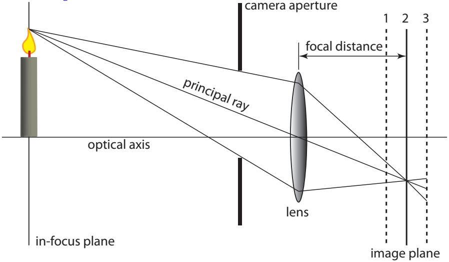
Real Cameras and their Calibration

COMPSCI 527 — Computer Vision

Outline

1. Real Cameras
2. Camera Calibration
   - A Camera Model
   - Parameter Optimization
   - Lab Setup and Imaging

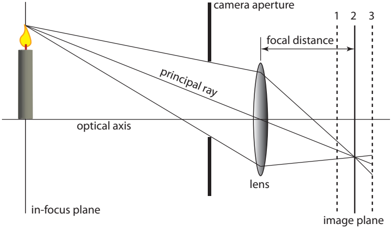
Depth of Field



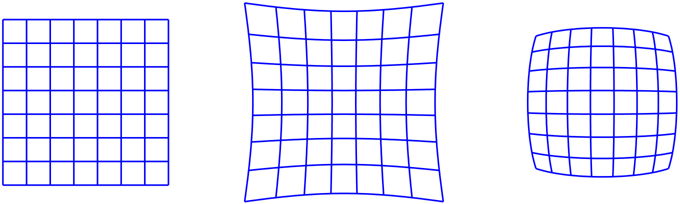
- Focal *length*: focal *distance* when an object at ∞ is in focus
- Focal length is a lens property
- Focal distance can be changed by rotating the focusing ring
- Nothing to do with *zoom*, which changes focal length
- Alas, f is often used for either focal distance or focal length

Changing Depth of Field

- *Aperture*: diameter of the hole in front of the lens
- Measured in *stops*, or *f-numbers* $n = \frac{f}{a}$
  (a is aperture diameter, f is focal length)
- Area (light flux) is proportional to *square* of diameter
- Small aperture (big f-number) $\Rightarrow$ great depth of field
- A shallow depth of field is sometimes desirable



Lens Distortion



pincushion barrel

Camera Calibration

- Cameras have *intrinsic parameters*: focal distance, pixel size, principal point, lens distortion parameters
- ... and *extrinsic parameters*: Rotation, translation relative to some world reference system
- *Camera calibration* is a combination of lab measurements and algorithms aimed at determining both types of parameters
- We do not calibrate for finite depth of field

  (stop down the aperture, flood the scene with light)

Calibration as Learning

- There are many specific variants of calibration, but the general idea is the same
- Looks very much like machine learning:
  1. Make a parametric model of what a camera does: Inputs are world points $\mathbf{W}$ in world coordinates, outputs are image points ξ in image pixel coordinates ("predictor architecture")
  2. Collect a sufficiently large set S of input-output pairs $(\mathbf{W}_n, \xi_n)$ ("training set")
  3. Fit the parameters to S by numerical optimization ("training")
- We even have generalization requirements: The parameters should be correct for pairs $(\mathbf{W}, \xi)$ not in S

  (However, the "hypothesis space" is very small here, so just use "enough points" and do fitting)
- We already know how to do 1, 3. Need to figure out 2

Camera Model

$$\mathbf{X} = R(\mathbf{W} - \mathbf{t})$$

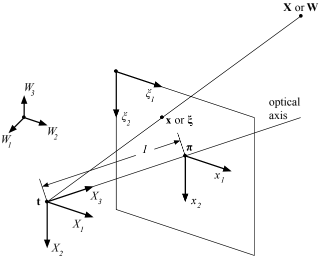
$$\mathbf{x} = p(\mathbf{X}) = \frac{1}{X_3}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\mathbf{y} = d(\mathbf{x}) \qquad \text{(lens distortion)}$$

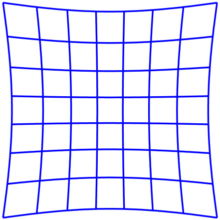
$$\xi = S\mathbf{y} + \pi$$

$$S = f\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Can only determine *products* $f\,s_x$ and $f\,s_y$, not f, s_x, s_y individually



Lens Distortion Model



- Distortion is radial around the principal point:

  $\mathbf{y} = d(\mathbf{x}) = \delta(r)\,\mathbf{x} \quad \text{where} \quad r = \|\mathbf{x}\|$
- *Radial distortion function* $\delta(\cdot)$ is nonlinear
- Must be analytical everywhere (Maxwell). Implication:
  - Restrict to x axis: $\delta(r(\mathbf{x})) = \delta(|x|)$
  - Odd powers of $|x|$ have a cusp at the origin
  - Therefore, $\delta(r) = 1 + k_1 r^2 + k_2 r^4 + \ldots$
- Large powers of r only affect peripheral areas and cannot be determined well
- Typically, $\delta(r) = 1 + k_1 r^2 + k_2 r^4$

Camera Parameters

$$
\begin{aligned}
\mathbf{X} &= R\,(\mathbf{W} - \mathbf{t}) \\
\mathbf{x} &= p(\mathbf{X}) = \frac{1}{X_3}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\
\mathbf{y} &= \mathbf{x}\left(1 + k_1\|\mathbf{x}\|^2 + k_2\|\mathbf{x}\|^4\right) \\
\xi &= S\mathbf{y} + \pi
\end{aligned}
$$

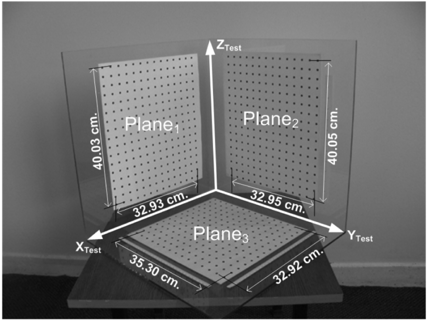
- Extrinsic parameters: R, $\mathbf{t}$ (6 degrees of freedom)
- Intrinsic parameters: π, $f\,s_x$, $f\,s_y$, k_1, k_2 (6 numbers)

$\xi = \mathbf{c}(\mathbf{W}; \mathbf{p})$ where $\mathbf{p} \in \mathbb{R}^{12}$

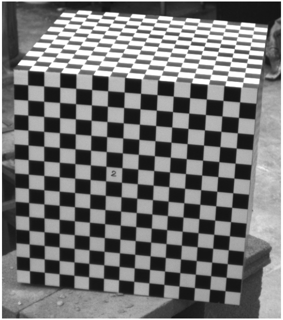
Data Fitting

- Collect input-output pairs $(\mathbf{W}_n, \xi_n)$ for $n = 1, \ldots, N$
  $\xi = \mathbf{c}(\mathbf{W}; \mathbf{p})$ where $\mathbf{p} \in \mathbb{R}^{12}$
  $\mathbf{p}^* = \arg\min_{\mathbf{p}} e(\mathbf{p}) \quad \text{where} \quad e(\mathbf{p}) = \frac{1}{N} \sum_{n=1}^{N} \|\xi_n - \mathbf{c}(\mathbf{W}_n; \mathbf{p})\|^2$
- e is nonlinear
- To initialize: clamp $k_1 = k_2 = 0$, solve a linear system
- Approximate because of clamping *and* because the residual is different from $e(\mathbf{p})$
- Use any optimization algorithm to refine

Calibration Target

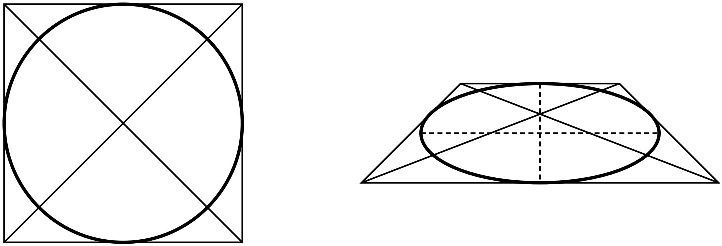


http://www.mdpi.com/1424-8220/9/6/4572/htm



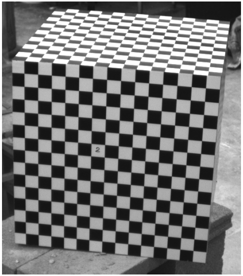
Duke Computer Vision Lab

Circles are Problematic



Calibration Protocol Summary

- Place calibration target in front of camera (fill the image)
- Measure image coordinates (with software help?)
- Make a file with $(\mathbf{W}_n, \xi_n)$ pairs
- Fit parameters by numerical optimization
- Redo if you touch the lens or the camera!



An Example for Distortion Only