Question 1

a. Given the following knowledge base:

$$\forall x \ [Knows(Bill, x) \implies Loves(Bill, x)]$$
$$\forall x, y \ [Friend(x, y) \implies Friend(y, x)]$$
$$\forall x, y \ [Friend(x, y) \implies Knows(x, y)]$$
$$Friend(Ted, Bill)$$

Prove:

$$Loves(Bill, Ted)$$

b. Given the following knowledge base:

$$\forall x\ [Knows(Bill, x) \implies Loves(Bill, x)]$$
$$\forall x, y\ [Friend(x, y) \implies Friend(y, x)]$$
$$\forall x, y\ [Friend(x, y) \implies Knows(x, y)]$$
$$\exists x\ Friend(x, Bill)$$

Prove:

$$\exists x\ Loves(Bill, x)$$

c. Given the following knowledge base:

$$\forall x \; [Knows(Bill, x) \implies Loves(Bill, x)]$$
$$\forall x, y \; [Friend(x, y) \implies Friend(y, x)]$$
$$\forall x, y \; [Friend(x, y) \implies Knows(x, y)]$$
$$\forall x \, [\exists y \; Friend(x, y)]$$

Prove:

$$\exists x \; Loves(Bill, x)$$

Question 2

Let A, B and C be three discrete random variables with joint probability distribution $P(\cdot)$ which is a *function* of three variables.

Notation: Capital letters denote random variables, small letters are numbers that denote possible realizations of the random variables (i.e. values it can take). The letter P is overloaded notation and means different things dependeing on the arguments passed. So $P(ABC)$ is the joint probability over A, B, C and is a function that takes three arguments. Similarly, $P(BC)$ is the joint probability over B, C and is a function that takes two arguments and $P(A)$ is the marginal probability of A, which is a function of a single variable. Also, $P(abc)$, $P(bc)$ and $P(a)$ are *numbers* that denote the probability of the events $(A = a \wedge B = b \wedge C = c)$, $(B = b \wedge C = c)$ and $(A = a)$ respectively.

Write the following in terms of the joint probability distribution.

a. $P(BC)$

b. $P(A)$

c. $P(A \vee B)$