Due date: April 11, 2022

Problem 1: [10 pts] Let L be a set of n lines in $\mathbb{R}^2$. For each cell C in the arrangement $A(L)$, let $|C|$ denote the complexity of the cell. Prove that

$$\sum_{C \in A(L)} |C|^2 = O(n^2).$$

(**Hint:** *Use the zone theorem.*)

Problem 2: [10 pts] Let C be a convex m-gon, and let S be a set of n line segments in $\mathbb{R}^2$ whose endpoints lie on C. Give an $O(m + n \log n)$-time algorithm to count the number of intersection points of segments in S. (**Hint:** *Traverse the boundary of* C.)

Problem 3: [20 pts] Let S be a set of n line segments in $\mathbb{R}^2$. Describe an $O(n^{4/3} \log^{O(1)} n)$-time algorithm to count the number of of intersection points of segmnts in S. (**Hint:** *Construct a geometric cutting* Ξ *of lines supporting the triangles. Count the number of intersection points within each triangle* $\Delta \in \Xi$ *using the first problem and the one from the previous assignment.*)

Problem 4: [10 pts] Let S be a set of points in $\mathbb{R}^d$. Show that the randomized LP algorithm discussed in the class can be extended to compute the smallest ball containing S in $O(n)$ expected time.