Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM M is defined by M=(Q,Σ,Γ,δ,q₀,B,F) where

- Q is finite set of states
- Σ is input alphabet
- Γ is tape alphabet
- B∈Γ is blank
- q₀ is start state
- F is set of final states
- δ is transition function

δ(q,a) = (p,b,R) means “if in state q with the tape head pointing to an ’a’, then move into state p, write a ’b’ on the tape and move to the right”.
**TM as Language recognizer**

**Definition:** Configuration is denoted by $\vdash$.

If $\delta(q,a) = (p,b,R)$ then a move is denoted

$$abaqabba \vdash abapbba$$

**Definition:** Let $M$ be a TM, $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$. $L(M) = \{w \in \Sigma^* | q_0 w \vdash x_1 q_f x_2 \text{ for some } q_f \in F, \}$

**TM as language acceptor**

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second 'a' by a 'b' if string is even length.

- Algorithm
Example:

$L = \{ a^n b^n c^n | n \geq 1 \}$

Is the following TM Correct?
**Definition:** A function with domain D is *Turing-computable* or *computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0w \xrightarrow{*} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

start with: 111

↑

end with: 111111

↑

Is the following TM correct?
Example:

$L = \{ww \mid w \in \Sigma^+\}$, $\Sigma = \{a, b\}$