Due date: February 3, 2022

Problem 1: [10 pts] Let $\mathcal{R}$ be a set of n (possibly intersecting) orthogonal rectangles in $\mathbb{R}^2$. Describe an $O(n \log n)$ time algorithm to compute the area of the union of rectangles in $\mathcal{R}$.

Problem 2: [15 pts] Let $\mathcal{I}$ be a set of intervals in $\mathbb{R}^1$. Given an arbitrary interval $\delta \subset \mathbb{R}^1$, an interval $I \in \mathcal{I}$ intersecting δ is called *short* (resp. *long*) if δ contains at least one endpoint of I (resp. $\delta \subset I$). Show that $\mathbb{R}^1$ can be partitioned, in $O(n \log n)$ time, into a family $\Delta = \{\delta_1, \ldots, \delta_m\}$ of intervals so that for each $i \leq m$, $||L_i| - |S_i|| \leq 1$, where L_i (resp. S_i) is the subset of intervals of $\mathcal{I}$ that are long (resp. short) in δ_i. Use this partition to build a linear-size data structure that returns the subset of all k intervals of $\mathcal{I}$ containing a query point $x \in \mathbb{R}^1$ in $O(\log n + k)$ time. What is the preprocessing time of your data structure?

Problem 3: [15 pts]

(i) A kd-tree can also be used when querying with other ranges than rectangles. Show that the query time for range queries with triangles is linear in the worst case, even if no answers are reported at all.

(ii) Suppose that a data structure is needed that can answer triangular range queries, but only for triangles whose edges are horizontal, vertical, or have slope $+1$ or -1. Develop a linear-size data structure that answers such range queries in $O(n^{3/4} + k)$ time, where k is the number of points reported.

(iii) Describe an $O(n \log^2 n)$ size data structure for the above problem that answers a query in $O(\log^2 n + k)$ time. (You can use fractional cascading to shave off a $\log n$ factor from the query time.)

Problem 4: [10 pts] Let S be a set of n pairwise-disjoint segments in $\mathbb{R}^2$. Show that S can be preprocessed, in $O(n \log n)$ time, into a linear-size data structure so that all k segments of S that intersect a query vertical segment (i.e., a segment whose endpoints are (a, b_1) and (a, b_2) for some $a, b_1, b_2 \in \mathbb{R}$) can be reported in $O(\log n + k)$ time. (**Hint:** *Use a persistent data structure.*)