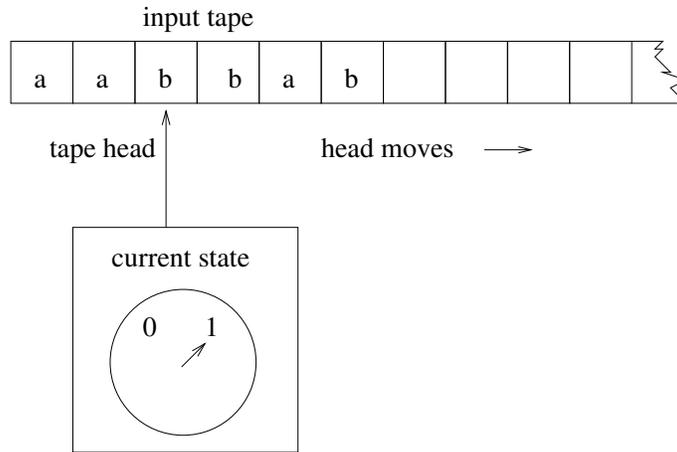
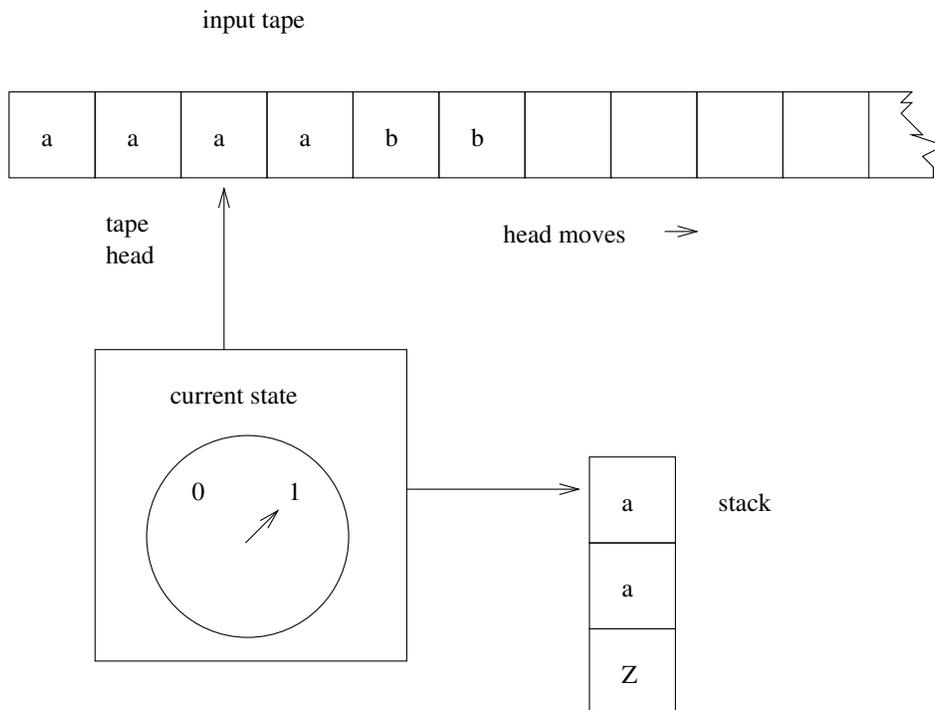


Ch. 7 - Pushdown Automata

A DFA = $(Q, \Sigma, \delta, q_0, F)$



Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).



Definition: Nondeterministic PDA (NPDA) is defined by

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

where

Q is finite set of states

Σ is tape (input) alphabet

Γ is stack alphabet

q_0 is initial state

z - start stack symbol, (bottom of stack marker), $z \in \Gamma$

$F \subseteq Q$ is set of final states.

$\delta: Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow$ finite subsets of $Q \times \Gamma^*$

Example of transitions

$$\delta(q_1, a, b) = \{(q_3, b), (q_4, ab), (q_6, \lambda)\}$$

Meaning: If in state q_1 with “a” the current tape symbol and “b” the symbol on top of the stack, then pop “b”, and either

move to q_3 and push “b” on stack

move to q_4 and push “ab” on stack (“a” on top)

move to q_6

Transitions can be represented using a transition diagram.

The diagram for the above transitions is:

Each arc is labeled by a triple: $x,y;z$ where x is the current input symbol, y is the top of stack symbol which is popped from the stack, and z is a string that is pushed onto the stack.

Instantaneous Description:

$$(q, w, u)$$

Notation to describe the current state of the machine (q), unread portion of the input string (w), and the current contents of the stack (u).

Description of a Move:

$$(q_1, aw, bx) \vdash (q_2, w, yx) \\ \text{iff}$$

Definition Let $M=(Q, \Sigma, \Gamma, \delta, q_0, z, F)$ be a NPDA. $L(M)=\{w \in \Sigma^* \mid (q_0, w, z) \vdash^* (p, \lambda, u), p \in F, u \in \Gamma^*\}$. The NPDA accepts all strings that start in q_0 and end in a final state.

Example: $L=\{a^n b^n \mid n \geq 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$

Another Definition for Language Acceptance

NPDA M accepts $L(M)$ by empty stack:

$$L(M)=\{w \in \Sigma^* \mid (q_0, w, z) \vdash^* (p, \lambda, \lambda)\}$$

Example: $L = \{a^n b^m c^{n+m} \mid n, m > 0\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$

Examples for you to try on your own: (solutions are at the end of the handout).

- $L = \{a^n b^m \mid m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$
- $L = \{a^n b^{n+m} c^m \mid n, m > 0\}$, $\Sigma = \{a, b, c\}$,
- $L = \{a^n b^{2n} \mid n > 0\}$, $\Sigma = \{a, b\}$

Definition: A PDA $M=(Q,\Sigma,\Gamma,\delta,q_0,z,F)$ is *deterministic* if for every $q \in Q$, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

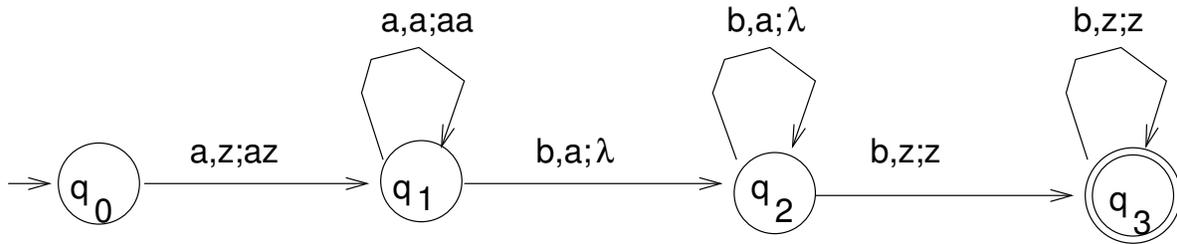
1. $\delta(q, a, b)$ contains at most 1 element
2. if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b)=\emptyset$ for all $c \in \Sigma$

Definition: L is DCFL iff \exists DPDA M s.t. $L=L(M)$.

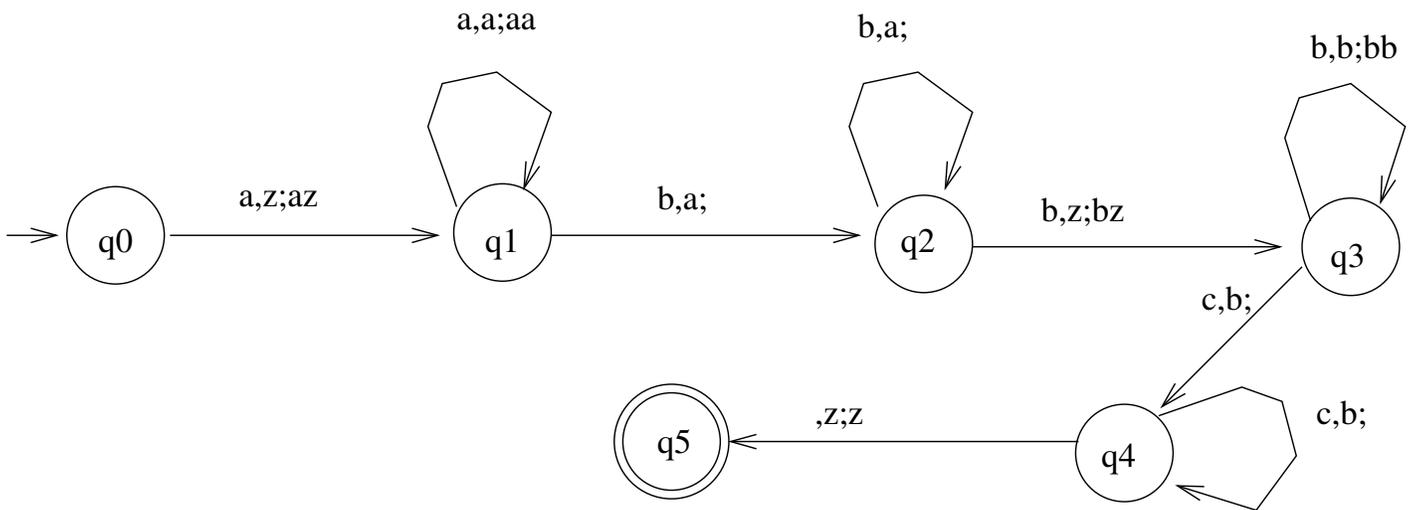
Examples:

1. Previous pda for $\{a^n b^n | n \geq 0\}$ is deterministic?
2. Previous pda for $\{a^n b^m c^{n+m} | n, m > 0\}$ is deterministic?
3. Previous pda for $\{ww^R | w \in \Sigma^+\}, \Sigma = \{a, b\}$ is deterministic?

Example: $L = \{a^n b^m \mid m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$



Example: $L = \{a^n b^{n+m} c^m \mid n, m > 0\}$, $\Sigma = \{a, b, c\}$,



Example: $L = \{a^n b^{2n} \mid n > 0\}$, $\Sigma = \{a, b\}$

