- optimization problem
  - set of variables (parameters)
  - constraints
  - objective
- linear program:
  - variables: n real numbers $x_1, x_2, x_3, \dots, x_n \in \mathbb{R}$
  - constraints: linear inequalities

    $2x_1 \geq x_2 - x_3$ ✓

    $x_1 \leq x_5 + 10$ ✓

    $\underline{c}\, x_1 \leq x_2$ ✓ as long as c is a known constant

    $x_1 x_2 \leq 1$ ✗ $\quad \log x_1 + \log x_2 \leq 3$ ✗

    $\underline{\log x_1 \leq 10} \iff \underline{x_1 \leq e^{10}}$
  - objective linear function over the variables
- Example:

$$\boxed{\begin{array}{ll} \text{variables:} & x, y \\ \text{constraints:} & x \geq 0 \\ & y \geq 0 \\ & x + y \leq 1 \\ \text{objective:} & \max \underline{2x + y} \end{array}}$$

- solutions to linear program
  - Solution: an assignment of the variable

  feasible solution: a solution that satisfies all constraints.

  $(x, y) = (0, 0), (1, 0), (\frac{1}{2}, \frac{1}{2}), (\frac{1}{4}, \frac{1}{2})$ ✓

  $(x, y) = (-1, -1), (2, 0)$ ✗

  - optimal solution: a feasible solution that achieves the $\underline{\text{best}}$ objective value

    $\underline{(x, y) = (1, 0)}$ optimal value $2x + y = 2$

    $$\underline{2x + y \underset{\substack{\uparrow \\ y \geq 0}}{\leq} 2x + 2y \underset{\substack{\uparrow \\ x+y \leq 1}}{\leq} 2}$$

- geometric interpretation
  - variables $x_1, \dots, x_n \iff$ point in n-dimensional space
  - constraint

    [illegible] $\iff$ half planes (half spaces)

- constraint

linear inequality $\Longleftrightarrow$ half planes (half spaces)

group of linear inequalities $\Longleftrightarrow$ intersection of half planes.

feasible region

- objective

rewrite objective as $\max \vec{C} \cdot X$

$\sum_{i=1}^{n} c_i x_i$

the point $\vec{C}$ represents a direction of gravity

when $\vec{c}$ is pointing down, lowest point of feasible region gives the optimal solution

- canonical form
$$\begin{aligned} &\min \ \langle c, x \rangle \\ \text{s.t.} \ &Ax \geq b \\ &x \geq 0 \end{aligned}$$

$\Longleftrightarrow$

$\min \sum_{i=1}^{n} c_i x_i$

for every $j = 1, 2, \cdots, m$

$\sum_{i=1}^{n} A_{j,i} x_i \geq b_j$

for every $i = 1, 2, \cdots, n$

$x_i \geq 0$

$m+n$ constraints

$$\boxed{\begin{aligned} &x, y \\ &x \geq 0 \\ &y \geq 0 \\ &x + y \leq 1 \\ &\max 2x + y \end{aligned}} \Longrightarrow \boxed{\begin{aligned} &\min -2x - y \\ &-x - y \geq -1 \\ &x \geq 0 \\ &y \geq 0 \end{aligned}}$$

$c = (-2, -1)$

$A = \begin{pmatrix} -1 & -1 \end{pmatrix}$ $\quad b = -1$

- applying linear programming
  - fractional Knapsack.

    n items, item i has weight w_i, value v_i

    one Knapsack. capacity C

    can put fractions of items into Knapsack.

goal: pack the Knapsack so that it has max value variables: let X_i be $\boxed{\text{the fraction}}$ of item i in knapsack.

constraints: capacity $\underbrace{\sum_{i=1}^{n} w_i \cdot X_i}_{\text{total weight}} \leq C$

fraction: for every $i = 1, 2, \cdots, n$

$$0 \leq X_i \leq 1$$

objective: $\max \sum_{i=1}^{n} V_i X_i$