Due on March 23, 2020
65 points total

General directions: All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (9+1 points)
Function $f : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^+ \times (-\pi/2, \pi/2)$ is defined as follows:

$$f(x, y) = (\sqrt{x^2 + y^2}, \arctan(y/x)).$$

Show that f is a bijective function.

Problem 2 (18+2 points)
Let f and g be functions from a set X to itself. Prove or disprove the following statement in the two cases below: *if $f \circ g$ is a bijective function, then f and g must both be bijective functions.* (Recall that $(f \circ g)(x) = g(f(x))$.)

(a) (9+1 points) X is a finite set.

(b) (9+1 points) X is an infinite set.

Problem 3 (14+1 points)
Relation R is defined on the set of integers $\mathbb{Z}$ as follows: $(x, y) \in R$ if and only if

$$(ay + b) \mod p \;=\; (ax + b) \mod p,$$

where a, b, p are fixed integers satisfying $1 \leq a \leq p - 1$, $0 \leq b \leq p - 1$, and p a prime number.

Show that R is an equivalence relation and state the equivalence classes of the relation.

Problem 4 (18+2 points)
Let R be a relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$ defined as follows: $((a, b), (c, d)) \in R$ if and only if $ad = bc$.

(a) (9+1 points) Prove that R is an equivalence relation.

(b) (9+1 points) Define a bijective function from the set of equivalences classes induced by R to the set of positive rationals, and prove that this function is indeed bijective.