Experimenting with Grammars to Generate L-Systems – in JFLAP

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L-Systems

- Grammatical systems introduced by Lyndenmayer
- Model biological systems and create fractals
- Similar to Chomsky grammars, except all variables are replaced in each step, not just one!
- Successive strings are interpreted as strings of render commands and displayed graphically
Parts of an L-System
(a type of grammar)

• Defined over an alphabet
• Three parts
  – Axiom (starting place)
  – Replacement rules (replaces all variables at once)
  – Geometric rules (for drawing)
    • g  means move forward one unit with pen down
    • f  means move forward one unit with pen up
    • +  means turn right by the default angle
    • -  means turn left by the default angle
L-System

An L-system is composed of three parts \((\Sigma, h, w)\)

\(\Sigma\) finite alphabet set of symbols
\(h\) rewriting rules each symbol is replaced by string of symbols
\(w\) axiom starting point

\(h\) is finite substitutions, \(h: \Sigma \rightarrow \Sigma^*\).
$h(w)$

$h(w)$ is computed by replacing every symbol in $w$ that has a rewrite rule by that rule.

A language $L$ of an $L$-system is the word sequence generated by

$\bullet$ $h^0(w) = w$
$\bullet$ $h^1(w) = h(w)$
$\bullet$ $h^2(w) = h(h(w))$
$\bullet$ …

$L = \{h^i(w) \mid i \geq 0\}$
NOTE: If $h(a) = bb$ we will write this as a rule

$$a \rightarrow bb$$
Example:

\[ \Sigma \text{ alphabet: } \{a, b\} \]
\[ h \text{ rules: } \quad a \rightarrow aa \]
\[ \quad b \rightarrow ab \]
\[ w \text{ axiom: } \quad ab \]

What is the language \( L \) of strings represented by this L-system?

\[ L = \]
Drawing a picture of an L-system

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
- Geometric rules: Defines colors, widths, etc.
Graphically represent

Symbols for drawing and moving:

- **g**: draw a line one step in the current direction
- **f**: move forward one step in the current direction
Example: example1

axiom X

X -> g f g X

distance 15
lineWidth 5
color black

L =
What does this draw?
Geometric rules

• + change direction to the right
• – change direction to the left
• % change direction 180 degrees
• ~ decrement the width of the next lines
• [ save in stack current state info
• ] recover from stack state info
• { start filled in polygon
• } end filled in polygon
Example – lsys-samp1

• Axiom

• Replacement Rules

• Geometric Rules

NOTE: Must use spaces as separator between symbols
Example – lsys-samp1 (cont)

- Derivation of strings

\[
\begin{align*}
  &X \\
  &gggX+Y \\
  &gggggggX + Y + g \\
  &gggggggggggX+Y+g+g \\
  &gggggggggggggX+Y+g+g+g
\end{align*}
\]

Note: replace both X and Y each time
Example – lsys-samp2

[Image of a software interface showing an example of lsys-samp2 with an axiom and parameters listed in a table.]
Example – lsys-samp2 (cont)

$g[\sim+Yg]gX$

$g[\sim++Yg]gg[\sim+Yg]gX$

$g[\sim+++Yg]gg[\sim++Yg]gg[\sim+Yg]gX$

$\ldots$

$\ldots$
Example - tree
Example – tree rendered
Stochastic Tree

- Add a rule $T \rightarrow T$
- Now there is a choice for $T$, draw a line or don’t
Same Stochastic L-System

- Rendered 3 times, each at 8th derivation
JFLAP

- JFLAP is available for free:
  www.jflap.org
- Duke School of Environment uses L-systems to model pine needles in Duke Forest
Classwork - Exercise 1

- Write an L-system for the picture below.
- Symbols needed are: g, + and one variable
- Distance of the line is 100, rendering at 1 draws the first line, each additional render draws another line.
Exercise 2

• Write an L-system for the picture below.
• Symbols may need: g, %, +
• Distance set to 15, angle set to 45, side of square is length 30, first diagonal line is 60
• 1st, 2nd and 6th renderings shown
Exercise 3

• Write an L-system for the picture below.
• Symbols may need: g, +, -, [ ]
• Angle set to 90, distance set to 15
• Shows 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} renderings