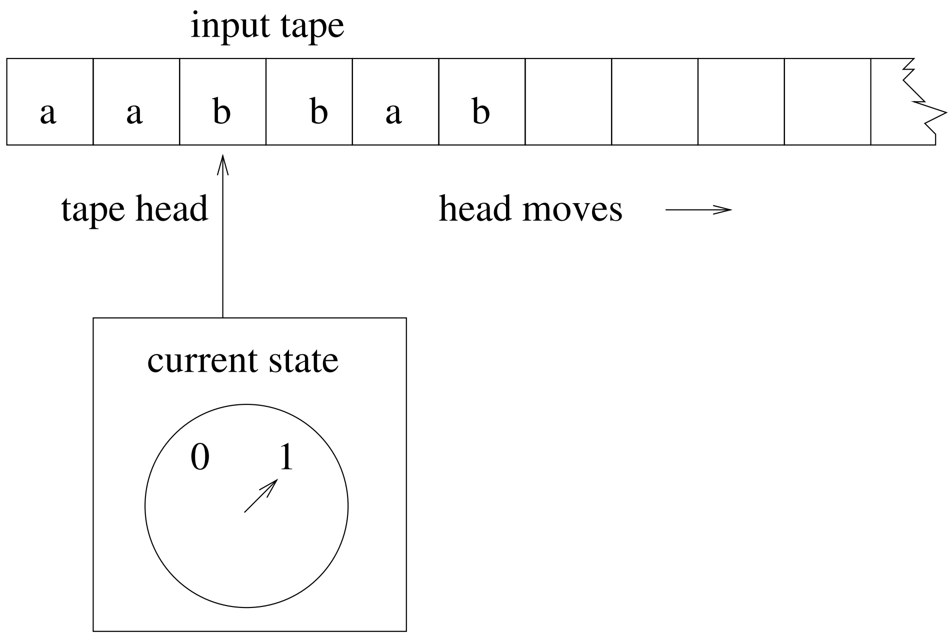
Section: Finite Automata

Deterministic Finite Accepter (or Automata)

A DFA=(Q,Σ,δ,q_0,F)



where

Q is finite set of states
Σ is tape (input) alphabet
q_0 is initial state
F $\subseteq$ Q is set of final states.
δ:Q$\times\Sigma\rightarrow$Q

Example: DFA that accepts even binary numbers.

Transition Diagram:

M=(Q,Σ**,**δ**,**q_0**,F) =**

Tabular Format

	0	1
q0		
q1		

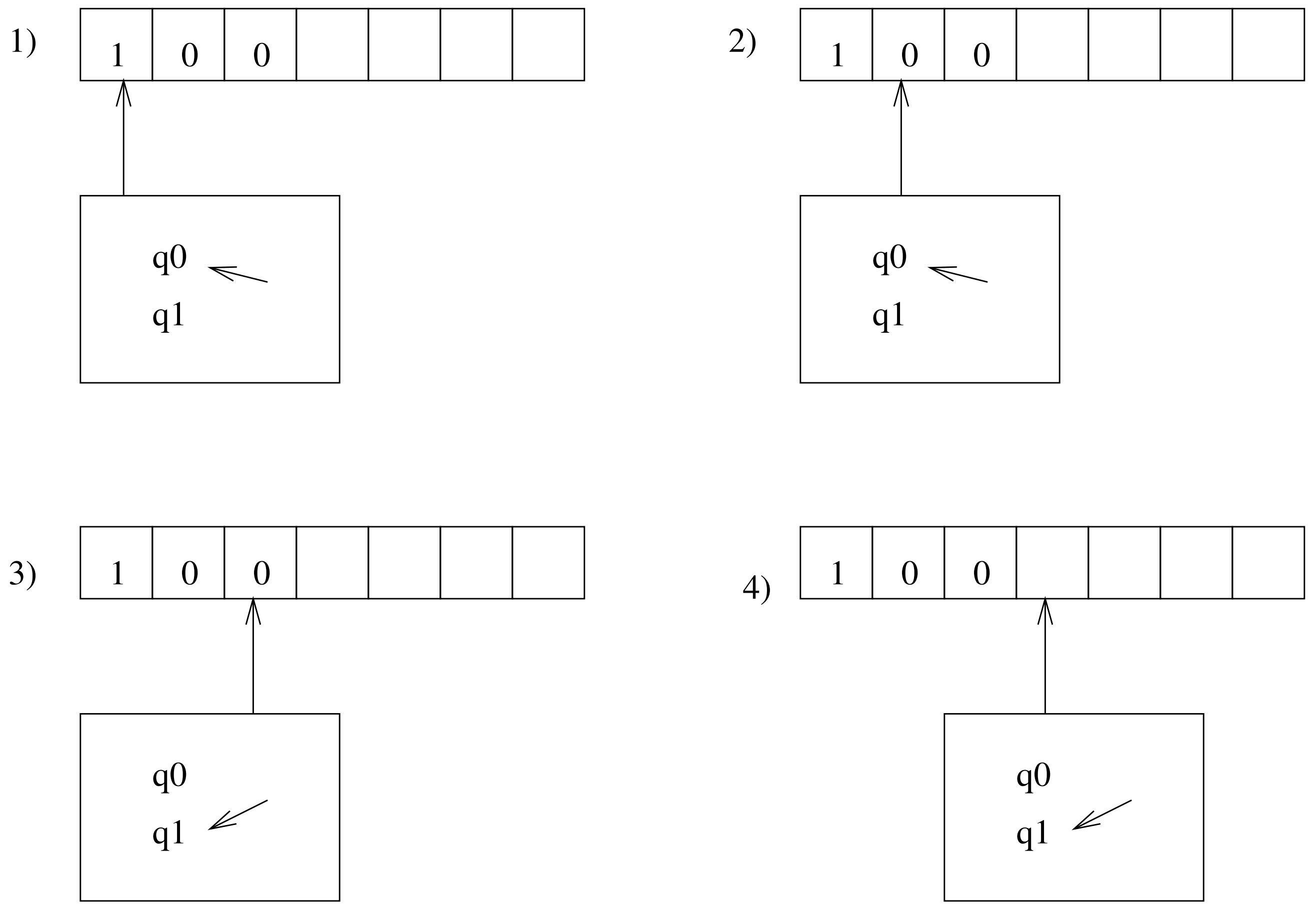
Example of a move: δ**(q0,1)=**

Algorithm for DFA:

> **Start in start state with input on tape**
> **q = current state**
> **s = current symbol on tape**
> **while (s != blank) do**
>   **q = δ(q,s)**
>   **s = next symbol to the right on tape**
> **if q$\in$F then accept**

Example of a trace: 11010

Pictorial Example of a trace for 100:



Definition:

$$\delta^*(q, \lambda) = q$$

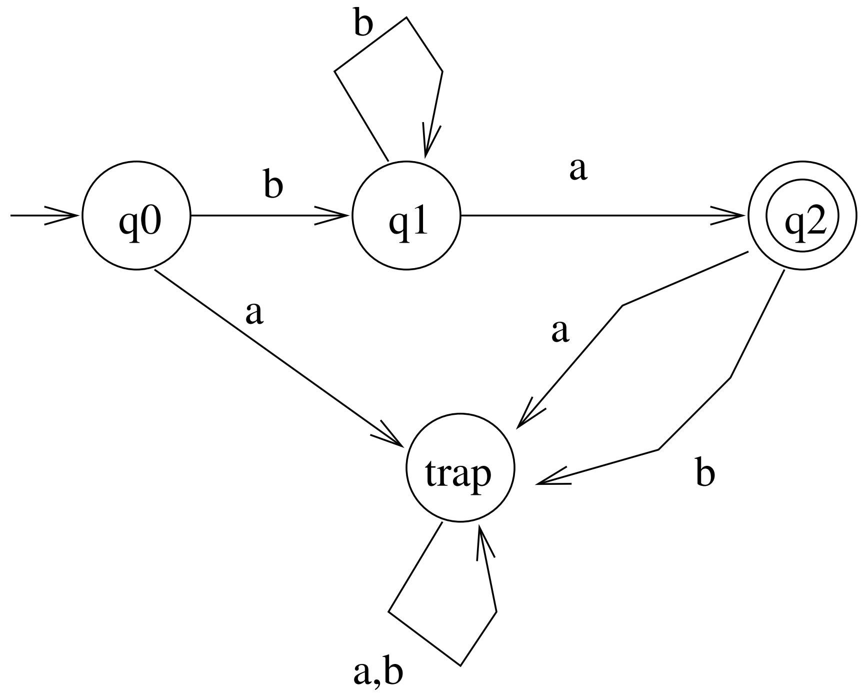
$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$

Definition The language accepted by a DFA M=(Q,Σ,δ,q_0,F) is set of all strings on Σ accepted by M. Formally,

L(M)=$\{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$

Trap State

Example: L(M) =



Example:

L $=\{w \in \Sigma^* \mid$ **w has an even number of a's and an even number of b's**$\}$

Example: DFA that accepts even binary numbers that have an even number of 1's.

Definition A language is regular iff there exists DFA M s.t. L=L(M).

Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

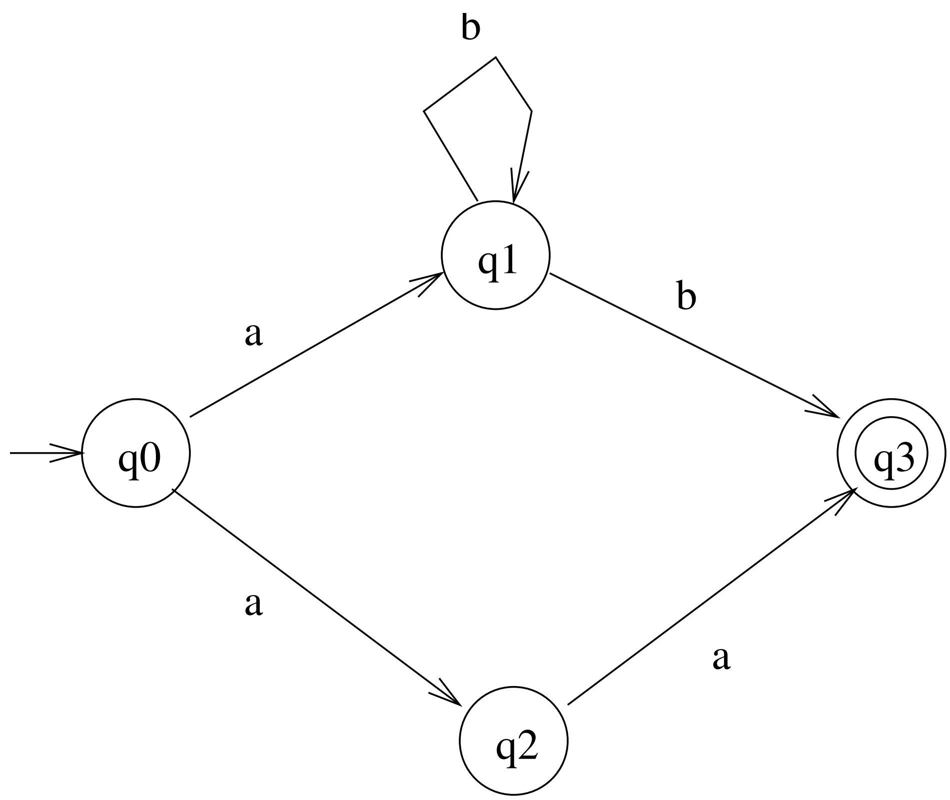
Definition

An NFA=(Q,Σ,δ,q_0,F)

where

Q is finite set of states
Σ **is tape (input) alphabet**
q_0 **is initial state**
F $\subseteq$ Q is set of final states.
δ**:Q**$\times(\Sigma \cup \{\lambda\}) \rightarrow 2^Q$

Example



Note: In this example $\delta(q_0, a) =$

L=

Example

L=$\{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$

Definition $q_j \in \delta^*(q_i, w)$ **if and only if there is a walk from** q_i **to** q_j **labeled** w**.**

Example From previous example:

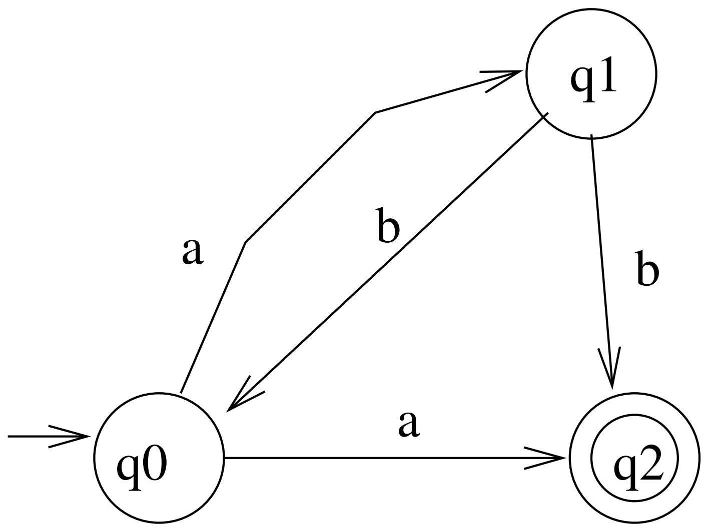
$\delta^*(q_0, ab)=$

$\delta^*(q_0, aba)=$

Definition: For an NFA M,
L(M)=$\{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$

2.3 NFA vs. DFA: Which is more powerful?

Example:



Theorem Given an NFA M_N=$(Q_N, \Sigma, \delta_N, q_0, F_N)$, then there exists a DFA M_D=$(Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(M_N) = L(M_D)$.

Proof:

We need to define M_D based on M_N.

$Q_D =$

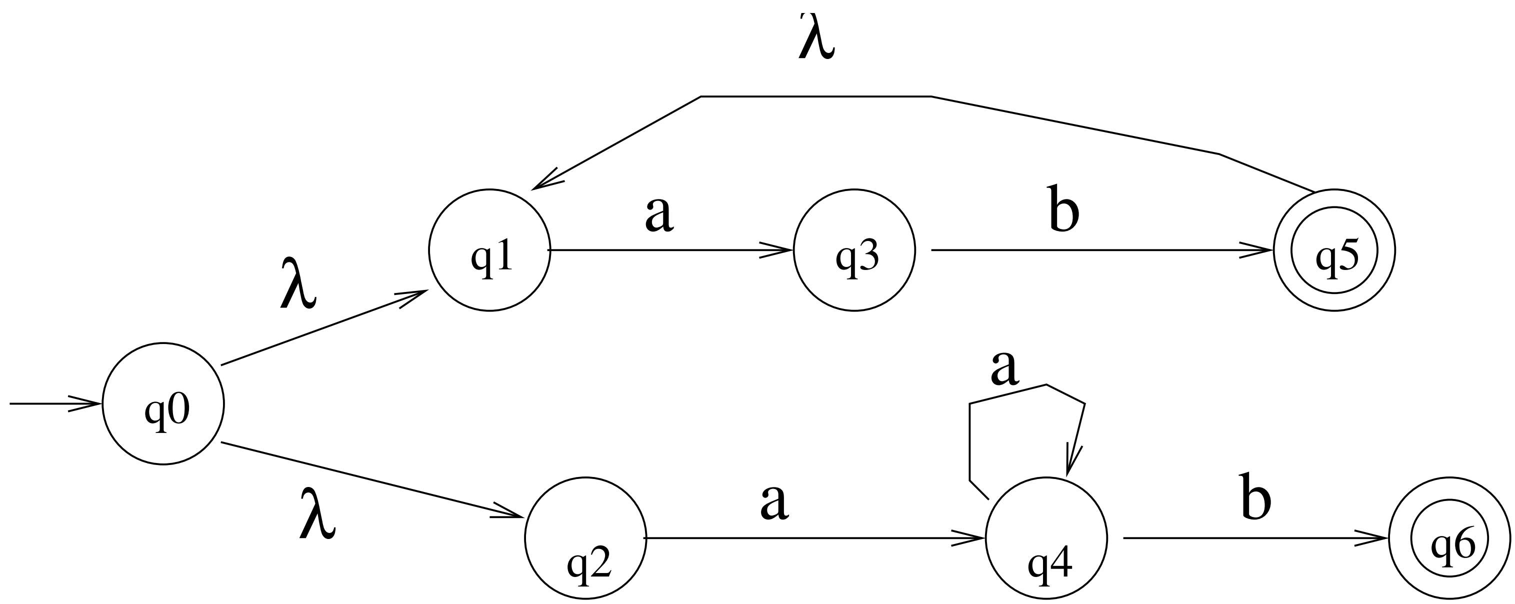
$F_D =$

$\delta_D :$

Algorithm to construct M_D

1. **start state is** $\{q_0\} \cup$ **closure(**q_0**)**
2. **While can add an edge**
   (a) **Choose a state A=**$\{q_i, q_j, ...q_k\}$ **with missing edge for** $a \in \Sigma$
   (b) **Compute B =** $\delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   (c) **Add state B if it doesn't exist**
   (d) **add edge from A to B with label a**
3. **Identify final states**
4. **if** $\lambda \in L(M_N)$ **then make the start state final.**

Example:



Properties and Proving - Problem 1

Consider the property Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with a b. If a string does not have an a, then the string is not in R1awb(L).

Example 1: Consider L=$\{aaab, bbaa\}$

R1awb(L)=

Example 2: Consider $\Sigma = \{a, b\}$**, L =** $\{w \in \Sigma^* \mid$ **w has an even number of a's and an even number of b's**$\}$

R1awb(L)=

Proof:

Properties and Proving - Problem 2

Consider the property Truncate_all_preceeding_b's or TruncPreb for short. If L is a regular, prove TruncPreb(L) is regular.

The property TruncPreb applied to a language L removes all preceeding b's in each string. If a string does not have an preceeding b, then the string is the same in TruncPreb(L).

Example 1: Consider L=$\{aaab, bbaa\}$

TruncPreb(L)=

Example 2: Consider L = $\{(bba)^n \mid n > 0\}$

TruncPreb(L)=

Proof:

Minimizing Number of states in DFA

Why?

Algorithm

- **Identify states that are indistinguishable**

  These states form a new state

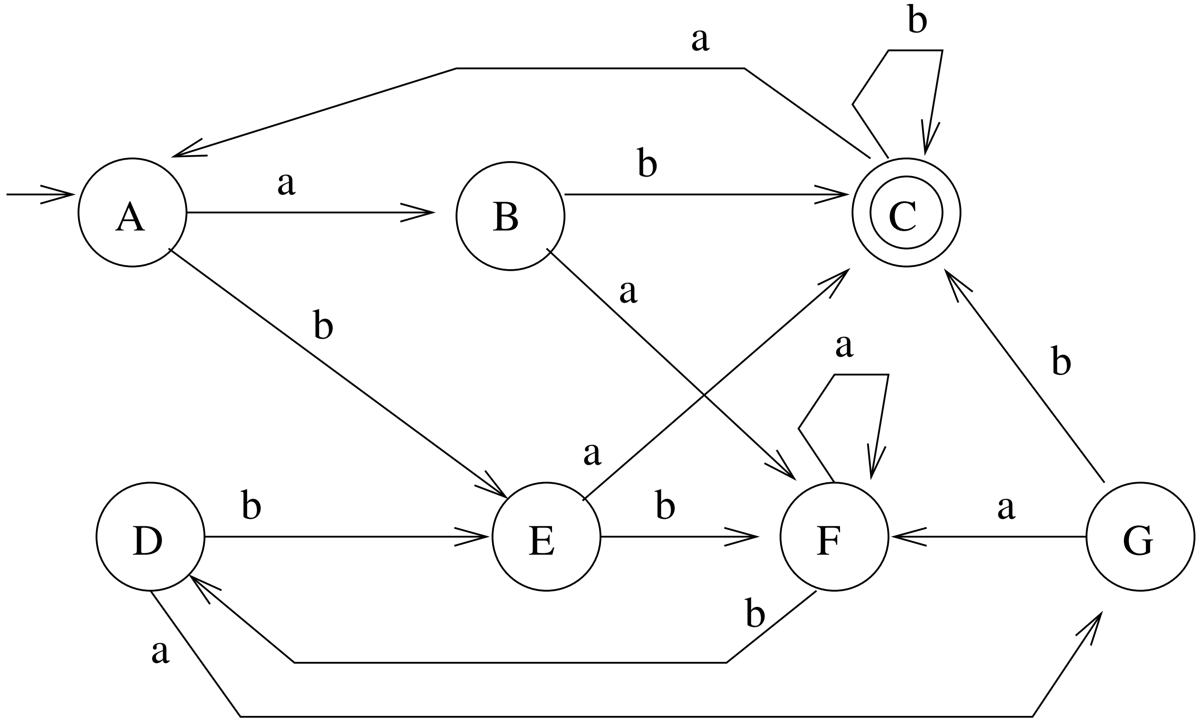
Definition Two states p and q are indistinquishable if for all $w \in \Sigma^*$

$$\begin{array}{c} \delta^*(q,w) \in F \Rightarrow \delta^*(p,w) \in F \\ \delta^*(p,w) \notin F \Rightarrow \delta^*(q,w) \notin F \end{array}$$

Definition Two states p and q are distinquishable if $\exists\ w \in \Sigma^*$ **s.t.**

$\delta^*(q,w) \in F \Rightarrow \delta^*(p,w) \notin F$ **OR**
$\delta^*(q,w) \notin F \Rightarrow \delta^*(p,w) \in F$

Example:



Example: