Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

1. Given Turing Machines $M_1$ and $M_2$
   Notation for
   - Run $M_1$
   - Run $M_2$

2. Given Turing Machines $M_1$ and $M_2$
   Notation for
   - Run $M_1$
   - If $x$ is current symbol
     - then Run $M_2$
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose $\Gamma=\{a,b,c,B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$
- $y$ is any element except $x$ from $\Gamma$
- $z$ is any element from $\Gamma$

1. $s$ - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. R_a - move right until you see an a

6. L_a - move left until you see an a

7. R_{\sim a} - move right until you see anything that is not an a

8. L_{\sim a} - move left until you see anything that is not an a

9. h - halt in a final state

10. \( a, b \rightarrow w \)

   If the current symbol is a or b, let w represent the current symbol.
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example

Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( abbabb \), output: \( abbabbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).

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**Turing’s Thesis** Any computation that can be carried out by a mechanical means can be performed by a TM.

**Definition:** An *algorithm* for a function \( f: D \rightarrow R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).

**Example:** \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

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start with: 111+1111
         ↑
end with:  1111111
         ↑
```
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{ a, b, c \} \)

Denoted by \( C \)

- start with: \( abac \)
- end with: \( abac0\)abac

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

\[ \begin{array}{c}
\text{s R 0 L R} \\
B \quad B
\end{array} \quad \text{a,b,c} \quad \begin{array}{c}
w \quad B \\
R \quad w \quad B \\
B \quad B
\end{array} \]
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with:} & \quad \text{aaBbabca} \\
\text{end with:} & \quad \text{aaBBbaca}
\end{align*}
\]

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
**Example:** Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

- start with: $\text{babcaBba}$
- end with: $\text{bacaBBba}$

(similar to $S_R$)
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, \( f(x \cdot y) = x \cdot y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

start with: \( 1111 \times 11 \)
\[ \uparrow \]

end with: \( 11111111 \)
\[ \uparrow \]