Discussion Section 5

Question 1

Prove this expanded version of Bayes' theorem: $P(A|BC) = \frac{P(B|AC)P(A|C)}{P(B|C)}$.

Question 2

Problem 16.17 from the textbook. (Adapted from Pearl (1988).) A used-car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic), and then, depending on the outcome of the tests, decide which car to buy. We will assume that the buyer is deciding whether to buy car c_1, that there is time to carry out at most one test, and that t_1 is the test of c_1 and costs $50.
A car can be in good shape (quality q^+) or bad shape (quality q^-) and the tests might help indicate what shape the car is in. Car c_1 costs $1500 and its market value is $2000 if it is in good shape; if not, $700 in repairs will be needed to make it in good shape. The buyer's estimate is that c_1 has a 70% chance of being in good shape.

1. Calculate the expected net gain from buying c_1, given no test.

2. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

$$P(pass(c_1, t_1)|q^+(c_1)) = 0.8 \tag{1}$$

$$P(pass(c_1, t_1)|q^-(c_1)) = 0.35 \tag{2}$$

   Use Bayes' theorem to calculate the probability that the car will pass (or fail) its test and hence the probability that it is in good (or bad) shape given each possible test outcome.

3. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

4. Calculate the value of information of the test, and derive an optimal conditional plan for the buyer.