Due date: April 11, 2022

Problem 1: [10 pts] Let $L$ be a set of $n$ lines in $\mathbb{R}^2$. For each cell $C$ in the arrangement $A(L)$, let $|C|$ denote the complexity of the cell. Prove that

$$\sum_{C \in A(L)} |C|^2 = O(n^2).$$

(Hint: Use the zone theorem.)

Problem 2: [10 pts] Let $C$ be a convex $m$-gon, and let $S$ be a set of $n$ line segments in $\mathbb{R}^2$ whose endpoints lie on $C$. Give an $O(m + n \log n)$-time algorithm to count the number of intersection points of segments in $S$. (Hint: Traverse the boundary of $C$.)

Problem 3: [20 pts] Let $S$ be a set of $n$ line segments in $\mathbb{R}^2$. Describe an $O(n^{4/3} \log^{O(1)} n)$-time algorithm to count the number of intersection points of segments in $S$. (Hint: Construct a geometric cutting $\Xi$ of lines supporting the triangles. Count the number of intersection points within each triangle $\Delta \in \Xi$ using the first problem and the one from the previous assignment.)

Problem 4: [10 pts] Let $S$ be a set of points in $\mathbb{R}^d$. Show that the randomized LP algorithm discussed in the class can be extended to compute the smallest ball containing $S$ in $O(n)$ expected time.