Lecture 18 Linear Programming Algorithms

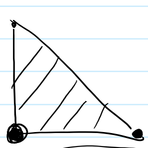
Wednesday, April 1, 2020 2:55 PM

- Complinentary slackness
  - if $a^T x \geq b$ is a primal constraint, and y is the corresponding dual variable, then for any pair of optimal solutions, $y(a^T x - b) = 0$

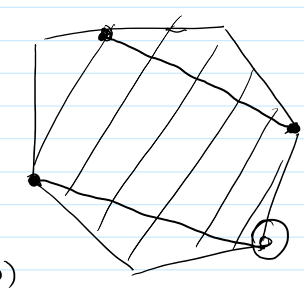
    if $a^T x > b$ (primal constraint is not tight) then
    $y = 0$ (dual variable $= 0$)

    if $y > 0$ (dual variable positive) then
    $a^T x = b$ (primal constraint is tight.)

- simplex algorithm
  - basic feasible solution
    - a basic feasible solution of a linear program with n variables is a feasible solution equal to the solution of a system of n linear equations where each equation is a tight constraint.
  - relies on convexity



$x \geq 0$
$y \geq 0$
$x + y \leq 1$



$x = 0$
$y = 0$
$(0,0)$

$x + y = 1$
$y = 0$
$(1,0)$

$x = 0$
$x + y = 1$
$(0,1)$

- a linear program has n variables m constraints (includes $x_i \geq 0$) $m > n$

  a basic feasible solution is
  a feasible solution where n out of m constraints are set to equalities.

- ellipsoid algorithm

- separation oracle: given a candidate solution (assignment of x) decide x is feasible or output a constraint which x violates.

- LP for min spanning tree

  $x_{i,j}$ variable for edge i-j $\quad x_{i,j} = \begin{cases} 1 & \text{if } (i,j) \text{ is in MST} \\ 0 & \end{cases}$

  $0 \le x_{i,j} \le 1$

  $\boxed{\text{for every cut } C \;\; \sum_{(i,j)\in C} x_{i,j} \ge 1}$

- interior point
  - barrier function $\qquad x+y \le 1$

  $x \ge 0 \Rightarrow \begin{cases} \frac{1}{x} \\ e^{\frac{1}{x}} - 1 \end{cases} \qquad 1-x-y \ge 0 \Rightarrow \begin{cases} \frac{1}{1-x-y} \\ e^{\frac{1}{1-x-y}} - 1 \end{cases}$