Lecture 7 Greedy Algorithms 1

Monday, February 3, 2020 2:24 PM

- Interval Scheduling

$n = 5$

$(1, 3)$ $(2, 4)$ $(4, 5)$ $(4, 6)$ $(8, 10)$

$(1,3)$ $(4,5)$ $(8,10)$

- Proof of correctness.

assume towards contradiction that the alg is not correct.

then there is an instance with n meetings, and meeting time $\{(s_i, t_i)\}_{i=1,2,\dots,n}$ where the alg is incorrect.

without loss of generality (wlog) assume $t_1 \le t_2 \le t_3 \le \dots \le t_n$

for this instance, let ALG schedule meeting

$$(i_1, i_2, i_3, \dots, i_k)$$

let OPT schedule meeting

$$(j_1, j_2, j_3, \dots, j_l)$$

if ALG is incorrect, then $k < l$.

① there is an index $1 \le p \le k$ st. $i_p \ne j_p$

let p be the first index where $i_p \ne j_p$

by design of the algorithm, i_p has the earliest ending time among all meetings that can be scheduled together with $(i_1, i_2, \dots, i_{p-1})$

$$\Longrightarrow t_{i_p} \le t_{j_p}$$

consider an alternative solution

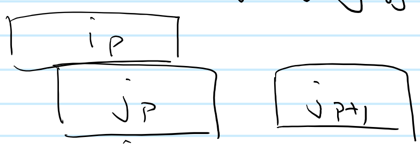
$$(i_1, i_2, \dots, \underset{\uparrow\, j_p}{i_p}, j_{p+1}, j_{p+2}, \dots, j_l)$$

this solution is also valid because

① $(i_1, \cdots, i_p)$ is valid (by design of ALG)

② $(\hat{j}_{p+1}, j_{p+2}, \cdots, j_l)$ is valid (by validity of OPT)

③ $t_{i_p} \le t_{\hat{j}_p} \le S_{\hat{j}_{p+1}}$ $\underbrace{\hphantom{t_{\hat{j}_p} \le S_{\hat{j}_{p+1}}}}_{\text{validity of OPT}}$



– repeating this procedure $\le k$ times, can get an alternative optimal solution $(\hat{j}_1', \hat{j}_2', \cdots, \hat{j}_l')$ s.t. for every $1 \le p \le k$ $i_p = \hat{j}_p'$.

② if for every $1 \le p \le k$, $i_p = \hat{j}_p$.

consider $\hat{j}_{k+1}$, by design of the algorithm

$$\underline{t_{\hat{j}_{k+1}}} > t_{\hat{j}_k} = \underline{t_{i_k}}$$

alg will consider meeting $\hat{j}_{k+1}$ after scheduling i_k

since $\hat{j}_{k+1}$ does not conflict with $i_k(\hat{j}_k)$, alg should have scheduled this meeting. This is a contradiction.

Therefore the algorithm is always correct.