Compsci 334-Mathematical Foundations of CS Dr. S. Rodger<br>Section: Turing Machines (handout)

## Review

Regular Languages

- FA, RG, RE
- recognize


## Context Free Languages

- PDA, CFG
- recognize

DFA:


Turing Machine:


## Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms


## Definition of TM

- Storage
- tape
- actions
- write symbol
- read symbol
- move left (L) or right (R)
- computation
- initial configuration
* start state
* tape head on leftmost tape square * input string followed by blanks
- processing computation
* move tape head left or right
* read from and write to tape
- computation halts
* final state


## Formal Definition of TM

A TM M is defined by $\mathrm{M}=\left(\mathrm{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathrm{~B}, \mathrm{~F}\right)$ where

- Q is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $\mathrm{B} \in \Gamma$ is blank
- $q_{0}$ is start state
- $F$ is set of final states
- $\delta$ is transition function
$\delta(\mathrm{q}, \mathrm{a})=(\mathrm{p}, \mathrm{b}, \mathrm{R})$ means "if in state q with the tape head pointing to an 'a', then move into state p, write a ' $b$ ' on the tape and move to the right".


## TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.
if $\delta(\mathrm{q}, \mathrm{a})=(\mathrm{p}, \mathrm{b}, \mathrm{R})$ then a move is denoted

$$
\text { abaqabba } \vdash \text { ababpbba }
$$

Definition: Let M be a TM, $\mathrm{M}=\left(\mathrm{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathrm{~B}, \mathrm{~F}\right) . \mathrm{L}(\mathrm{M})=\left\{w \in \Sigma^{*} \mid q_{0} w \stackrel{*}{\vdash} x_{1} q_{f} x_{2}\right.$ for some $q_{f} \in \mathrm{~F}$, $\left.x_{1}, x_{2} \in \Gamma^{*}\right\}$

## TM as language acceptor

$M$ is a $T M, w$ is in $\Sigma^{*}$,

- if $w \in L(M)$ then $M$ halts in final state
- if $\mathrm{w} \notin \mathrm{L}(\mathrm{M})$ then either
- M halts in non-final state
- M doesn't halt


## Example

$\Sigma=\{a, b\}$
Replace every second 'a' by a 'b' if string is even length.

- Algorithm


## Example:

$\mathrm{L}=\left\{a^{n} b^{n} c^{n} \mid n \geq 1\right\}$
Is the following TM Correct?


TM as a transducer
TM can implement a function: $f(w)=w$,


Definition: A function with domain D is Turing-computable or computable if there exists TM $\mathrm{M}=\left(\mathrm{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathrm{~B}, \mathrm{~F}\right)$ such that

$$
q_{0} w \stackrel{*}{\vdash} q_{f} f(w)
$$

$q_{f} \in \mathrm{~F}$, for all $w \in \mathrm{D}$.

## Example:

$\mathrm{f}(\mathrm{x})=2 \mathrm{x}$
x is a unary number

| start with: | 111 |
| :--- | :--- |
|  | $\uparrow$ |
| end with: | 111111 |
|  | $\uparrow$ |

Is the following TM correct?


## Example:

$\mathrm{L}=\left\{w w \mid w \in \Sigma^{+}\right\}, \Sigma=\{a, b\}$

