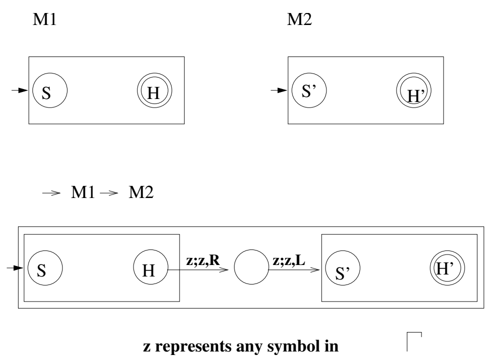
Compsci 334 - Mathematical Foundations of CS
Dr. S. Rodger
Section: Turing Machines - Building Blocks (handout)

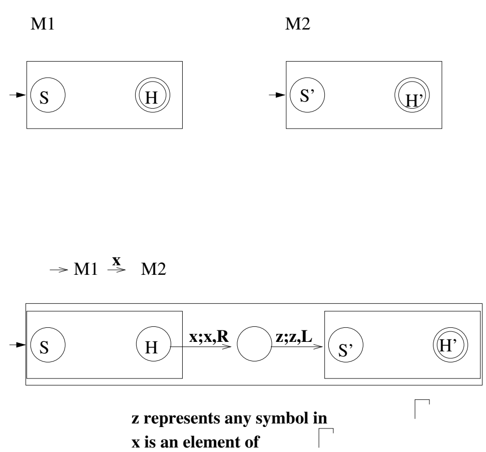
Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

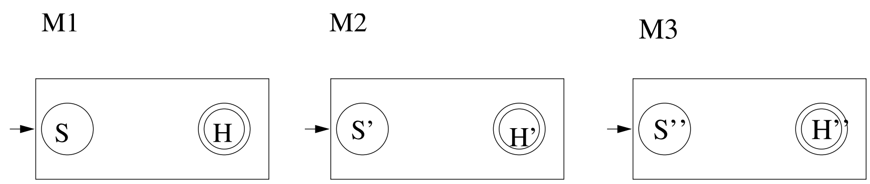
1. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - Run M2

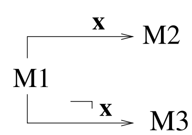


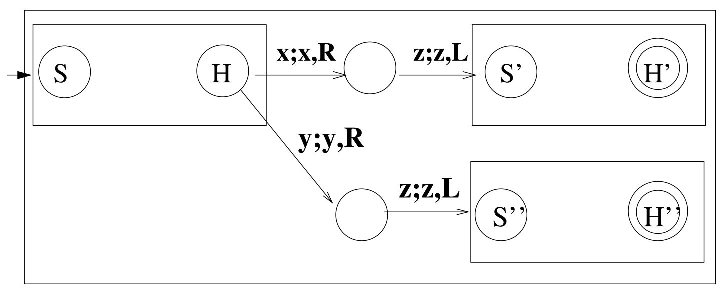
2. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - If x is current symbol
     - then Run M2



3. Given Turing Machines M1, M2, and M3
   Notation for
   - Run M1
   - If x is current symbol
     - then Run M2
     - else Run M3







x is an element of Γ
y is any element except x from Γ
z is any element from Γ

More Notation for Simplifying Turing Machines

Suppose Γ={a,b,c,B}

z is any symbol in Γ

x is a specific symbol from Γ

1. s - start
2. R - move right

3. L - move left

4. x - write x (and don't move)

5. R_a - move right until you see an a

6. L_a - move left until you see an a

7. $R_{\neg a}$ - move right until you see anything that is not an a

8. $L_{\neg a}$ - move left until you see anything that is not an a

9. h - halt in a final state

10. $\left.\overset{a,b}{\rightarrow}\right\} \overset{w}{\rightarrow}$

    If the current symbol is a or b, let w represent the current symbol.

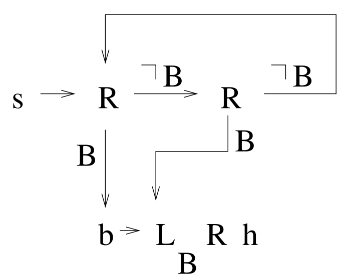
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a b at the end of the string. The tape head should finish pointing at the leftmost symbol of w.

input: bab, output: babb

input: ba, output: ba



What is the running time?

Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each a in the string, append a b to the end of the string.

input: *abbabb*, output: *abbabbbb*

The tape head should finish pointing at the leftmost symbol of w.

Turing's Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An *algorithm* for a function f:D→R is a TM M, which given input d∈D, halts with answer f(d)∈R.

Example: $f(x + y) = x + y$, x and y unary numbers.

start with:	111+1111 ↑
end with:	1111111 ↑

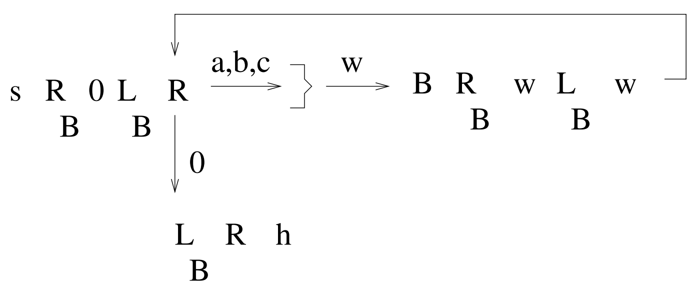
Example: Copy a String, f(w)=w0w, w$\in \Sigma^*$, $\Sigma = \{a, b, c\}$

Denoted by C

start with:	abac ↑
end with:	abac0abac ↑

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol



Example: Shift the string that is to the left of the tape head to the right,

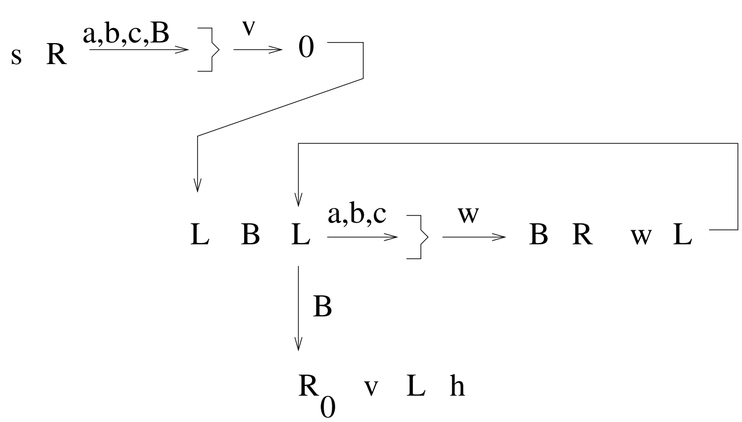
denoted by S_R (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with:	aaBbabca
	↑ (under the second b)
end with:	aaBBbaca
	↑ (under the a after "bb")

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position



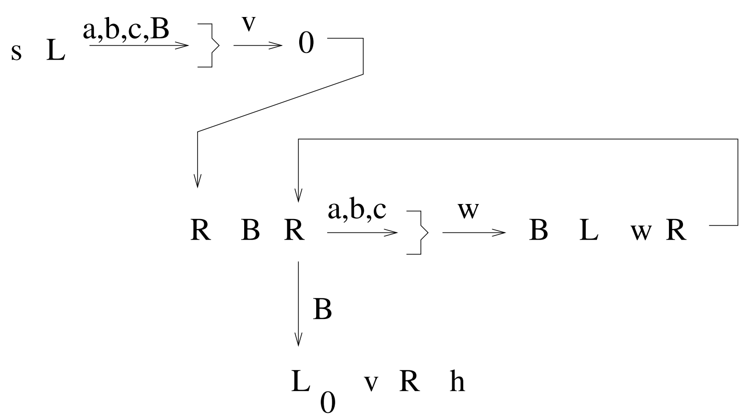
Example: Shift the string that is to the right of tape head to the left,

denote by S_L (shift left)

start with: babcaBba
↑

end with: bacaBBba
↑

(similar to S_R)



Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, f(x∗y)=x∗y, x and y unary numbers. Assume x,y>0.

start with:	1111∗11 ↑
end with:	11111111 ↑