

Section: Turing Machines

Review

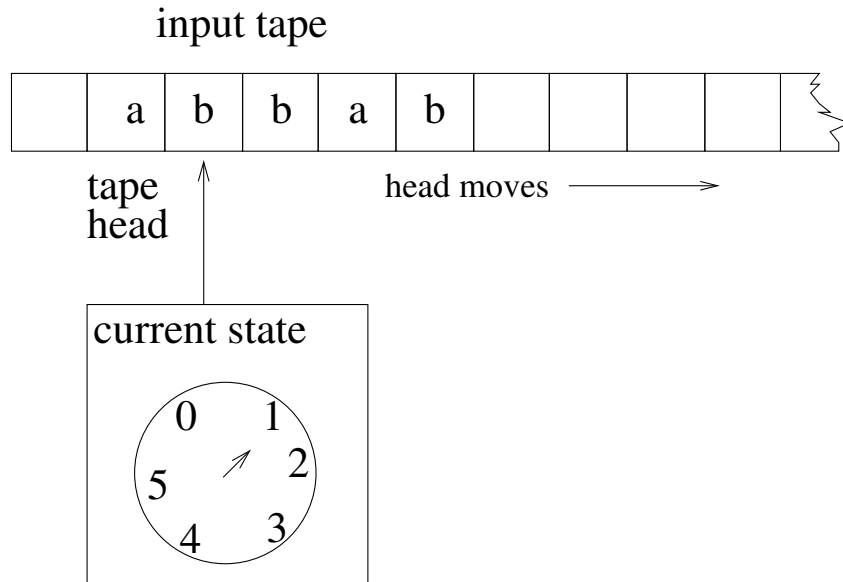
Regular Languages

- FA, RG, RE
- recognize

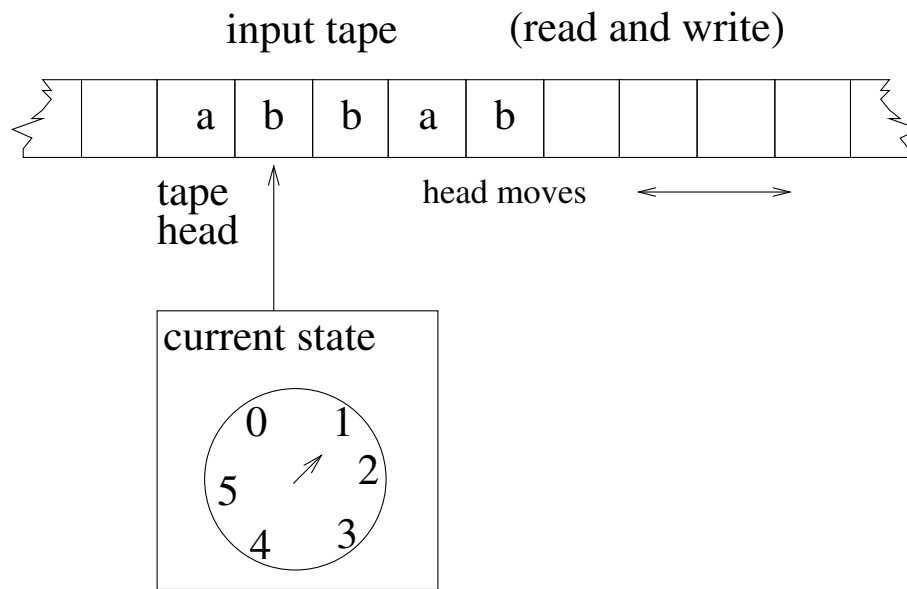
Context Free Languages

- PDA, CFG
- recognize

DFA:



Turing Machine:



Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
 - tape
- actions
 - write symbol
 - read symbol
 - move left (L) or right (R)

- computation
 - initial configuration
 - * start state
 - * tape head on leftmost tape square
 - * input string followed by blanks
 - processing computation
 - * move tape head left or right
 - * read from and write to tape
 - computation halts
 - * final state

Formal Definition of TM

A TM M is defined by
 $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

- Q is finite set of states
- Σ is input alphabet
- Γ is tape alphabet
- $B \in \Gamma$ is blank
- q_0 is start state
- F is set of final states
- δ is transition function
 $\delta(q, a) = (p, b, R)$ means

TM as Language recognizer

Definition: Configuration is denoted by \vdash .

if $\delta(q,a) = (p,b,R)$ then a move is denoted

abaqabba \vdash ababpbba

Definition: Let M be a TM,
 $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) =$
 $\{w \in \Sigma^* \mid q_0 w \overset{*}{\vdash} x_1 q_f x_2 \text{ for some } q_f \in F,$
 $x_1, x_2 \in \Gamma^*\}$

TM as language acceptor

M is a TM, w is in Σ^* ,

- if $w \in L(M)$ then M halts in final state
- if $w \notin L(M)$ then either
 - M halts in non-final state
 - M doesn't halt

Example

$$\Sigma = \{a, b\}$$

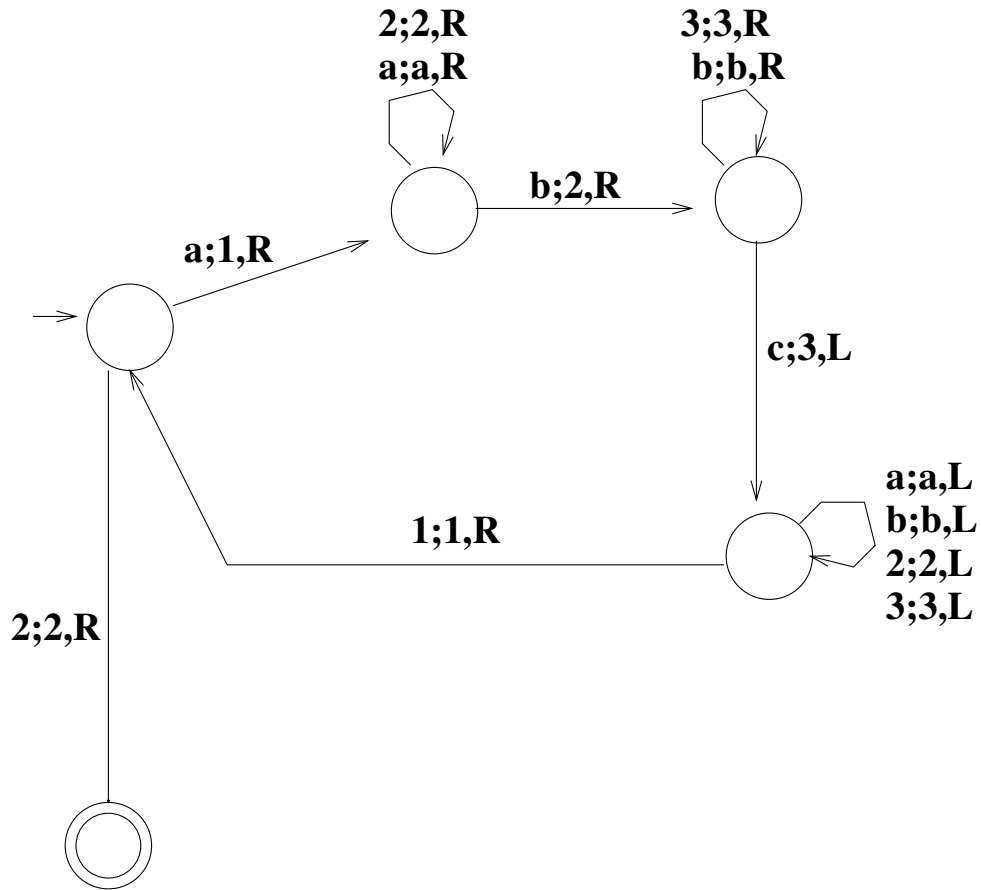
Replace every second 'a' by a 'b' if string is even length.

- Algorithm

Example:

$$L = \{a^n b^n c^n \mid n \geq 1\}$$

Is the following TM Correct?



TM as a transducer

TM can implement a function:

$$f(w) = w'$$

start with: w
 \uparrow

end with: w'
 \uparrow

Definition: A function with domain D is *Turing-computable* or *computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0w \stackrel{*}{\vdash} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

Example:

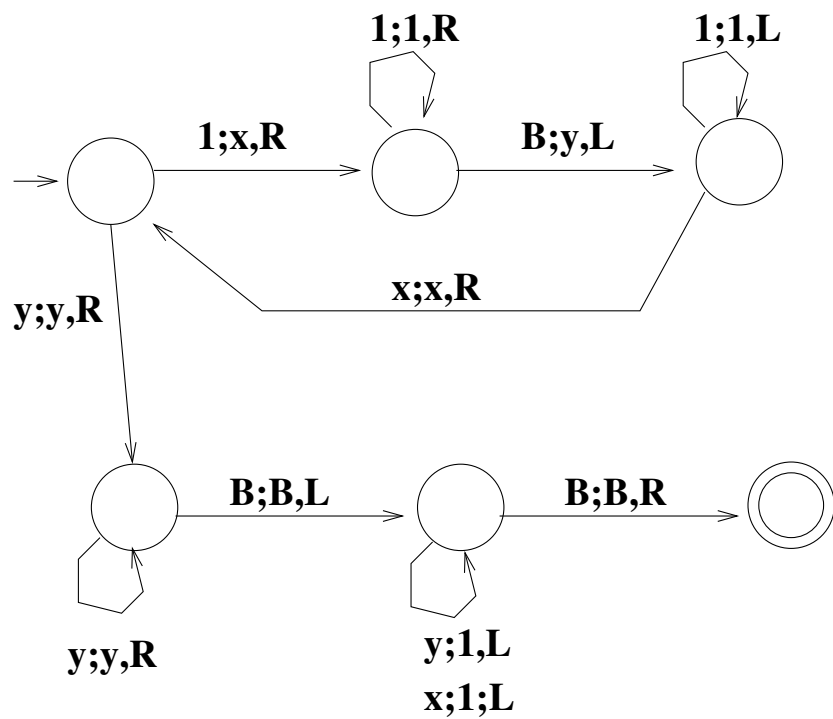
$$f(x) = 2x$$

x is a unary number

start with: 111
 ↑

end with: 111111
 ↑

Is the following TM correct?



Example:

$$\mathbf{L} = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$$