CompSci 370
Other Search Paradigms

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Searching with Partial Information

(not a focus of this class, but good to be aware of)

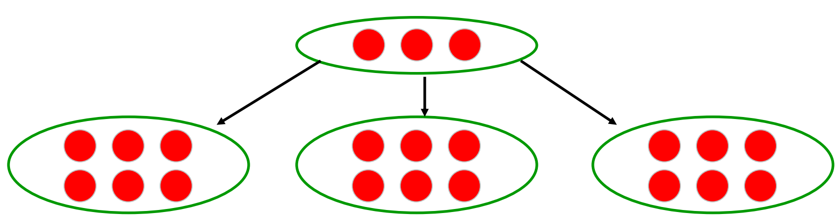
- Multiple state problems
  - Several possible initial states
- Contingency problems
  - Several possible outcomes for each action
- Exploration problems
  - Outcomes of actions not known *a priori*, must be discovered by trying them

Example

- Initial state may not be detectable
  - Suppose sensors for a nuclear reactor fail
  - Need *safe* shutdown sequence despite ignorance of some aspects of state
- This complicates search *enormously*
- In the worst case, contingent solution could cover the entire state space

State Sets

- Idea:
  - Maintain a set of candidate states
  - Each search node represents a set of states
  - Can be hard to manage if state sets get large
- If states have probabilistic outcomes, we maintain a probability distribution over states



Searching in Unknown Environments

(not a focus of this class, but good to be aware of)

- What if we don't know the consequences of actions before we try them?
- Often called on-line search
- Goal: Minimize competitive ratio
  - Actual distance/distance traveled if model known
  - Problematic if actions are irreversible
  - Problematic if links can have unbounded cost

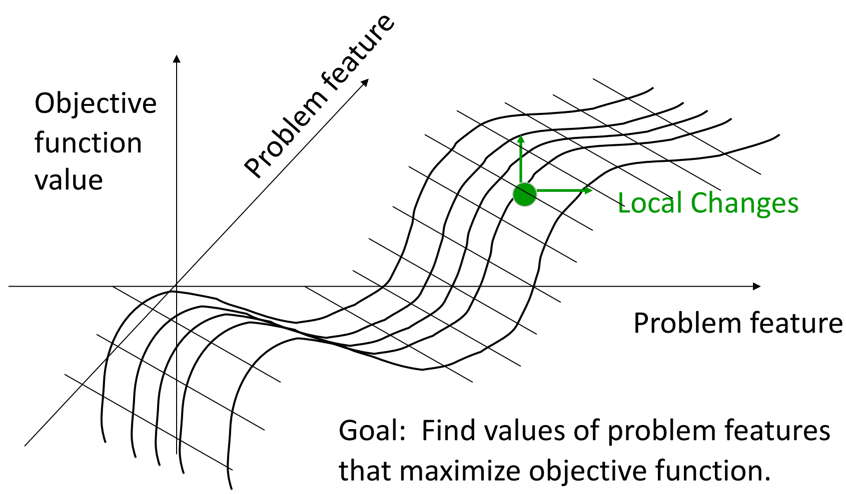
Optimization

(Not directly a topic of this class, but used later)

- Want to find the "best" state
- Solution is more important than path, but
- Some solutions are better than others
- Interested in minimizing or maximizing some function of the problem state
  - Find a protein with a desirable property
  - Optimize circuit layout

- History of search steps not worth the trouble

State Space Landscape



Goal: Find values of problem features that maximize objective function.

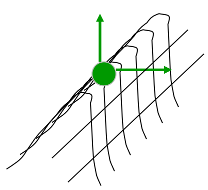
Note: This is conceptual. Often this function is not smooth.

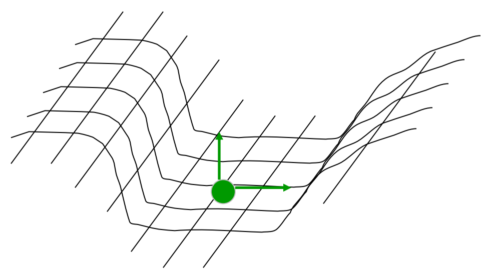
Hill Climbing

- Idea: Try to climb up the state space landscape (often in axis-parallel directions) to find a setting of the problem features with high value.
- Approaches:
  - Steepest ascent
  - Stochastic – pick one of the good ones
  - First choice
- This is a *greedy* procedure

Limitations of Hill Climbing

- Local maxima
- Ridges – direction of ascent is at 45 degree angle to any of the local changes
- Plateaux – flat expanses





Getting Unstuck

- Random restarts
- Simulated annealing for minimization (maximization)
  - Take uphill (downhill) moves with small probability
  - Probability of moving uphill (downhill) decreases with
    - Number of iterations
    - Steepness of uphill (downhill) move
  - If system is "cooled" slowly enough, will find global optimum w.p. 1
  - Motivated by the annealing of metals and glass, where annealing reduces potential energy stored in chemical/physical structures, making substance more ductile and less brittle

Genetic Algorithms

- GAs run hot and cold (cold now, hotish in 90’s)
- Biological metaphors to motivate search
- Organism is a word from a finite alphabet (organisms = states)
- Fitness of organism measures its performance on task (fitness = objective)
- Uses multiple organisms (parallel search)
- Uses mutation (random steps)

Crossover

Crossover is a distinguishing feature of GAs:

Randomly select organisms for “reproduction” in accordance with their fitness. More “fit” individuals are more likely to reproduce.

Reproduction is sexual and involves *crossover*:

Organism 1: 1 1 0 0 1 0 0 1 0

Organism 2: 0 0 0 1 0 1 1 1 0

Offspring: 1 1 0 0 1 1 1 1 0

Is this a good idea?

- Has worked well in some examples
- Can be very brittle
  - Representations must be carefully engineered
  - Sensitive to mutation rate
  - Sensitive to details of crossover mechanism
- For the same amount of work, stochastic variants of hill climbing sometimes do better
- Hard to analyze; needs more rigorous study

- Compare with neural network hype cycle

Continuous Spaces

- In continuous spaces, we don't need to "probe" to find the values of local changes

- If we have a closed-form expression for our objective function, we can use the calculus

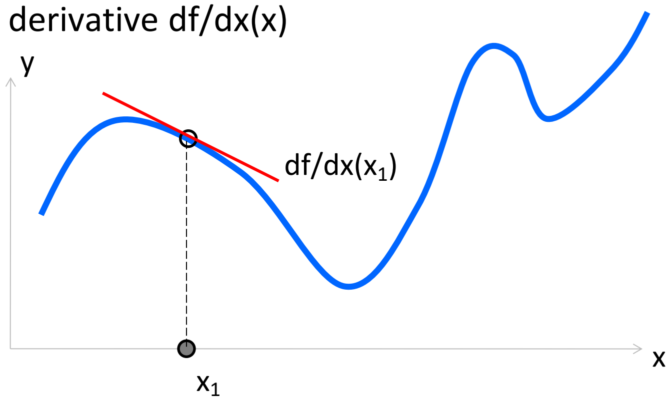
- Suppose objective function is: $f(x_1, y_1, x_2, y_2, x_3, y_3)$

- Gradient tells us direction and steepness of change

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

Gradient Descent in Continuous Space

- Minimize y=f(x)
- Move in opposite direction of derivative df/dx(x)



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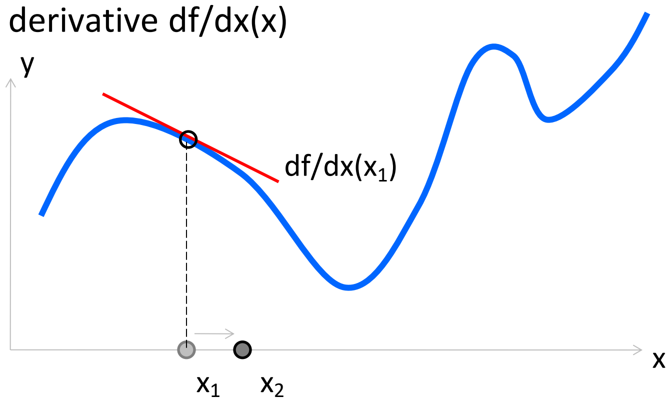

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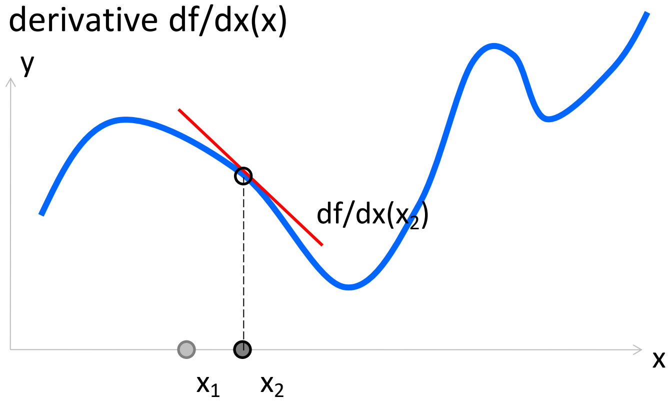

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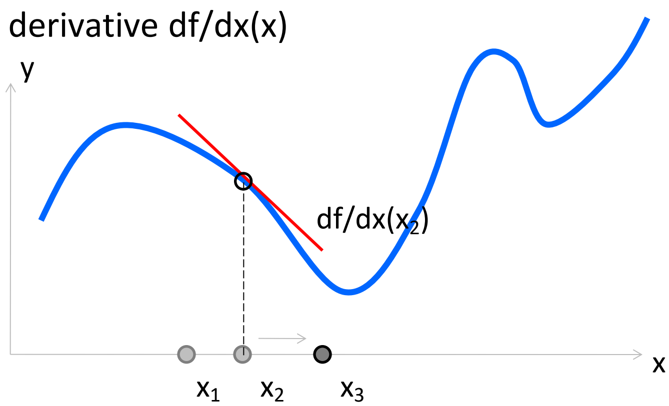

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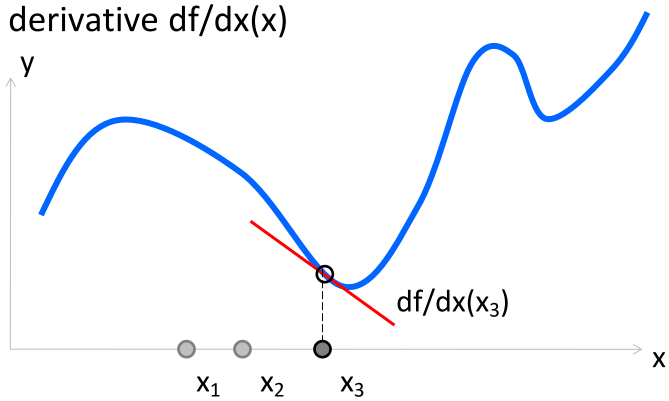

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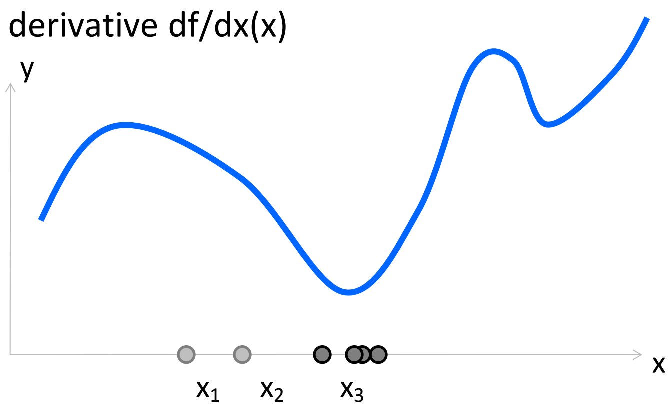
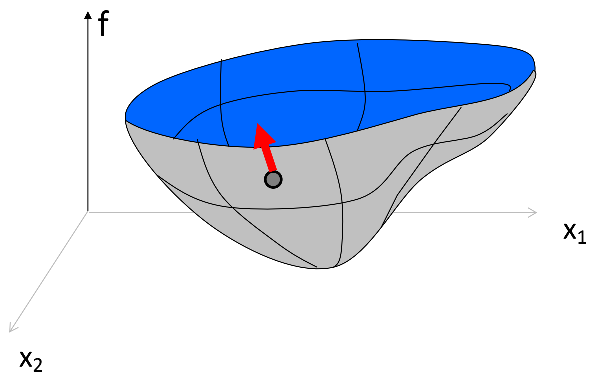


Gradient: analogue of derivative in multivariate functions $f(x_1,\ldots,x_n)$

Direction that you would move $x_1,\ldots,x_n$ to make the steepest increase in f



Algorithm for Gradient Descent

- Input: continuous *objective function* f, initial point $\mathbf{x}^0=(x_1^0,\ldots,x_n^0)$
- For t=0,…,N-1:

  Compute gradient vector $\mathbf{g}^t=(\partial f/\partial x_1(\mathbf{x}^t),\ldots,\partial f/\partial x_n(\mathbf{x}^t))$

  If the length of $\mathbf{g}^t$ is small enough [convergence]

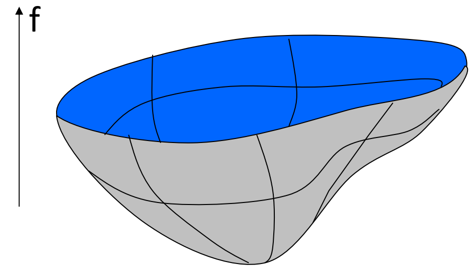
  Return $\mathbf{x}^t$

  Pick a *step size* α^t

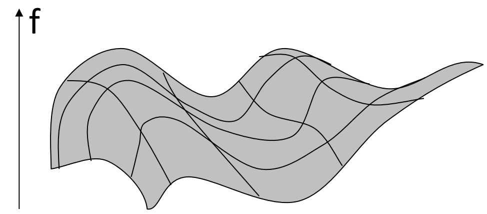
  Let $\mathbf{x}^{t+1}= \mathbf{x}^t -\alpha^t\mathbf{g}^t$

"Industrial strength" optimization software uses more sophisticated techniques to use higher derivatives, handle constraints, deal with particular function classes, etc.

GD works well



GD works poorly



Search Conclusions

- Search = most general purpose technique in existence
- Everything can be formulated as a search problem, from sorting to curing cancer
- Search techniques have been specialized to match different types of problems

- Be a smart consumer of search:
  - Specifying your problem clearly
  - Find the technique that matches your problem