Announcements:

- **This is a math course with applications. Prereq: Compsci 230 or equivalent, CompSci 201.**
- **Course web page:**

  www.cs.duke.edu/courses/

  spring22/compsci334

  Familiarize yourself with all parts of the web page.
- **Read Chapter 1 in the Linz book for next time.**
- **Complete the reading quiz on Sakai.**
- **Course bulletin board: Ed Discussion.**

What will we do in Compsci 334?

Questions

- **Can you write a program to determine if a string is an integer?**

  9998.89 8abab 789342

- **Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)**

- **Can you write a program to determine if the following are correct arithmetic expressions?**

$$((34 + 7 * (18/6)))$$

$$(((((((a + b) + c) * d(e + f)))))$$

- **Can you do this if your machine had no additional memory other than the program?**

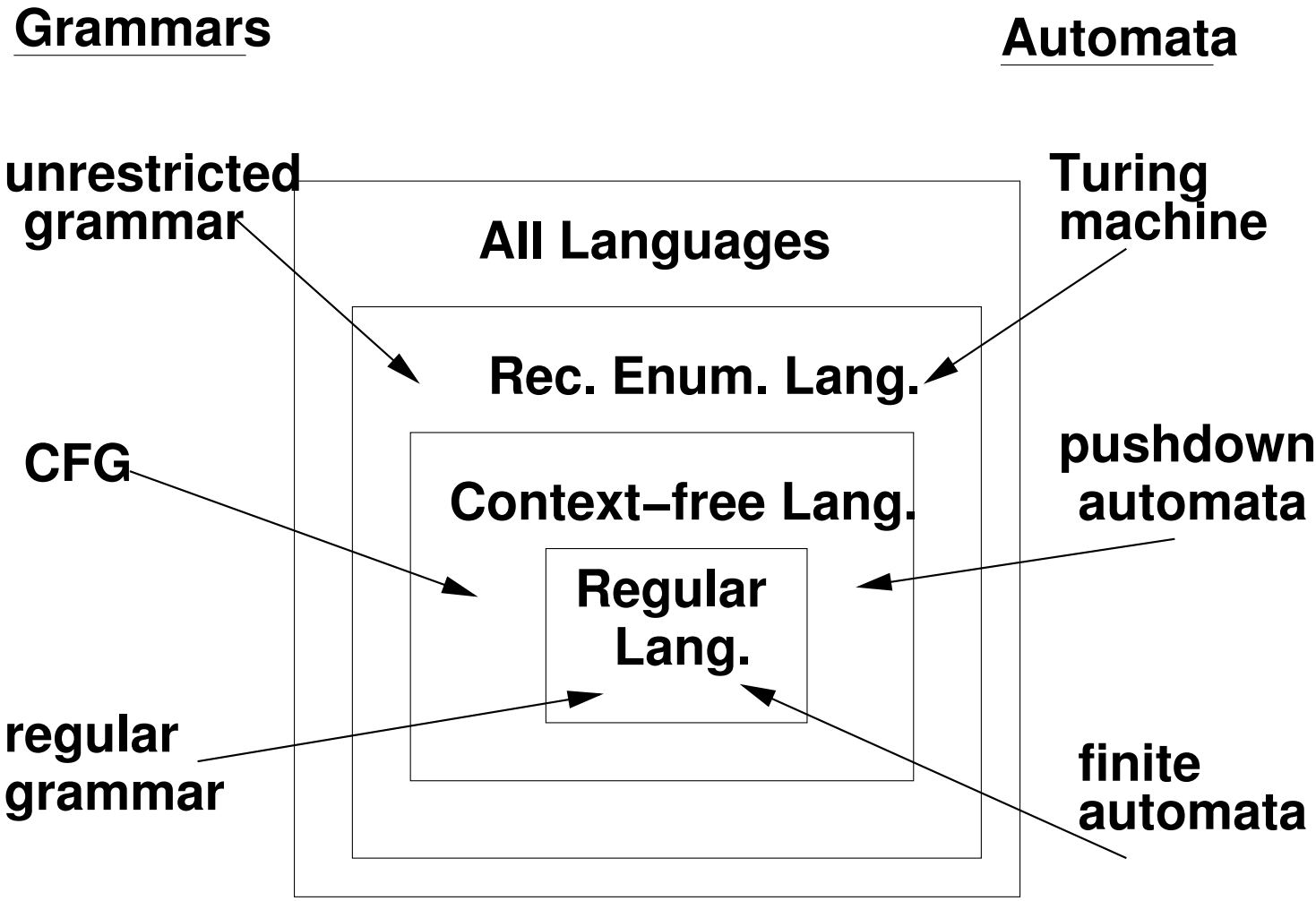
- **Can you write a program to determine the value of the following expression?**

$$((34 + 7 * (18/6)))$$

- **Can you write a program to determine if a file is a valid Java program?**

- **Can you write a program to determine if a Java program given as input will ever halt?**

Language Hierarchy



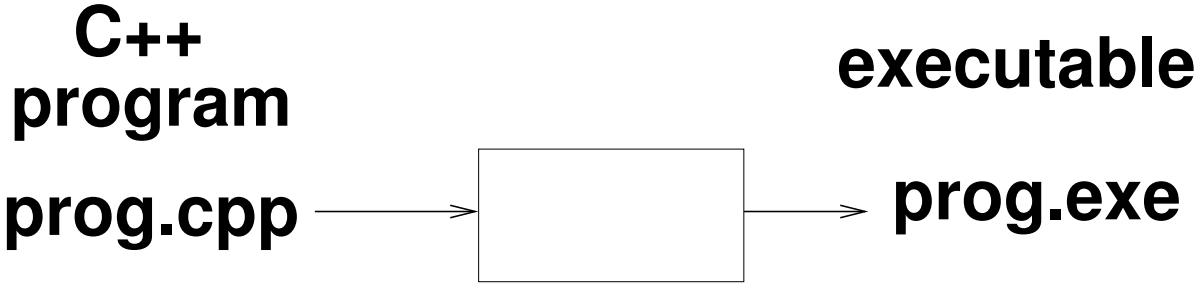
Power of Machines

automata	Can do?	Can't do?
FA (no memory)	**integers**	**arith expr**
PDA (stack)	**arith expr**	**compute expr**
TM (infinite)	**compute expr**	**decide if halts**

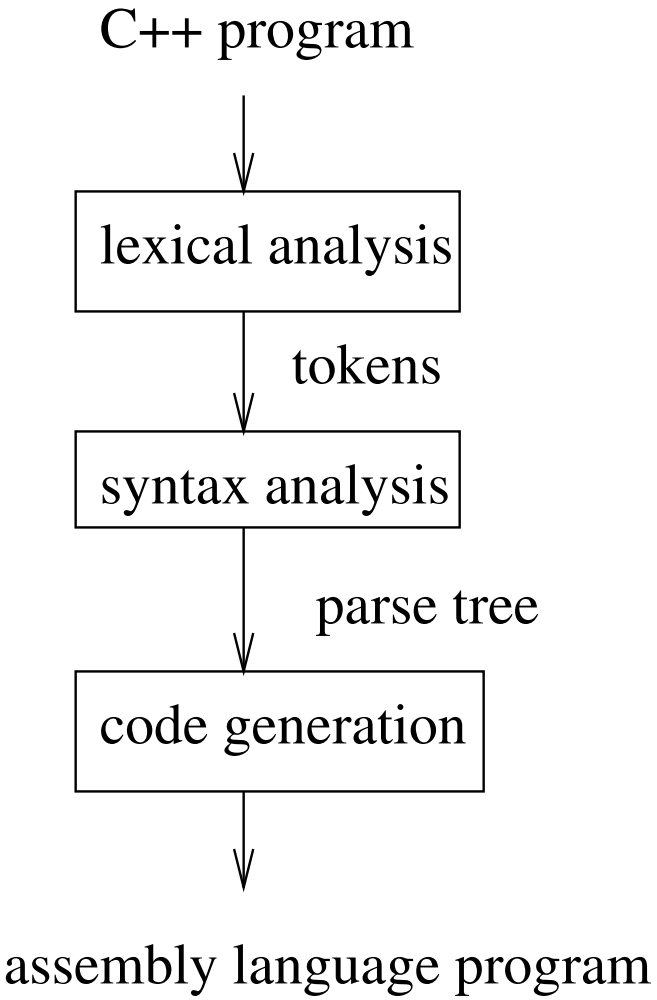
Application

Compiler

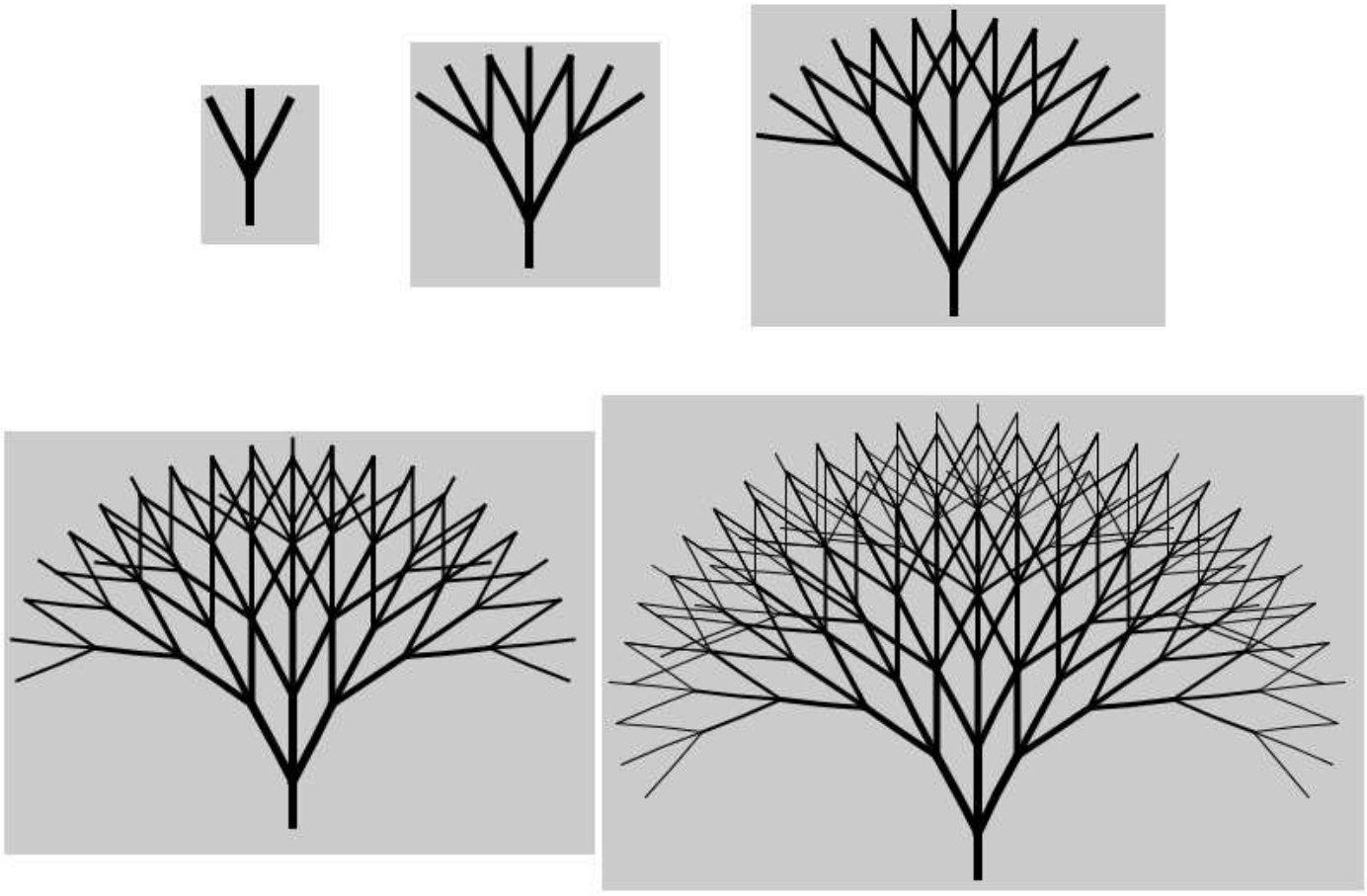
- Our focus - Question: Given a program in some language (say Java or C++) - is it valid?
- Question: language L, program P - is P valid?
- Other things to consider, how is the compilation process different for different programming languages? (Java vs C++?)



Stages of a Compiler



L-Systems - Model the Growth of Plants



Chapter 1 - Set Theory

A Set is a collection of elements.

A={1,4,6,8}, B={2,4,8},

C={3,6,9,12,...}, D={4,8,12,16,...}

- **(union) A∪B=**
- **(intersection) A∩B=**
- **C∩D=**
- **(member of) 42 ∈ C?**
- **(subset) B⊂C?**
- **B∩A ⊆D?**
- **|B|=**
- **(product) A×B=**
- **|A×B|=**
- **∅ ∈ B∩C?**
- **(powerset) 2^B=**

Example What are all the subsets of $\{3, 5\}$?

How many subsets does a set S have?

$\lvert S \rvert$	number of subsets
0	
1	
2	
3	
4	

How do you prove? Set S has $2^{|S|}$ subsets.

Technique: Proof by Induction

1. **Basis: P(1)?**
2. **I.H.**
   Assume P(n) is true for 1,2,...,n
3. **I.S.**
   Show P(n+1) is true (using I.H.)

Proof of Example:

1. Basis:

2. I.H. Assume

3. I.S. Show

Ch. 1: 3 Major Concepts

- **languages**
- **grammars**
- **automata**

Languages

- **Σ - set of symbols, alphabet**
- **string - finite sequence of symbols**
- **language - set of strings defined over Σ**

alphabet Σ

Examples

- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  L=$\{0, 1, 2, ..., 12, 13, 14, ...\}$
- Σ=$\{a, b, c\}$
  L=$\{ab, ac, cabb\}$
- Σ=$\{a, b\}$
  L=$\{a^n b^n \mid n > 0\}$

Notation

- **symbols in alphabet: a, b, c, d, ...**
- **string names: u,v,w,...**

Definition of concatenation

Let w=$a_1 a_2 \ldots a_n$ **and v=**$b_1 b_2 \ldots b_m$

Then $w \circ v$ **OR wv=**

String Operations

strings: w=abbc, v=ab, u=c

- **size of string**
  $|w| + |v| =$
- **concatenation**
  $v^3 =$ **vvv** = **v**$\circ$**v**$\circ$**v** =
- $v^0 =$
- $w^R =$
- $|vv^Rw|=$
- **ab** $\circ$ $\lambda=$

Definition

Σ^* **concatenate 0 or more**

Example

$\Sigma = \{a, b\}$

$\Sigma^* =$

$\Sigma^+ =$

Examples

$\Sigma = \{a, b, c\}$, L_1=$\{ab, bc, aba\}$,
$L_2 = \{c, bc, bcc\}$

- $L_1 \cup L_2 =$
- $L_1 \cap L_2 =$
- $\overline{L_1} =$
- $\overline{L_1 \cap L_2} =$
- $L_1 \circ L_2 = \{xy \mid x \in L_1 \textbf{ and } y \in L_2\} =$

Definition

$L^0 = \{\lambda\}$

$L^2 = L \circ L$

$L^3 = L \circ L \circ L$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots$

$L^+ = L^1 \cup L^2 \cup L^3 \ldots$

Grammars

Grammar for english

<sentence> →
<subject><verb><d.o.>

<subject> → <noun> |
<article><noun>

<verb> → hit | ran | ate

<d.o.> → <article><noun> | <noun>

<noun> → Fritz | ball

<article> → the | an | a

Examples (derive a sentence)

Fritz hit the ball.

```
<sentence> -> <subject><verb><d.o>
           -> <noun><verb><d.o>
           -> Fritz <verb><d.o.>
           -> Fritz hit <d.o.>
           -> Fritz hit <article><noun>
           -> Fritz hit the <noun>
           -> Fritz hit the ball
```

Can we also derive the sentences?

`The ball hit Fritz.`

`The ball ate the ball`

Syntactically correct?

Semantically correct?

Grammar

G=(V,T,S,P) where

- **V - variables (or nonterminals)**
- **T - terminals**
- **S - start variable (S$\in$V)**
- **P - productions (rules)**
  x$\rightarrow$y x$\in$(V$\cup$ T)$^+$, y$\in$(V$\cup$T)*

Definition

w $\Rightarrow$ z **w derives z**
w $\stackrel{*}{\Rightarrow}$ z **derives in 0 or more steps**
w $\stackrel{+}{\Rightarrow}$ z **derives in 1 or more steps**

Definition

G=(V,T,S,P)

L(G)={w$\in T^*$ | S $\stackrel{*}{\Rightarrow}$ w}

Example

G=({S}, {a,b}, S, P)

P={S→ aaS, S→ b}

L(G)=

Example

L(G) = $\{a^n ccb^n \mid n > 0\}$

G =

Example

G=({S}, {a,b}, S, P)

P={S→ aSb, S→ SS, S→ ab}

Which of these strings $aabb, abab, abba, babab$ **can be generated by this grammar? Show the derivations.**

L(G) =

Automata

Abstract model of a digital computer