## Section: Turing Machines

Review

Regular Languages

- FA, RG, RE
- recognize


## Context Free Languages

- PDA, CFG
- recognize


## DFA:



## Turing Machine:



## Turing Machine (TM)

- invented by Alan M. Turing (1936) - computational model to study algorithms


## Definition of TM

- Storage
- tape
- actions
- write symbol
- read symbol
- move left (L) or right (R)


## - computation

- initial configuration
* start state
* tape head on leftmost tape
square
* input string followed by blanks
- processing computation
* move tape head left or right * read from and write to tape
- computation halts
* final state

Formal Definition of TM
A TM M is defined by
$\mathbf{M}=\left(\mathbf{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathbf{B}, \mathbf{F}\right)$ where

- Q is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $\mathrm{B} \in \Gamma$ is blank
- $q_{0}$ is start state
- $F$ is set of final states
- $\delta$ is transition function $\delta(\mathbf{q}, \mathbf{a})=(\mathbf{p}, \mathbf{b}, \mathbf{R})$ means

TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.
if $\delta(\mathbf{q}, \mathbf{a})=(\mathbf{p}, \mathbf{b}, \mathbf{R})$ then a move is denoted
abaqabba $\vdash$ ababpbba

Definition: Let M be a TM,
$\mathbf{M}=\left(\mathbf{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathbf{B}, \mathbf{F}\right) . \mathbf{L}(\mathbf{M})=$ $\left\{w \in \Sigma^{*} \mid q_{0} w \stackrel{*}{\vdash} x_{1} q_{f} x_{2}\right.$ for some $q_{f} \in \mathbf{F}$, $\left.x_{1}, x_{2} \in \Gamma^{*}\right\}$

TM as language acceptor
M is a TM, w is in $\Sigma^{*}$,

- if $\mathbf{w} \in \mathbf{L}(\mathrm{M})$ then M halts in final state
- if $\mathbf{w} \notin \mathrm{L}(\mathrm{M})$ then either
- M halts in non-final state
- M doesn't halt


## Example

$\Sigma=\{a, b\}$
Replace every second 'a' by a 'b' if string is even length.

- Algorithm


## Example:

$\mathbf{L}=\left\{a^{n} b^{n} c^{n} \mid n \geq 1\right\}$
Is the following TM Correct?


## TM as a transducer

## TM can implement a function: $\mathbf{f}(\mathbf{w})=\mathbf{w}$,

start with:
end with:
w,
$\uparrow$

Definition: A function with domain D
is Turing-computable or computable if there exists $\mathbf{T M} \mathbf{M}=\left(\mathbf{Q}, \Sigma, \Gamma, \delta, q_{0}, \mathbf{B}, \mathbf{F}\right)$ such that

$$
q_{0} w \stackrel{*}{\vdash} q_{f} f(w)
$$

$q_{f} \in \mathbf{F}$, for all $w \in \mathbf{D}$.

## Example:

$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=2 \mathrm{x} \\
& \mathrm{x} \text { is a unary number }
\end{aligned}
$$

start with:
end with:

111
$\uparrow$
111111
$\uparrow$

## Is the following TM correct?



## Example:

$$
\mathbf{L}=\left\{w w \mid w \in \Sigma^{+}\right\}, \Sigma=\{a, b\}
$$

