COMPSCI330 Design and Analysis of Algorithms
Assignment 6

Due Date: Wednesday, April 8, 2020

Guidelines

- **Describing Algorithms** If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time. There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice. If the running time of your algorithm is worse than the suggested running time, you might receive partial credits.

- **Typesetting and Submission** Please submit the problems to GradeScope. You will be asked to **label your solution for individual problems**. Failing to label your solution can cost you 5% of the total points (3 points out of 60 for this homework).

- LaTeX is preferred, but answers typed with other software and converted to pdf is also accepted. Please make sure you submit to the correct problem, and your file can be opened by standard pdf reader. **Handwritten answers or pdf files that cannot be opened will not be graded. (Exceptions apply to students who don't have regular access to computers.)**

- **Timing** Please start early. The problems are difficult and they can take hours to solve. The time you spend on finding the proof can be much longer than the time to write. If you need more time for your homework please use this form and submit a STINF.

- **Collaboration Policy** Please check this page for the collaboration policy. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**
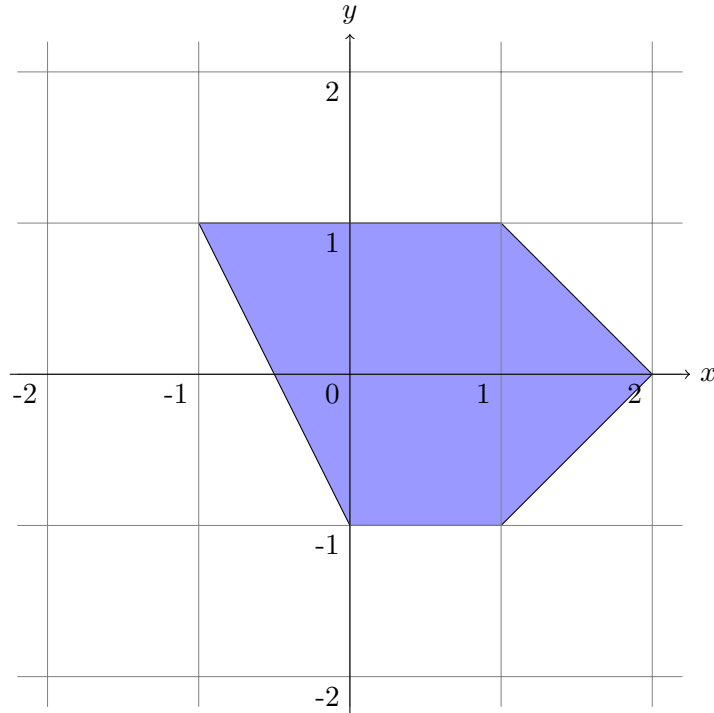
Figure 1: Feasible Region of a Linear Program

Problem 1 (Linear Programming Basics)**.** (15 points)

Consider a linear program with two variables x and y. The set of feasible solutions is the blue region in Figure 1.

(a) (10 points) Write out the constraints for x, y so that the feasible region is exactly equal to the blue region in Figure 1.

(b) (5 points) If the objective function is $\max x + 2y$, what is the optimal solution? If the objective function is $\max x + y$, what is the optimal solution?

Problem 2 (Fractional Knapsack)**.** (20 points) For the fractional knapsack problem, recall there are n items with weights w_i and value v_i, the goal is to put items into a knapsack of capacity c and achieve the maximum value. The problem can be solved by a linear program, where we have a variable x_i for each item,

$$\max \sum_{i=1}^{n} v_i x_i$$
$$\forall i = 1, 2, ..., n \quad x_i \leq 1,$$
$$\sum_{i=1}^{n} w_i x_i \leq c,$$
$$\forall i = 1, 2, ..., n \quad x_i \geq 0.$$

(a) (8 points) Prove that every basic feasible solution of this linear program has at most one x_i that is not equal to 0 or 1.

(b) (12 points) Write the dual of this linear program.

Hint: This linear program looks more similar to the dual linear program we did in class. The dual of a dual linear program is the primal linear program

Problem 3 (Cutting Vertices). (25 points) Given a (directed) graph G and vertices s, t, design an algorithm to find the minimum number of vertices to remove (other than s and t) such that after these vertices are removed s and t are no longer connected. You should try to solve this problem by converting it to a min cut problem on a new graph. Prove the correctness of your algorithm.

Hint: 1. Since min cut problem talks about cutting edges, and this problem cares about removing vertices, you should first convert each vertex (other than s, t) to two vertices connected by an edge. Think about how the original edges should be connected. 2. You are not allowed to cut the original edges, so think about how to ensure that by changing their capacity in the new graph.