Proof of Problem 1. Consider the property Rlawb. If L is regular, prove $Rlawb(L)$ is regular.

Proof

Assume L is regular

$\exists$ a DFA M s.t. $L = L(M)$.

~~$M = (Q, \Sigma, \delta, q_0$~~

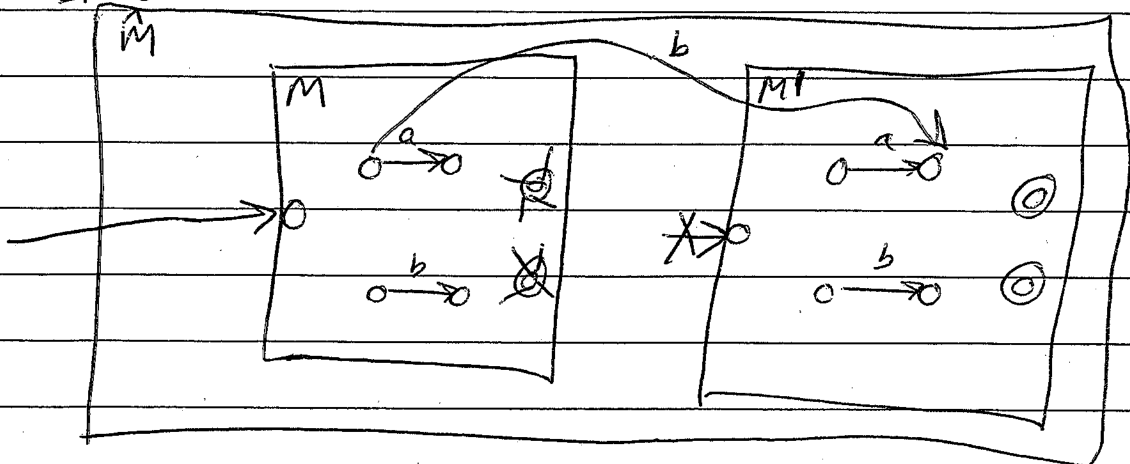
$$M = (Q, \Sigma, \delta, q_0, F)$$

Construct an NFA $\hat{M}$ from M s.t.

$$L(\hat{M}) = Rlawb(L)$$

To construct $\hat{M}$, make a copy of M called $M' = (Q', \Sigma, \delta', q_0', F')$ where M' is an exact copy with everything primed.

Idea:



Now describe the construction/changes.

$\hat{M} = (\hat{Q}, \Sigma, \hat{\delta}, q_0, \hat{F})$

$\hat{Q} = Q \cup Q'$ (states in $\hat{Q}$ are states from M & M')

$\hat{F} = F'$ (final states in $\hat{Q}$ are final states from M')

$\hat{\delta} = \delta \cup \delta' \cup \{ \hat{\delta}(q,b) = p'$ for every a arc

$\delta(q,a) = p$ where $q, p \in Q$ & $p' \in Q' \}$

(for every 'a' arc in M, add a 'b' arc to the corresponding destination in M')

Let $w = uav$ Show that is $w \in L$, then $w' = ubv \in Rlawb(L)$

Suppose $w \in L$ $\quad \delta^*(q_0, uav) = p \in F$

$\delta^*(q_0, u) = r \in Q, \ \delta(r,a) = s \in Q, \ \delta^*(s,v) = p \in F$

Thus, $\hat{\delta}^*(q_0, u) = r \in Q, \ \hat{\delta}(r, b) = s' \in Q', \ \hat{\delta}^*(s', v) = p' \in F'$

so $w' = ubv \in F'$, thus $w' = ubv \in Rlawb(L)$

Suppose $w \notin L$ $\quad \delta^*(q_0, uav) = p \notin F$

$\delta^*(q_0, u) = r \in Q, \ \delta(r,a) = s \in Q, \ \delta^*(s,v) = p \notin F$

Thus $\hat{\delta}^*(q_0, u) = r \in Q, \ \hat{\delta}(r, b) = s' \in Q', \ \delta^*(s', v) = p' \notin F$

so $w' = ubv \notin F'$, thus $w' = ubv \notin Rlawb(L)$