Deterministic Finite Accepter (or Automata)

A DFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.
δ: Q × Σ → Q

Example: Create a DFA that accepts even binary numbers.

Transition Diagram:

M = (Q, Σ, δ, q₀, F) =

Tabular Format

<table>
<thead>
<tr>
<th>q₀</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>q₀</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>q₁</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example of a move: δ(q₀, 1) =
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = δ(q,s)
    s = next symbol to the right on tape
if q∈F then accept

Example of a trace: 11010

Pictorial Example of a trace for 100:

1) 1 0 0
   q0
   q1
2) 1 0 0
   q0
   q1
3) 1 0 0
   q0
   q1
4) 1 0 0
   q0
   q1

Definition:

δ*(q,λ) = q
δ*(q,wa) = δ(δ*(q, w), a)

Definition The language accepted by a DFA M=(Q,Σ,δ,q0,F) is set of all strings on Σ accepted by M. Formally,

L(M)={w ∈ Σ* | δ*(q0, w) ∈ F}
**Trap State**

Example: $L(M) =$

![Diagram of a DFA

You don’t need to show trap states! Any arc not shown will by default go to a trap state.

**Example:**

$L = \{w \in \Sigma^* \mid w \text{ has an even number of } a\text{'s and an even number of } b\text{'s}\}$

**Example:** Create a DFA that accepts even binary numbers that have an even number of 1’s.

**Definition** A language is regular iff there exists DFA $M$ s.t. $L = L(M)$. 
Chapter 2.2

Nondeterministic Finite Automata (or Acceptor)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

- Q is a finite set of states
- Σ is the tape (input) alphabet
- q₀ is the initial state
- F ⊆ Q is the set of final states
- δ: Q × (Σ ∪ {λ}) → 2^Q

Example

Note: In this example δ(q₀, a) =

L =

Example

L = {ab^n | n > 0} ∪ {a^n b | n > 0}

Definition: For an NFA M, L(M) = {w ∈ Σ* | δ*(q₀, w) ∩ F ≠ ∅}

The language accepted by nfa M is all strings w such that there exists a walk labeled w from the start state to final state.
2.3 NFA vs. DFA: Which is more powerful?

Example:

Theorem Given an NFA \( M = (Q_N, \Sigma, \delta_N, q_0, F_N) \), then there exists a DFA \( M_D = (Q_D, \Sigma, \delta_D, q_0, F_D) \) such that \( L(M_N) = L(M_D) \).

Proof:

We need to define \( M_D \) based on \( M_N \).

\( Q_D = \)

\( F_D = \)

\( \delta_D: \)

Algorithm to construct \( M_D \)

1. start state is \( \{q_0\} \cup \text{closure}(q_0) \)
2. While can add an edge
   (a) Choose a state \( A = \{q_i, q_j, \ldots, q_k\} \) with missing edge for \( a \in \Sigma \)
   (b) Compute \( B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a) \)
   (c) Add state \( B \) if it doesn’t exist
   (d) add edge from \( A \) to \( B \) with label \( a \)
3. Identify final states
4. if \( \lambda \in L(M_N) \) then make the start state final.
Properties and Proving - Problem 1

Consider the property Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with b. If a string does not have an a, then the string is not in R1awb(L).

Example 1: Consider L = \{aaab, bbba\}

R1awb(L) =

Example 2: Consider \(\Sigma = \{a, b\}\), \(L = \{w \in \Sigma^* \mid w \) has an even number of a’s and an even number of b’s\} \n
R1awb(L) =

Proof:
Properties and Proving - Problem 2

Consider the property Truncate_all_preceeding_b's or TruncPreb for short. If L is a regular, prove TruncPreb(L) is regular.

The property TruncPreb applied to a language L removes all preceeding b’s in each string. If a string does not have an preceeding b, then the string is the same in TruncPreb(L).

Example 1: Consider L={aaab,bbaa}

TruncPreb(L)=

Example 2: Consider L = {(bba)^n \mid n > 0}

TruncPreb(L)=

Proof:
Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable
  These states form a new state

**Definition** Two states p and q are indistinguishable if for all \( w \in \Sigma^* \)

\[
\begin{align*}
\delta^*(q, w) &\in F \Rightarrow \delta^*(p, w) \in F \\
\delta^*(p, w) &\notin F \Rightarrow \delta^*(q, w) \notin F
\end{align*}
\]

**Definition** Two states p and q are distinguishable if \( \exists \ w \in \Sigma^* \) s.t.

\[
\begin{align*}
\delta^*(q, w) &\in F \Rightarrow \delta^*(p, w) \notin F \text{ OR} \\
\delta^*(q, w) &\notin F \Rightarrow \delta^*(p, w) \in F
\end{align*}
\]
Example:
Example: