Due on April 13, 2020
40 points total

General directions:

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (14+1 points)
An *Eulerian circuit* in an undirected graph is a cycle that traverses each edge exactly once. Prove that an undirected connected graph has an Eulerian circuit if and only if the degree of every vertex is even.

(Hint: Use induction over the number of vertices.)

Problem 2 (14+1 points)
Let $G = (V, E)$ be a directed graph, and let u, v be two vertices of G. We say that v is *reachable* from u if there exists a directed path from u to v in G. Furthermore, u and v are *strongly connected* if v is reachable from u and u is reachable from v. Now recall that the relation $\{(u, v) :$ u and v are strongly connected$\}$ is an equivalence relation on V, and we call each equivalence class induced by this relation a *strongly connected component* of G.

Let C be a strongly connected component of G. We say that C is a *source component* if C does not contain a vertex that is reachable from a vertex outside C. Now recall that when we run DFS on G, every vertex u is assigned two positive integers $pre(u)$ and $post(u)$.

Prove the following: in any run of DFS on G, the vertex with the largest $post$-value belongs to a source component of G.

(Hint: Consider the DAG formed by the strongly connected components of G.)

Problem 3 (9+1 points)
Suppose there are $n \geq 3$ people who each simultaneously flip a coin that returns H with probability p and T with probability $1 - p$. Compute the probability that there exists a person whose result is unique among all results, and justify your result.