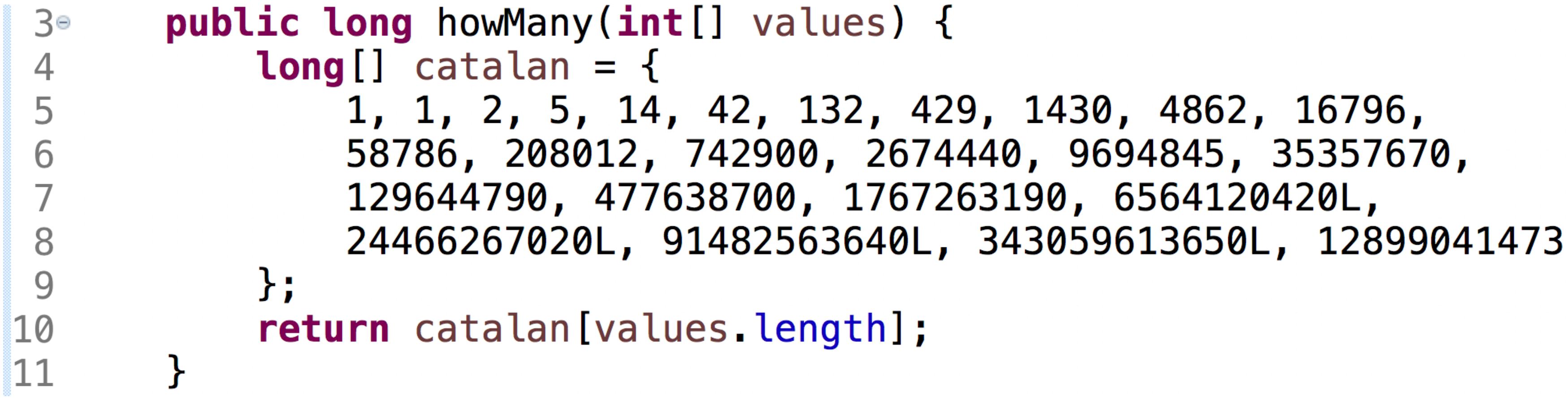
- This solution will time out, too many helper calls
  - Use memoization to get all green
  - Add array or map, store, re-use



```java
    public long helper(int n) {
        if (n == 0 || n == 1) return 1;
        long total = 0;
        for(int leftCount = 0; leftCount < n; leftCount++) {
            total += helper(leftCount)*helper(n-leftCount-1);
        }
        return total;
    }
```

All Green? Do NOT turn this in

- Catalan via Wikipedia: this should NOT be used.
  - Notice **`6564120420L`**, long constant



```
    public long howMany(int[] values) {
        long[] catalan = {
            1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796,
            58786, 208012, 742900, 2674440, 9694845, 35357670,
            129644790, 477638700, 1767263190, 6564120420L,
            24466267020L, 91482563640L, 343059613650L, 12899041473
        };
        return catalan[values.length];
    }
```

WOTO

http://bit.ly/201spring20-0417-2