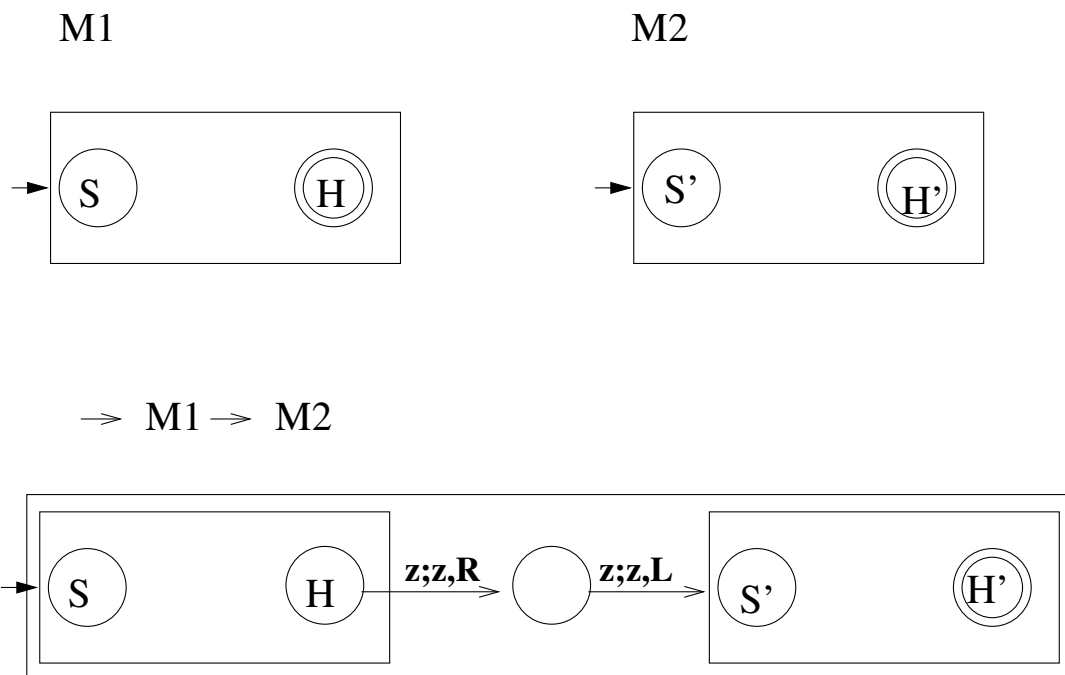


# Section: Turing Machines - Building Blocks

## 1. Given Turing Machines M1 and M2 Notation for

- Run M1
- Run M2

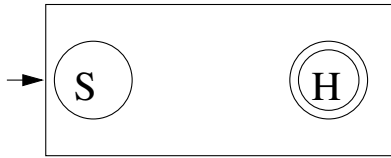


**z represents any symbol in**

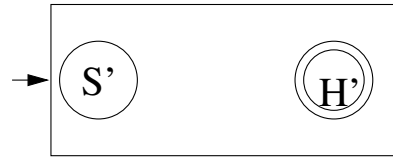


## 2. Given Turing Machines M1 and M2

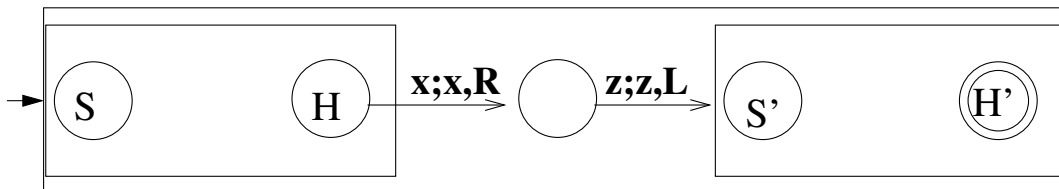
M1



M2

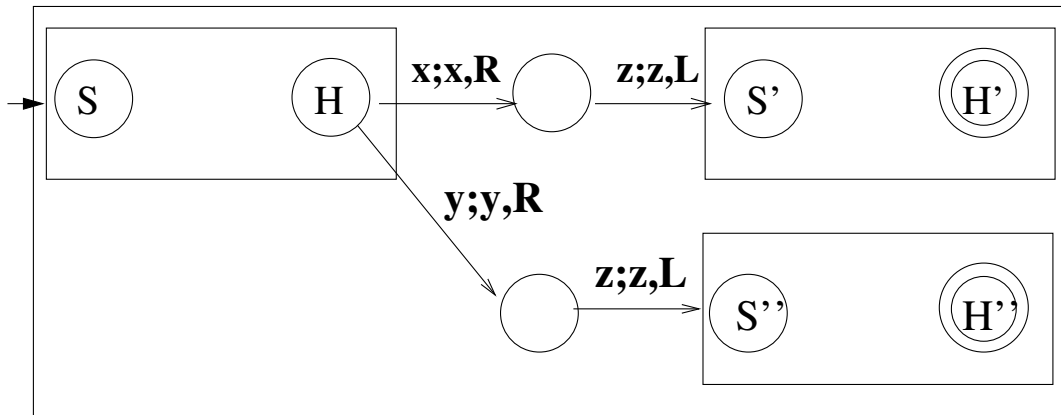
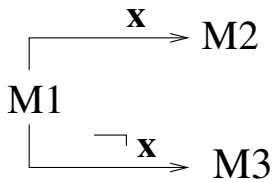
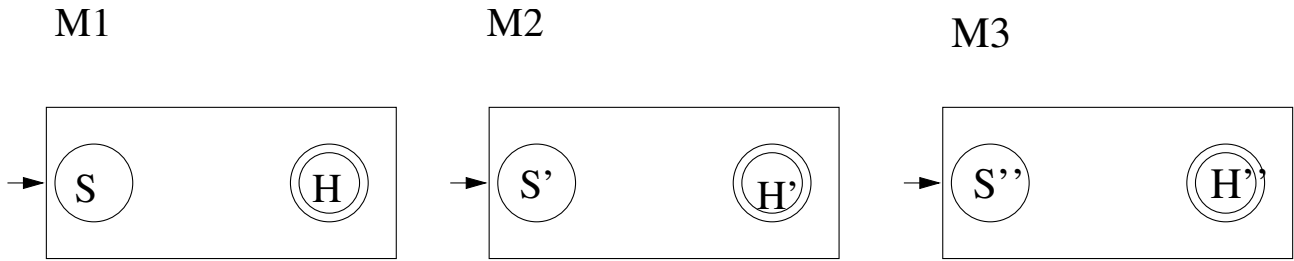


$\Rightarrow M1 \xrightarrow{x} M2$



**z represents any symbol in**  
**x is an element of**

### 3. Given Turing Machines M1, M2, and M3



**x** is an element of  $\Sigma$

**y** is any element except **x** from  $\Sigma$

**z** is any element from  $\Sigma$

# More Notation for Simplifying Turing Machines

Suppose  $\Gamma = \{a, b, c, B\}$

$z$  is any symbol in  $\Gamma$

$x$  is a specific symbol from  $\Gamma$

1.  $s$  - start

2.  $R$  - move right

3.  $L$  - move left

4.  $x$  - write  $x$  (and don't move)

5.  $R_a$  - move right until you see an  $a$

6.  $L_a$  - move left until you see an  $a$

7.  $R_{\neg a}$  - move right until you see anything that is not an  $a$

8.  $L_{\neg a}$  - move left until you see anything that is not an  $a$

9.  $h$  - halt in a final state

10.  $\left. \begin{array}{c} a, b \\ \rightarrow \end{array} \right\} \rightarrow w$

If the current symbol is  $a$  or  $b$ , let  $w$  represent the current symbol.

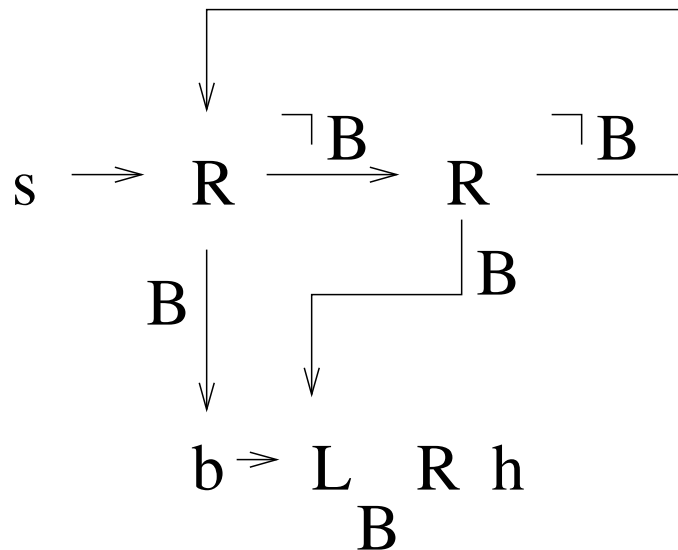
## Example

Assume input string  $w \in \Sigma^+$ ,  $\Sigma = \{a, b\}$ .

If  $|w|$  is odd, then write a  $b$  at the end of the string. The tape head should finish pointing at the leftmost symbol of  $w$ .

input: bab, output: babb

input: ba, output: ba



What is the running time?

## Example

Assume input string  $w \in \Sigma^+$ ,  $\Sigma = \{a, b\}$ ,  
 $|w| > 0$

For each  $a$  in the string, append a  $b$  to the end of the string.

input: *abbabb*, output: *abbabbbb*

The tape head should finish pointing at the leftmost symbol of  $w$ .

**Turing's Thesis** Any computation that can be carried out by a mechanical means can be performed by a TM.

**Definition:** An *algorithm* for a function  $f:D \rightarrow R$  is a TM  $M$ , which given input  $d \in D$ , halts with answer  $f(d) \in R$ .

**Example:**  $f(x + y) = x + y$ ,  $x$  and  $y$  unary numbers.

start with:            111+1111

↑

end with:            1111111

↑



**Example: Copy a String,  $f(w) = w0w$ ,**  
 $w \in \Sigma^*$ ,  $\Sigma = \{a, b, c\}$

**Denoted by C**

**start with:            abac**

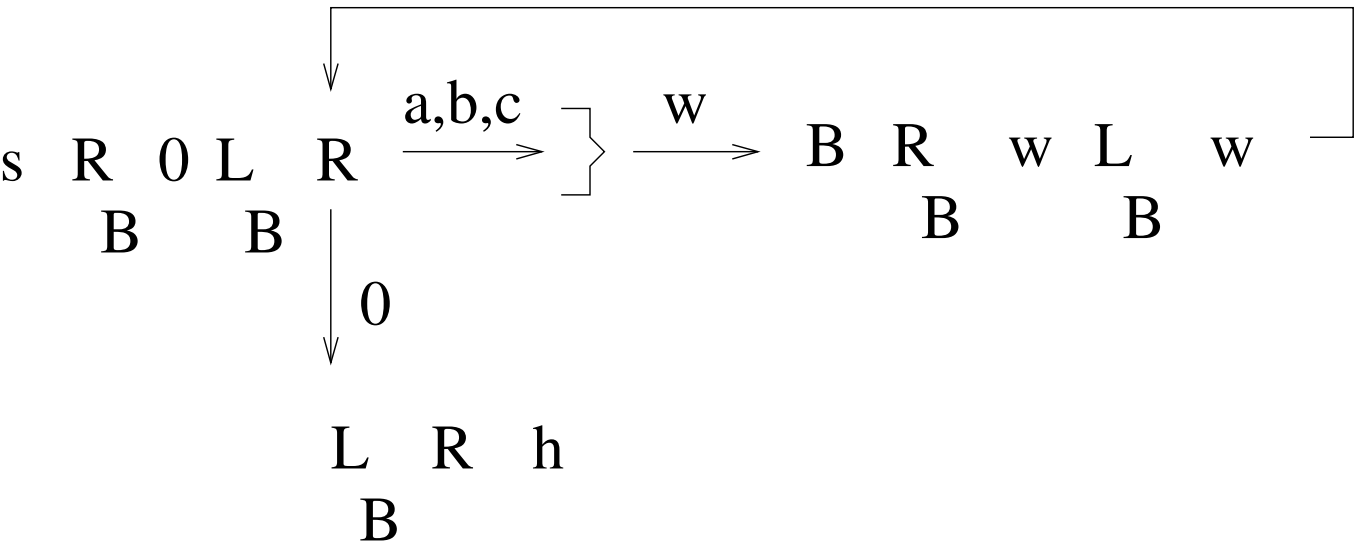
↑

**end with:             abac0abac**

↑

**Algorithm:**

- **Write a 0 at end of string**
- **For each symbol in string**
  - **make a copy of the symbol**



Example: Shift the string that is to the left of the tape head to the right, denoted by  $S_R$  (shift right)

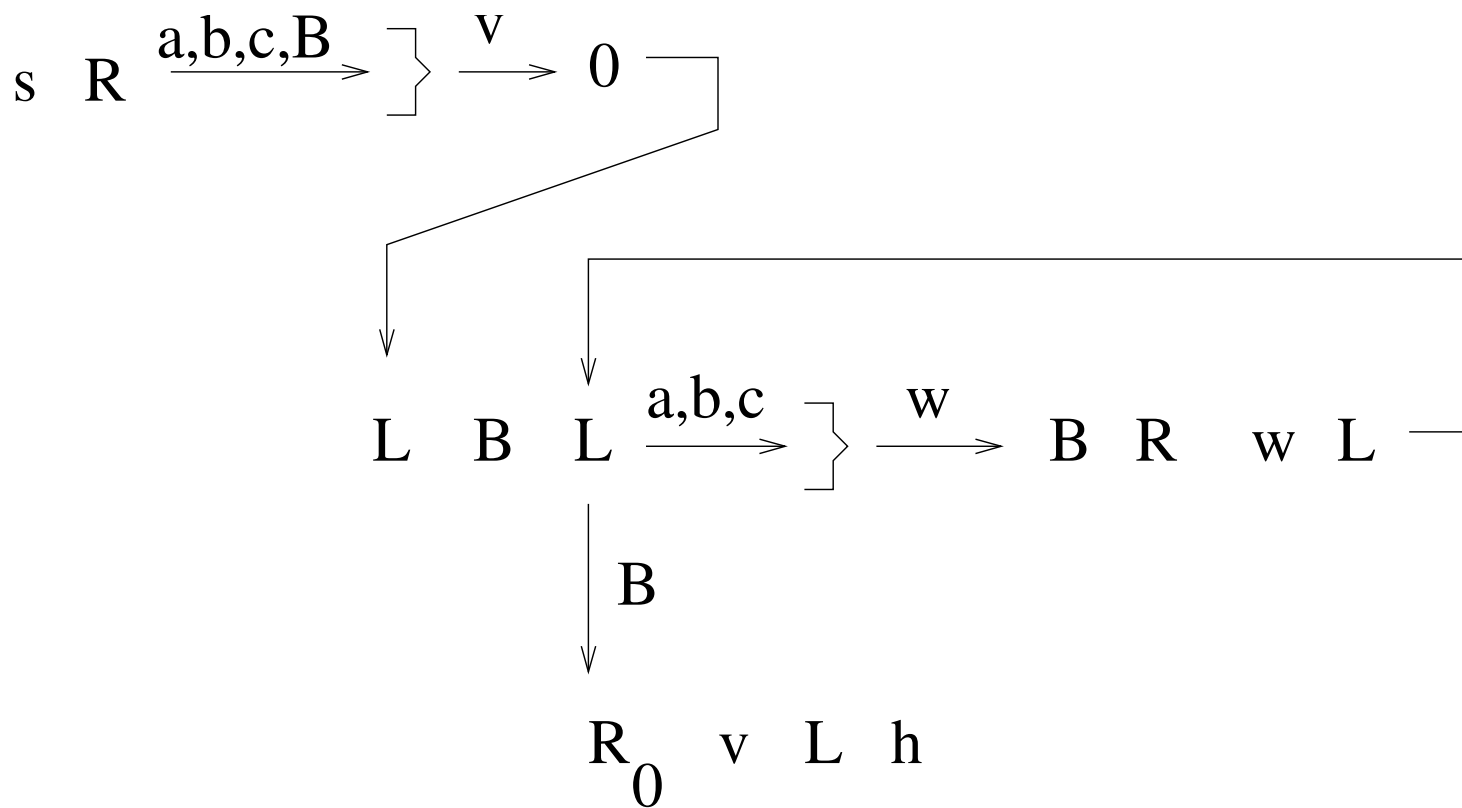
Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with:           aaBbabca  
  ↑

end with:             aaBBbaca  
  ↑

## Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position



**Example:** Shift the string that is to the right of tape head to the left, denote by  $S_L$  (shift left)

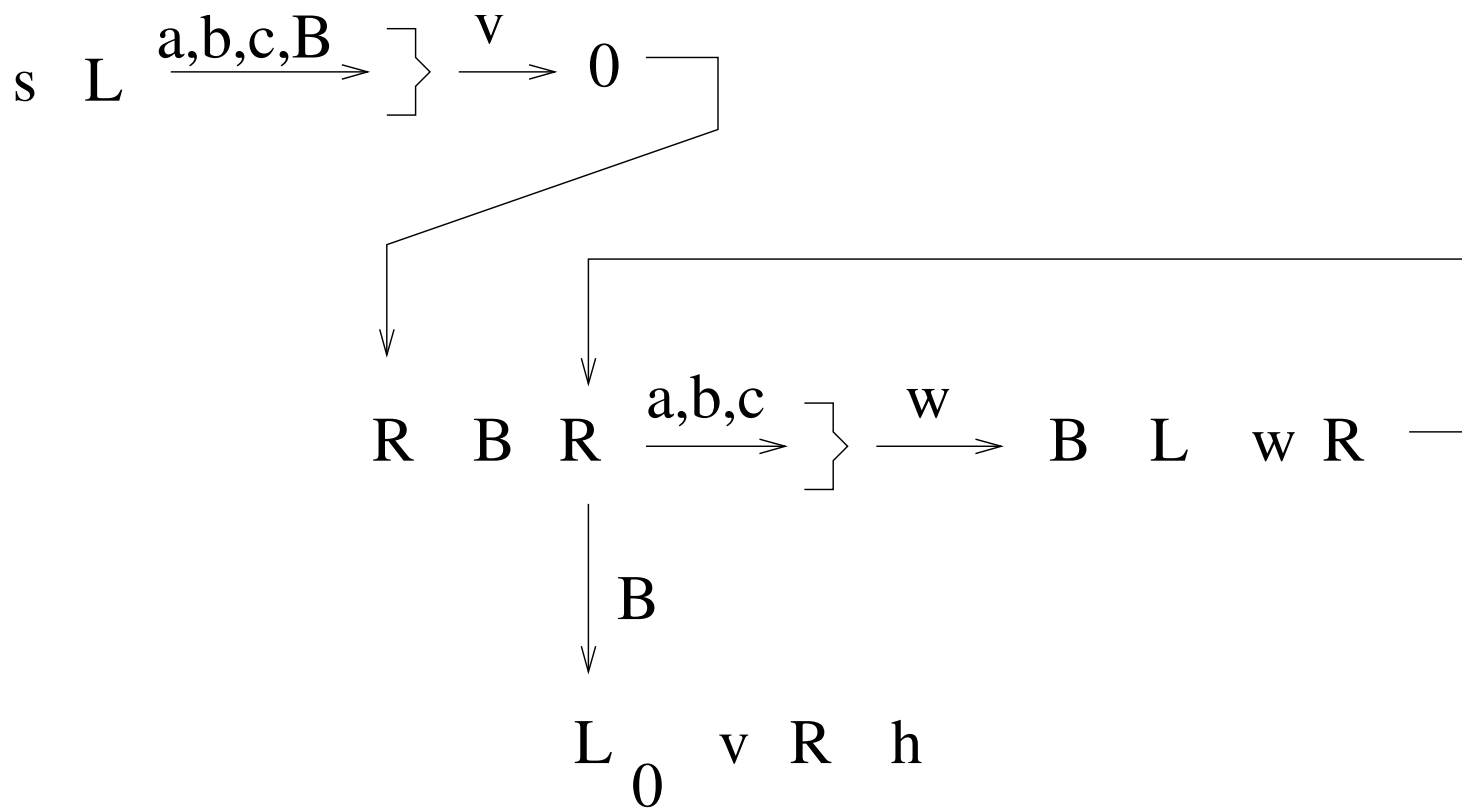
start with:            **babcaBba**

↑

end with:            **ba**ca**BBba**

↑

(similar to  $S_R$ )



**Example: Add unary numbers**

**This time use shift.**

**Example: Multiply two unary numbers,  $f(x*y)=x*y$ ,  $x$  and  $y$  unary numbers. Assume  $x,y>0$ .**

**start with:            1111\*11**  
                                  ↑

**end with:                11111111**  
                                  ↑