Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\[ \text{M1} \rightarrow \text{M2} \]

z represents any symbol in
2. Given Turing Machines $M_1$ and $M_2$

$M_1$

$M_2$

$\rightarrow S \quad H \rightarrow S' \quad H'$

$\rightarrow M_1 \xrightarrow{x} M_2$

$\rightarrow S \quad H \xrightarrow{x;x,R} z; z; L \rightarrow S' \quad H'$

$z$ represents any symbol in $x$ is an element of
3. Given Turing Machines M1, M2, and M3

M1

[Diagram of M1]

M2

[Diagram of M2]

M3

[Diagram of M3]

x is an element of

y is any element except x from

z is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{¬a}$ - move right until you see anything that is not an $a$

8. $L_{¬a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\overrightarrow{a,b} \{w\} \overrightarrow{w}$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \).

If \(|w|\) is odd, then write a \( b \) at the end of the string. The tape head should finish pointing at the leftmost symbol of \( w \).

input: bab, output: babb
input: ba, output: ba

What is the running time? \( \Theta(n) \)
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $ab abb b$, output: $ab abb b bb b$

The tape head should finish pointing at the leftmost symbol of $w$.  

\[ S \rightarrow \mathbf{b} \mathbf{R} a \rightarrow \mathbf{R} b b \mathbf{L} \mathbf{a} \]

\[ \mathbf{c} \mathbf{h} \rightarrow \mathbf{B} \]

\[ \rightarrow \mathbf{L} \mathbf{B} \mathbf{R} \mathbf{h} \]

input: $ab abb b$    $\Theta(n^2)$ cleanup
Turing’s Thesis  Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f: D \rightarrow R \) is a TM M, which given input \( d \in D \), halts with answer \( f(d) \in R \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\text{end with:} & \quad 1111111
\end{align*}
\]
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{align*}
\text{start with:} & \quad \text{abac} \\
\quad & \uparrow \\
\text{end with:} & \quad \text{abac0abac} \\
\quad & \uparrow 
\end{align*}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: $\underbrace{aaBbabca}_\uparrow$

end with: $\underbrace{aaBBbaca}_\uparrow$
Algorithm:

• remember symbol to the right and erase it

• for each symbol to the left do
  – shift the symbol one cell to the right

• replace first symbol erased

• move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: $\text{babcaBba}$

end with: $\text{bacaBBba}$

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

\[
\begin{array}{c}
1111 \\
\uparrow \\
11111111 \\
\uparrow
\end{array}
\]

Example: Multiply two unary numbers, \( f(x\cdot y) = x \cdot y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\begin{array}{c}
\text{start with:} \\
1111*11 \\
\uparrow \\
\text{end with:} \\
11111111 \\
\uparrow
\end{array}
\]