

Review

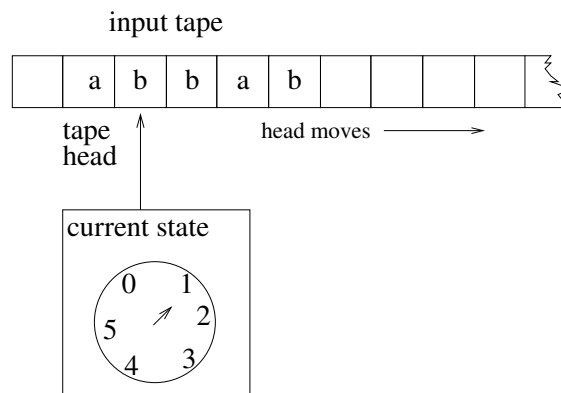
Regular Languages

- FA, RG, RE
- recognize

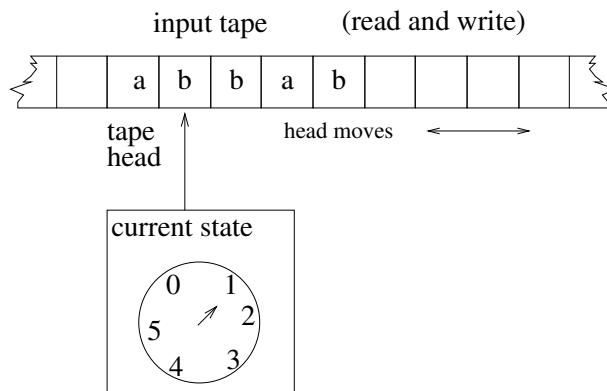
Context Free Languages

- PDA, CFG
- recognize

DFA:



Turing Machine:



Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
 - tape
- actions
 - write symbol
 - read symbol
 - move left (L) or right (R)
- computation
 - initial configuration
 - * start state
 - * tape head on leftmost tape square
 - * input string followed by blanks
 - processing computation
 - * move tape head left or right
 - * read from and write to tape
 - computation halts
 - * final state

Formal Definition of TM

A TM M is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- Q is finite set of states
 - Σ is input alphabet
 - Γ is tape alphabet
 - $B \in \Gamma$ is blank
 - q_0 is start state
 - F is set of final states
 - δ is transition function
- $\delta(q,a) = (p,b,R)$ means “if in state q with the tape head pointing to an ‘ a ’, then move into state p , write a ‘ b ’ on the tape and move to the right”.

TM as Language recognizer

Definition: Configuration is denoted by \vdash .

if $\delta(q,a) = (p,b,R)$ then a move is denoted

abaqabba \vdash ababpbba

Definition: Let M be a TM, $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) = \{w \in \Sigma^* | q_0 w \vdash^* x_1 q_f x_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^*\}$

TM as language acceptor

M is a TM, w is in Σ^* ,

- if $w \in L(M)$ then M halts in final state
- if $w \notin L(M)$ then either
 - M halts in non-final state
 - M doesn't halt

Example

$\Sigma = \{a, b\}$

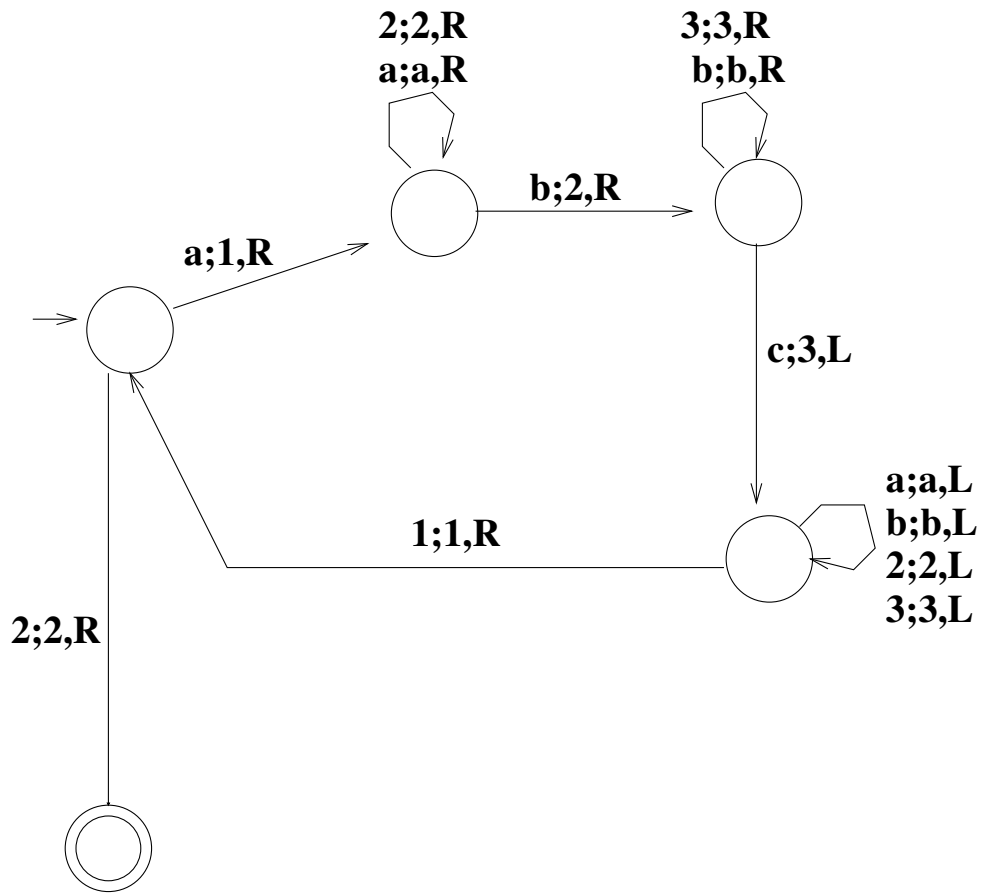
Replace every second 'a' by a 'b' if string is even length.

- Algorithm

Example:

$$L = \{a^n b^n c^n \mid n \geq 1\}$$

Is the following TM Correct?



TM as a transducer

TM can implement a function: $f(w) = w'$

start with:	w
	↑
end with:	w'
	↑

Definition: A function with domain D is *Turing-computable* or *computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0 w \vdash^* q_f f(w)$$

$q_f \in F$, for all $w \in D$.

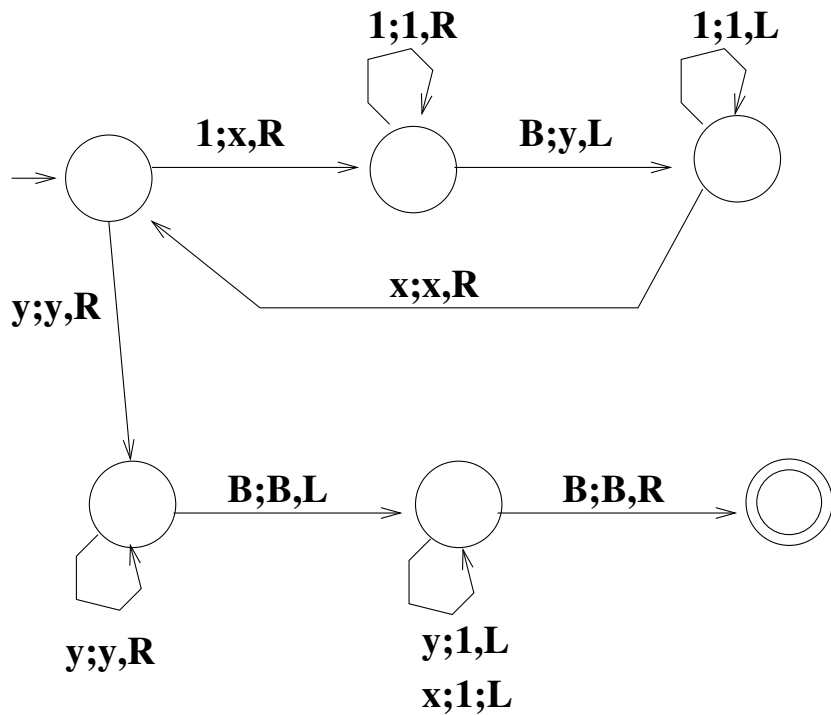
Example:

$$f(x) = 2x$$

x is a unary number

start with: 111
 \uparrow
 end with: 111111
 \uparrow

Is the following TM correct?



Example:

$$L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$$